

(An ISO 3297: 2007 Certified Organization) Website: <u>www.ijirset.com</u> Vol. 6, Issue 7, July 2017

# **Applications of Wavelet Transform and Tensor Decomposition**

Priya Kothre<sup>1</sup>, R.B Raut<sup>2</sup>

P.G. Student, Department of Electronics Engineering, Ramdeobaba College of Engineering, Nagpur, India<sup>1</sup>

Associate Professor, Department of Electronics and Comm. Engineering, Ramdeobaba College of Engineering,

Nagpur, India<sup>2</sup>

**ABSTRACT:** Hyperspectral Imaging (HSI) is a class of techniques commonly referred to as spectral imaging. It provide spectral information to identify and distinguish between spectrally similar (but unique) materials. Information in Hyper spectral images may vary over band to band. Fast computers and large data storage capacities are needed for processing and analysis of hyperspectral data. The aim of this paper is to present compression and reconstruction of hyperspectral images using Discrete Wavelet Transform and Tucker Decomposition (DWT-TD). In this, DWT is used to separate each Hperspectral band to sub-images and TD on DWT coefficient to compact the energy of sub-images. Objective of hyperspectral image compression is to reduce irrelevance data in order to be able to store or transmit data in efficient form.

KEYWORDS: Hyperspectral images (HSI), compression, wavelet transform, tucker decomposition (TD).

#### I. INTRODUCTION

Hyperspectral imaging provides a unique spectral signature of every pixel, which can be used by the processing techniques and allows more spectral-spatial models for more accurate classification of images. Hyperspectral imaging measures reflection, emission and absorption of radiation also provides greater depth of information about single pixel. Hyperspectral image compression has become very active trend in remote sensing application because of its large data. Image compression focus only on spectral bands reduction. Both spectral and spatial dimensionality reduction of hyperspectral image is possible by using this method. Our work is organized in different sections. Section II Presents the previous work. Section III discusses details of proposed algorithm. Section IV briefly describes DWT and TD. Section VI displays the result carried out on hyperspectral image, finally in section VII conclusion is highlited.

#### **II. PREVIOUS WORK**

[1] Q. Du and J. E. Fowler uses JPEG200 and Principal component analysis for compression of hyperspectral images. Principal component analysis deployed in JPEG2000 in order to achieve spectral dimensionality reduction. This methods results in Rate distortion performance better than spectral DWT.

[2] C.Huo, R. Zhang, and T. Peng proposed Lossless compression method based on searching optimal multibands. The searching model is constructed using tree structure and multiband having a strong correlation to chosen bands are founds and then used for prediction. At last images are then encoded by entropy encoder. This algorithm provides better compression performace compared with other algorithm.



(An ISO 3297: 2007 Certified Organization)

#### Website: www.ijirset.com

#### Vol. 6, Issue 7, July 2017

[3] A. Karami, M. Yazdi ,A.Zolghadre, proposed a hyperspectral image compression based on tucker decomposition and three dimensional discrete cosine transform and applies Tucker decomposition on DCT coefficient of image. It reduces the size of every dimensions of tensor and thus reduces the quantization depth and dimension.

[4] G. A. Licciardi, J. Chanussot, A. Piscini proposed a method for the compression of hyperspectral data to preserve original information. Proposed method is based on non linear generalization of standard Principal Component Analysis.for each image the compression rate depends only on nodes of bottleneck layer. Hence it is possible to increase the spectral information by operating on number of nodes. By this method, achieved compression rate is around 95%.

[5] Hrishikesh Tamhankar and James E. Fowler proposed a method for compression of hyperspectral image based on spectral-decorrelation strategy. They also uses wavelet based spatial decorrelation for compression of hyperspectral image. 3D-SPHIT algorithm is also used to code resulting transform coefficients.

[6] Lei Chen, Chao Bei, Yujia Zhai, Jing Wu used precompression algorithm for compression o hyperspectral image. In this algorithm they first find general realation between bands in accordance with high spectral correlation.based on the relationship ,Precompression algorithm is designed to segments the bands into group and remove spectral and spatial correlation of each group of images. After this 3D-DWT is applied for further compression. At low bit rate, this method produce significantly improved performance.

#### **III. PROPOSED ALGORITHM**

In this proposed method Compression of hyperspectral images is introduced using DWT and TD. 2-DWT using biorthogonal3.7 is applied on each band of hyperspectral image. After applying 2-DWT each hyperspectral band is divided into four sub-images. TD is applied to four sub-images which obtained after DWT to achieve higher compression ratio[7]. The proposed algorithm is given below in figure 1.



Fig1: Block diagram of Proposed Approach

The input image is a three dimensional hyperspectral image.First level compression is achieved by using DWT. Firstly 2DWT is applied to each hyperspectral band to obtained the four sub-images (Approximate, Horizontal, Vertical, Diagonal). Tucker decomposition is applied to four sub-images to compact the energy of sub-images. After this arithmetic encoding is applied to each band of hyperspectral image. Arithmetic coding is a form of entropy encoding used for data compression. Coding is used to transmit the elements of core tensor. AAC does not transmit codebook, hence achieve higher compression then other coding method. Inverse 2DWT is applied to reconstruct the original



(An ISO 3297: 2007 Certified Organization)

#### Website: www.ijirset.com

#### Vol. 6, Issue 7, July 2017

hyperspectral image. A program is developed in MATLAB whose objective is to find compression ratio, peak-signal-to-noise-ratio(PSNR) and mean-squared-error(MSE).

#### IV. DISCRETE WAVELET TRANSFORM (DWT) AND TENSOR DECOMPOSITION(TD)

#### A. DISCRETE WAVELET TRANSFORM(DWT)

Discrete wavelet transform has been successfully used for image processing application such as edge detection, image compression. Wavelet transform produces coefficients of image. These coefficients are then compressed more easily, because the information is focused on few coefficients. Wavelet transform has a advantage to separate the fine details in the signals. In DWT the signals which is to be analyzed is passed through the filters having different cut-off frequencies. DWT is computed as successive low pass and high pass filtering in time domain. At each level of compression, high pass filter produces the detailed information and low pass filter produces the approximate information.



Fig2: Illustrating concept of 2DWT.

We apply 2DWT on each band of hyperspectral image. If each iamge band has M row size and N column size then after applying 2DWT, we obtained four sub-images (A, H, V, D) each band having size M/2 rows and N/2 column. The approximation(A) sub-band having highest energy amongst all other three sub-bands. The 2DWT of function f(x, y) for size M×N is given by-

$$W_{\varphi}(m,n) = \frac{1}{\sqrt{MN}} \sum_{y=0}^{N-1} \sum_{x=0}^{M-1} f(x,y) \varphi_{m,n}(x,y)$$
(1)

$$W_{\psi}^{K}(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{y=0}^{N-1} \sum_{z=0}^{M-1} f(x,y) \psi_{j,m,n}^{K}(x,y)$$
(2)

$$K = \{H, V, D\}$$

 $\varphi$  is a scaling function.  $W\varphi(m,n)$  is the approximation of function f(x,y). For wavelet filtering, the biorthogonal 3.7 wavelet is used for better compression. Inverse 2DWT is given by-

$$f(x,y) = \frac{1}{\sqrt{MN}} \sum_{n} \sum_{m} W_{\varphi}(m,n)_{\varphi m,n}(x,y) + \frac{1}{\sqrt{MN}} \sum_{K} \sum_{j=0}^{J-1} \sum_{n} \sum_{m} W_{\psi}^{K}(m,n) \psi_{j,m,n}^{K}(x,y)$$
(3)

To achieve higher compression ratio we can decompose image into more levels.



(An ISO 3297: 2007 Certified Organization)

#### Website: www.ijirset.com

#### Vol. 6, Issue 7, July 2017

#### **B.TENSOR DECOMPOSITION**

The set of images that can be well represented as a 3-dimensional or multi-dimensional data array are referred to as tensor. Tensor can reduce both spectral and spatial correlation simultaneously. In this section brief review on tensor-tucker model is presented. Tucker decomposition decomposes a tensor into a unknown core tensor and its three component matrix. Figure 3 is illustrating basic concept of tensor-tucker-decomposition.



Fig3: Third order tensor-tucker decomposition

The third order tensor is given by  $\underline{\mathbf{Y}} \in \mathbf{R}^{\mathbf{M} \times \mathbf{N} \times \mathbf{C}}$  are decomposed into unknown core tensor of size  $\underline{\mathbf{G}} \in \mathbf{R}^{\mathbf{J} 1 \times \mathbf{J} 2 \times \mathbf{J} 3}$  and three unknown matrix component  $\mathbf{A}^{(n)}$ .

$$\underline{\mathbf{Y}} = \underline{\mathbf{G}} \times \{\mathbf{A}\} + \underline{\mathbf{E}}$$
<sup>(4)</sup>

$$\underline{\mathbf{Y}} = \hat{\mathbf{Y}} + \underline{\mathbf{E}} \tag{5}$$

$$\{\mathbf{A}\} = \mathbf{A}^{(1)} + \mathbf{A}^{(2)} + \mathbf{A}^{(3)}$$
(6)

# $\mathbf{A}^{(1)} \in \mathbf{R}^{M \times J1}$ , $\mathbf{A}^{(2)} \in \mathbf{R}^{N \times J2}$ , $\mathbf{A} \in \mathbf{R}^{C \times J3}$

Here the tensor  $\hat{\mathbf{Y}}$  is an approximation of tensor  $\mathbf{Y}$  and its depend on value of core tensor (J1 × J2 × J3). More about tucker decomposition is given in next section.

#### V. COMPRESSION USING DISCRETE WAVELET TRANSFORM AND TUCKER DECOMPOSITION

Compression of hyperspectral images is performed in four steps. In first step 2DWT is applied to each band of hyperspectral image in order to obtained four sub images i.e. Approximation, Horizontal, Vertical, Detailed (see fig.2). In Second step TD is applied to all four tensor. For each tensor the size of unknown core tensor  $(J1 \times J2 \times J3)$  is selected manually. Among all the tensors the approximate tensor  $(J_{1A}, J_{2A}, J_{3A})$  is having lowest frequency component and contains most of the energy, and hence set higher than other tensors. The diagoanl tensors which contains diagonal information were also set higher than vertical and horizontal tensors. In third step core tensor and matrices are transmitted. Arithmetic coding is used to transmit the core tensor and matrix components. Aithmetic coding is different than other type of coding, in a way that arithmetic coding encodes the entire message in a single number rather than separating the input into component symbol and replacing each with a code. After this applyin inverse DWT to reconstruct the original image.

#### A. DWT-TD ALGORITHM

1. Input is a hyperspectral image of size  $(M \times N \times C)$ 

2. Applying 2DWT to each band of hyperspectral image to obtained four sub-images (A,H,V,D) of M/2 rows and N/2 cloumn.



(An ISO 3297: 2007 Certified Organization)

#### Website: www.ijirset.com

#### Vol. 6, Issue 7, July 2017

3. Applying TD to four sub-images to obtained the tensor of size  $M/2 \times N/2 \times C$ . at the end of this step output will bethree component matrix  $A^{(n)}$  and core tensor of size  $\underline{G} \in \mathbf{R}^{\mathbf{J}\mathbf{1} \times \mathbf{J}\mathbf{2} \times \mathbf{J}\mathbf{3}}$ .

4. Encode core tensor and matrix component using Adaptive Arithmetic Coding.

5. Decoding the elements of  $\hat{\mathbf{Y}}$  i.e  $\mathbf{G} \times \{\mathbf{A}\}$ .

6. Computing inverse discrete wavelet transform (I2DWT) to reconstruct images.

#### VI. EXPERIMENTAL RESULT

Compression ratio of image is defined as a ratio of ammount of bits in original image to the bits in compressed image. In order to achieve better compression we consider only approximate information. It is given by-

$$CR = \frac{M \times N \times C}{Z}$$
$$Z = \frac{M}{2} \times J_{1A} + \frac{N}{2} \times J_{2A} + C \times J_{3A} + J_{1A} \times J_{2A} \times J_{3A}$$
(7)

Peak-signal-to-noise-ratio commonly used to measure the quality of reconstructed image in comparison with original one . The PSNR in dB is given defined as [12].

$$PSNR_{dB} = 10 \log_{10} \frac{|Z|}{|Z - \hat{Z}|}$$
(8)

Where |Z| is the original image and  $|\hat{Z}|$  is the reconstructed image after processing.

Experimentation of proposed algorithm is done in MATLAB Experimental results are carried out by taking Hyperspectral image of size  $255 \times 335 \times 33$ .the original image are shown in figure 5. First we separate all 33 bands from the hyperspectral image. Then apply 2D-DWT on each hyperspectral band as shown in figure 5. As we know after applying DWT we get four sub images i.e. Approximate, Horizontal, Vertical, Diagonal. We applied DWT on experimental image, we get four sub-images of size  $135 \times 175$ . Tucker decomposition is applied to four sub-images. Tucker decomposition decomposes tensor into unknown core tensor and three component matrix of size  $135 \times 175 \times 33$ . These core tensor and component matrix are encoded by using Adaptive Arithmetic Encoding(AAC). AAC does not require to transmit codebook, and hence achieved higher compression than Huffman coding. Inverse DWT is applied on four tensors to reconstruct the hyperspectral bands. Reconstructed image as shown in figure 6. By using this method we achieved a compression ratio 3.408. MSE and PSNR values are calculated between original image and reconstructed image. Experimentation results are shown in figure7.



(An ISO 3297: 2007 Certified Organization) Website: <u>www.ijirset.com</u>

Vol. 6, Issue 7, July 2017



Fig5: Original hyperspectral input image.



### Fig5: Separation of bands.



Fig6: Reconstructed image



(An ISO 3297: 2007 Certified Organization)

#### Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

	Band1	Band2	Band3	Band4	Band5	Band6	Band7	Band8	Band9	Band10
MSE	7.877178715×10 <sup>-15</sup>	5.028723455×10 <sup>-15</sup>	1.427244573×10 <sup>-14</sup>	1.177118088×10 <sup>-14</sup>	1.433099785×10 <sup>-14</sup>	1.185392518×10 <sup>-14</sup>	1.179275329×10 <sup>-14</sup>	1.464042863×10 <sup>-14</sup>	1.52514661×10 <sup>-14</sup>	1.602863338×10 <sup>-14</sup>
PSNR	37.5850	37.7799	37.8926	37.9762	37.8907	37.9732	37.9755	37.8815	37.8638	37.8422
	Band11	Band12	Band13	Band14	Band15	Band16	Band17	Band18	Band19	Band20
MSE	3.090353089×10 <sup>-15</sup>	1.507620214×10 <sup>-14</sup>	2.567888523×10 <sup>-14</sup>	3.708055094×10 <sup>-14</sup>	1.321234326×10 <sup>-14</sup>	1.608995031×10 <sup>-14</sup>	1.761966048×10 <sup>-14</sup>	1.858400367×10 <sup>-14</sup>	1.905026292×10 <sup>-14</sup>	1.780010394× <sup>-14</sup>
PSNR	37.5571	37.2853	37.0718	36.9122	36.7947	36.7091	36.6697	36.6465	36.6358	36.6650
	Band21	Band22	Band23	Band24	Band25	Band26	Band27	Band28	Band29	Band30
MSE	1.662518425×10 <sup>-14</sup>	1.5516695038×10 <sup>-14</sup>	1.564979865×10 <sup>-14</sup>	1.553039704×10 <sup>-14</sup>	1.450050254×10 <sup>-14</sup>	1.329499938×10 <sup>-14</sup>	4.36524014×10 <sup>-14</sup>	4.105568748×10 <sup>-14</sup>	4.001466012×10 <sup>-14</sup>	1.290672167×10 <sup>-14</sup>
PSNR	36.6949	36.7249	36.7211	36.7245	36.7543	36.7920	36.8414	36.8680	36.8791	36.8048
	Band31	Band32	Band33							
MSE	1.632437914×10 <sup>-14</sup>	1.959904741×10 <sup>-14</sup>	2.042103047×10 <sup>-14</sup>							
PSNR	36.7028	36.6234	36.6056							

Fig7: PSNR and MSE values of each hyperspectral band.

#### **VII. CONCLUSION**

Hyperspectral imaging is appear to be growing interest now-a-days because of their spectral-spatial-resolution raise rapidly, which results in huge size of dataset. To reduce the storage of these data sets effective compression becomes challenging issue. In this proposed method we used 2D-DWT for 1<sup>st</sup> level compression, 2<sup>nd</sup> level compression is done by applying TD on each sub-image to achieve more compression. Figure 8 shows the graph showing PSNR value of each band of image.



Fig8: Graph showing PSNR values with respect to band index.

#### REFERENCES

[1] Q. Du and J. E. Fowler, "Hyperspectral image compression using JPEG2000 and principal component analysis," IEEE Geosci. RemoteSens. Lett., vol. 4, no. 2, pp. 201–205, Apr. 2007.

[2] C.Huo, R. Zhang, and T. Peng, "Lossless compression of hyperspectralimages based on searching optimal multibands for prediction," IEEEGeosci. Remote Sens. Lett., vol. 6, no. 2, pp. 339–343, Apr. 2009.

[3] A. Karami, M. Yazdi ,A.Zolghadre "Hyperspectral Image Compression Based on Tucker Decomposition and Discrete Cosine Transform ", 2010, 2<sup>nd</sup> International Conference on Image Processing Theory Tools And Application(IPTA).

[4] G. A. Licciardi, J. Chanussot, A. Piscin "Spectral Compression of Hyperspectral Images by means of Nonlinear Principal Component Analysis Decorrelation", 2014 International Conference On Image Processing(ICIP).



(An ISO 3297: 2007 Certified Organization)

#### Website: www.ijirset.com

#### Vol. 6, Issue 7, July 2017

[5] Hrishikesh Tamhankar, James E. Fowler "Spectral-decorrelation strategies for the compression of hyperspectral imagery", IEEE International Geiscience And Remote Sensing Symposium-2007.

[6] Lei Chen, Chao Bei, Yujia Zhai, Jing Wu "Efficient Wavelet-based Compression for Hyperspectral Images Using Precompression", 2011 International Conference on Multimedia and Signal Processing.

[7] A. Karami, M. Yazdi, and G. Mercier, "Hyperspectral image compression based on tucker decomposition and wavelet transform," IEEE Journal Of Selected Topics In Applied Earth Observations And Remote Sensing, Vol. 5, No. 2, April 2012

[8] S. Mallat, AWavelet Tour of Signal Processing. San Diego, CA: Academic, 1998.

[9] R. C. Gonzalez and R. E. Woods, *Digital Image Processing*. Englewood Cliffs, NJ: Prentice Hall, 2002.

[10] I. H.Witten, R. M. Neal, and J. G. Cleary, "Arithmetic coding for datacompression," Commun. ACM, vol. 30, no. 6, pp. 520-540, 1987.

[11] Rajesh K. Yadav, S.P. Gangwar and Harsh V. Singh "Study and analysis of wavelet based image compression techniques", International Journal of Engineering, Science and Technology Vol. 4, No. 1, 2012, pp. 1-7

[12] K. Jain, Fundamentals of Digital Image Processing. Englewood Cliffs, NJ: Prentice-Hall, 1989.