

# Hydraulic Transient Flow Analysis using Method of Characteristics

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**ABSTRACT:** This paper presents formulation for the method of characteristics for the cases involved. In the ending sections, the conclusion of the study and future scope of work taking this study as a base is presented. A similar procedure is laid down for other cases involving more complex situations like transient initiation by pumps, and multiple Valves closure. Also, the model is compared with existing verified results for validation. In the first section; the need for studying the transient state is established. According to the relation provided by the *Juokowski* relation, the pressure fluctuations formed by abrupt changes in flow velocity in a standard water distribution system are of large magnitude and possess potential to cause damage to the components of system. By analyzing the transients caused by such sudden changes in flow conditions, the damage can be reduced. The study on transient state began when researchers like *Newton* and *Lagrange* studied about the propagation of waves in water and it came into light after *Juokowski* gave expression for pressure surge corresponding to an instantaneous change in flow velocity. Later, in early 20<sup>th</sup> century *Allievi* [3] presented the differential equation for transient state after which various methods have evolved for solving transient state problems. Then, after a half of century in 1950s *Gray* [11] presented the method of characteristics for computational analysis of the Water-hammer problem and then after a decade in 1962 *V.L. Streeter* and *C. Lai* [25] presented a paper which extended *Gray's* work and made the *method of characteristics* more popular. This section also talks about the evolution of methods for solving the transient problem, in a brief manner.

In second section, brief summaries for all the literature studied for establishing the final model for method of characteristics are put down, stating the purpose of study and the notable points. In the third section, the transient-state theory is established starting with the assumptions involved in classical transient theory, present in all the mathematical models formulated by researchers for transient state analysis. Then, the basic hyperbolic partial differential equations of *continuity* and *momentum*, which govern the flow conditions during transient state, are stated.

**KEYWORDS:** Hydraulic transient, pressure wave speed, Method of Characteristics, Quasi-steady friction, unsteady friction, *Juokowski* relation

## I. INTRODUCTION

**Hydraulic Transient** is a term used for referring to a state of flow, in which the pressure and flow velocity vary rapidly. In past it was also referred as **Water-hammer**, simply because of large variations in pressures involved. Hydraulic transients occur when the flow is forced from one stable condition to another for instance, they occur when a valve is closed abruptly in a pipe.[7]

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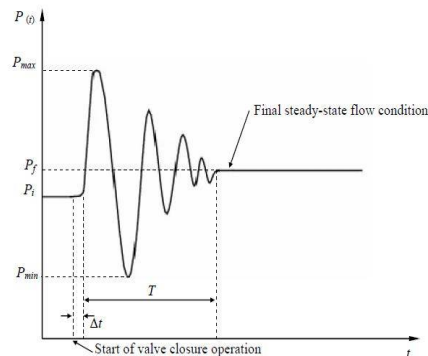


Figure 1: Transient State

As shown in the figure a damped sinusoidal behavior can be seen during the transient regime, indicating towards presence of pressure waves, or the so called **Water-hammer waves**.

Transient regions are impossible to avoid and their occurrence is generally caused by sudden action of pumps and valves. Most susceptible regions to the transients are areas with high elevations, with either low or high static pressures and areas distant from the overhead reservoir.[14]

According to *Juokowski* [13] when there is an instantaneous change in liquid's flow velocity a corresponding change in pressure intensity is also observed which are related as:

$$\Delta H = \frac{a}{g} \Delta V \quad \Delta H = \frac{a}{g} \Delta V \quad \Delta H = \frac{a}{g} \Delta V$$

Where  $a$  is pressure wave speed. If we consider the liquid to be *water* then  $\frac{a}{g} \approx 150$  sec therefore, the pressure rise or drop is 150 times the change in velocity. In case of industrial distribution systems, flow velocity can reach as high as 1.5 m/s therefore a maximum pressure variation of 225 m or 2205 kPa can be expected.

These extreme fluctuations in pressure and fluid velocity are a threat to the hydraulic system involved - the pipe walls, valves or other components. Thus, it is necessary to analyze the transient possibilities in a system and model them for achieving a safer design of the system.

The research on the transients or Water-hammer has been carried out for over hundred years, and the models and the methods to achieve transient state solution for flow characteristics have evolved greatly. However, the fundamental principles remain the same. The next section regards the basic assumptions, governing equations and the pressure wave theory which form the basis of every model or method.

## 1.1 Various Methods used for Transient analysis

Over the past hundred years, researchers have proposed methods for solving transient state problems under different conditions and assumptions. As stated in the previous section it was after the advent of 20th century when fairly accurate mathematical models came into picture.

In 1902 *Allievi* [3] proposed the **algebraic method** in which he solved the differential equations neglecting effects of friction, although the results of this method were inaccurate but this method served as the base for

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future research. Based on *Juokowski's* theory, *N.R. Gibson* [10] extended *Allievi's* method by including the nonlinear friction losses and called it *Gibson Method*, but the result were still approximate.

In 1928, *Löwy* [16] proposed the usage of **graphical method**, which was established by *Gespar Monge* [18] in 1798, for solving water-hammer problems. He was the first to include the friction terms in the basic partial differential equations. Later *L. Bergeron* [5] extended this method to determine the flow conditions at intermediary areas in the pipe, not just at the valve or reservoir end. The method was further improved by *J. Parmakian* [19] in 1950s, which he mentioned in his book *Waterhammer Analysis*. This method solves the problem assuming the quasi-steady friction model but for including the dynamic effects a "*correction term*" was added in solution. Although the method produced quick results but the method was inaccurate, it only gave accurate values for the first wave-period for later stages the friction *correction term* was not adequate

In the 1950s, with the advent of computers, the research became directed towards finding methods for computerized analysis of transients. The first one was *Gray* [11] who proposed an algorithm to implement **method of characteristics** to provide computational analysis of Water-hammer. His formulation was later improved by the works of *V.L. Streeter* and *C.Lai* [25]. In this method, the basic governing equations which are *hyperbolic* and *partial first order* in nature are converted into *ordinary first order*, along special lines called the *characteristic* lines. This method becomes fairly accurate when the unsteady friction models accommodated with it. It is, thus the most widely accepted model till date. However, the limitations of *stability* and *convergence* [7] of the finite-difference formulation pose a restriction to the *range of applicability* of this method.

In the present scenario, the frequency domain analysis of the transients has come forward. In 1989, *E.B. Wylie* along with *L. Suo* [30] published a paper presenting **impulse response method** which includes frequency dependent friction and wave speed. This method involves use of inverse Fourier transform (IFT) to solve for the flow conditions. This method is substantially faster than method of characteristics, but there is loss of accuracy because it linearizes the friction term which is not valid in every case.

## II. LITERATURE REVIEW

Prior studies indicate that **method of characteristics** is the most widely used method for numerically solving most transient state problems. Over the course of time, Researchers have evolved the usage of this method to obtain accurate and versatile computational procedures. The method fundamentally compacts the two basic *Partial Differential Equations* (PDEs) into *Ordinary Differential Equations* (ODEs) along the characteristic lines named  $C^+$  and  $C^-$ . Then an appropriate numerical difference procedure is applied to get the solution for the ODEs. But, the numerical approach restricts the stability of the solution, as shown by *Perkins et al.* [20] that the *time-increment* ( $\Delta t$ ) has a lower limit of  $a\Delta x$ . This condition is called **Courant's Stability** condition. Further, to incorporate different components like pumps and turbines, formulation of suitable boundary conditions is required. During the second half of 20<sup>th</sup> century, extensive research was carried out to improve the method. Following literature enumerates key improvements and advancements to the method of characteristics:

*E.L. Holmboe* and *W.T. Rouleau* [12] (1967) presented an experimental case study on the prevalence of effect of viscous shear on Water-hammer in a conduit. The experimental setup included a copper-tube with rapid closing valve attached at one end and a constant head reservoir at other. Pressure transducers were attached at the valve location and middle of the pipe for recording the data. The pipe was embedded in a concrete to nullify the pipe vibration, which is necessary to capture the transient conditions. In further sections the data is graphed and compared with frictionless analysis. The data provided by this experiment has been base for verifying numerical models by many researchers.

*E.B. Wylie* [28] (1983) laid down the basic formulation for the application of method of characteristics through a microcomputer, by applying a forward difference numerical procedure for getting computational solution. The paper also puts forward methodologies for efficient storage and computation of the flow conditions, in a microcomputer. A *staggered grid* procedure, which involves variable element length and time-increment to

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reduce computation time and shift focus to critical positions is also presented. In concluding sections, the numerical results for a test case are graphed and also compared with experimental data along with results of simple frictionless analysis.

B.W. Karney *et al.* [15] (1992) introduced the *linearization constant* "which provides a better forward difference approximation for the steady friction term in the formulation of characteristic equations. The paper discusses the variation of pressure and flow rate for a simple valve/orifice closure scenario, with a constant head reservoir upstream. Then, the procedure is implemented on a more complex network and an algorithm is put forward for reproducible results.

G. Pezzinga and P. Secunda [22] (1995) presented that the transient oscillations can be reduced by utilizing additional polymeric pipes to the standard pipes. The experiments verify that the pressure surges reduce and decay rate accelerates by addition of such pipes, for example *High-Density Polyethylene* (HDPE) which is utilized in the experimental setup. The paper presents a very elaborate description of experimental setup and also some numerical models are described which are able to predict flow characteristics. The data for both with and without the additional pipe is graphed. This paper has served as verification standard for various other papers like M.S. Ghidaoui *et al.* [8].

W.F. Silva-Araya *et al.* [23] (1997) estimated the radial variation of flow velocity for circular conduits. The paper presented a mathematical model for transient state and produced plots of pressure head at valve versus time and velocity versus radial distance at valve, for simple transient cases like rapid valve closure with an upstream constant head reservoir and compared the results with experimental data. This paper introduced the energy dissipation term in the method of characteristics formulation in the form of a *Dissipation Integral* and also established procedure for radial variation of flow velocity.

G. Pezzinga [21] (1999) established a Quasi-2D model for transient state. Firstly, a set of 2D PDEs for radial( $r$ ) and axial( $x$ ) variation of flow characteristics. Then the numerical scheme and computational procedure for solving the model, employing a process similar to method of characteristics, is presented. Then an experimental setup used for verification of the computed results is described. Then plots comparing the pressure head variation estimated by the Quasi-2D model and previous 1D models along with experimental data are presented. In concluding remarks, it is stated that the 2D model presented is very time consuming and takes about 3.5 hours of computational time, which is 30 times of the previous 1D models, however the accuracy is increased.

A. Bergant *et al.* [4] (2001) presented a review of different models used for the friction modeling during transient state. The paper begins with stating that the most commercial packages of transient flow modeling employ a basic friction model by assuming a constant *Darcy-Weisbach* friction factor at every time step, which is an erroneous assumption. Then a series of models is enumerated with different basic assumption. The paper describes two of the methods, starting with Zielke [31] (1966) who was the first to propose a time-varying (or *frequency-dependent*) friction factor, but only for laminar flows. Then the model formulated by Brunone (1991) but correctly formulated by Vitkovsky, co-author of the paper, in 1998 using the *Vardy-Brown* [27] model for shear-stress decay. Later, an experimental apparatus is shown, which is utilized for comparing the above-said models with experimental data.

A. Adamkowski [1] (2003) presented the implementation of method of characteristics for the case of pipes with varying cross-sectional area. In first few sections, a series of literature is presented which are used in development of model demonstrated in the paper. The model is based on the 1D theory of transient as developed by E.B. Wylie [28] and assumes that the cross section area of the conduit is varying with axial distance( $x$ ). Then, governing equation, applicable in this situation, are presented after which, the formulation of forward difference characteristic equations is demonstrated. The formulation involves calculation of parametric constants (depending on variation of area) for different types of cross-section variation. In concluding sections, the numerical results of the model is compared to those of the previous models like E.B. Wylie's to establish that

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numerical solution becomes more accurate with discussed model, although computational time is increased slightly. The comparison is one by plotting the pressure surge peaks at different time intervals for both the methods.

M.S. Ghidaoui et al. [9] (2005) presented a review article on the Water-hammer analysis. The article begins with some historical background on the transients, and then 1D mathematical model is presented for transient state analysis starting with the basic governing equations. Then, wall shear stress models are laid down stating improvements in a basic model from *quasi-steady* model to *unsteady* shear stress model. Then, in later sections various numerical schemes, including method of characteristics scheme, are discussed, stating limitations and benefits for each of them. Also, various methods of evaluating these schemes like *Energy Approach* and *Mass Balance Approach*. In following sections, a similar analysis is done for 2D model of transient state flow, starting with mathematical model, then discussing numerical methods and schemes to evaluate them. Next, an enumeration of various software packages, which invoke similar mathematical models and provide numerical solution of transient state by adopting one of the methods and schemes for evaluation, is presented. The list includes packages like *TRANSAM*, *LIQT* and *WHAMO*. Many of these packages can produce transient-state flow characteristic data for different causes of transient state like Valve closure, pump failure etc. In concluding sections, discussion on application of such analysis like *leak detection* in pipelines is done. Finally, scope for future research is laid down by stating particular areas like visualizing and modeling the physical mechanism behind formation *helical type vortex* in transient state flows.

A. Adamkowski et al. [2] (2006) presented an experimental comparative study of various unsteady friction models like Zielke's [31] model, Vardy-Brown [27] model and Brunone et al. [6] model. In initial sections, a standard method of characteristics procedure is presented with application of different unsteady friction models. Then, the experimental setup is described, an inclined pipeline with two constant head reservoirs located at both of its ends (High pressure reservoir at upstream end and low pressure reservoir at downstream end). A valve is placed at downstream end. Pressure transducers are located at four equidistant locations for data retrieval. Then, in later sections the experimental findings are compared with the numerical results produced by applying unsteady friction models. In concluding remarks, the paper regards the Vardy-Brown model as the most accurate model for transient flows.

A.K. Soares et al. [24] (2012) analyzed the transient state flows in a circular conduit caused by a pump failure/stoppage. The paper initially states an experimental setup which is used to produce data for comparison with numerical results. The setup includes a long pipeline attached to a constant head reservoir. A centrifugal pump is used to feed the pipeline with water extracted from the reservoir. The water is re-circulated to the reservoir for maintaining the constant pressure head. Then, in later sections the formulation for method of characteristics based computational procedure, which employs an unsteady friction model, is put forward. The pump characteristics are also accommodated in the Numerical approach by using pump data for various specific speeds ( $N_s$ ). Then, in following sections the numerical results are compared with experimental data and the comparison between pressure heads is graphed. In final section, a case study done on transient state formed in a local pumping station is presented, showing predicted and actual pressure head variation.

Other than the above stated literature, following books published in the second half of 20<sup>th</sup> century have served as great repository of various studies done on transient state flows namely- "*Applied Hydraulic Transients*" [7] by M.H. Chaudhary in 1979 and "*Fluid Transients*" [29] by E.B. Wylie and V.L. Streeter in 1978.

## III. THE TRANSIENT STATE THEORY

### 3.1 Classical Assumptions

The existing fundamental theory for the Transient flows in pipelines is drawn on following classical assumptions: [29] [7]



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The liquid-flow is single dimensional in other words, characteristic quantities are averaged over the cross section of pipe

The liquid is low compressible that is, it elastically deforms under high pressures with negligible relative changes in density

The Dynamic liquid-pipe interactions are neglected and a quasi-steady interaction between pipe and liquid is assumed i.e., **quasi-steady friction model\***

The liquid flow velocity is very small as compared to the pressure wave velocity

\*The quasi-steady friction model's accuracy is limited and therefore, the present theories include the dynamic interactions between liquid and pipe known as **unsteady friction model**.

## 3.2 Governing Equations

The fundamental equations of mass and momentum conservation, when subjected to the above assumptions, become the governing equations of the liquid flow in the transient regime. There are many forms of these equations, but the form most suitable for the *method of characteristics* formulation can be derived using the fundamental **Reynolds' Transport Theorem** followed by a series of assumptions.[7]

### Reynolds' Transport Theorem

This theorem relates time-rate of change of an extensive property of whole system with that of a small control volume. In mathematical form it can be written as:

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} \beta \rho dV + [\beta \rho AV_s]_{out} - [\beta \rho AV_s]_{in}$$

Where  $B_{sys}$  is the *extensive* property,  $\beta = \lim_{\Delta m \rightarrow 0} \frac{\Delta B_{sys}}{\Delta m}$  is corresponding *intensive* property  
And  $V_s$  is relative flow velocity i.e.,  $V_s = V - W$  and  $W$  is the control surface velocity.

### 3.2.1 Continuity Equation

In the equation (1) put  $B_{sys} = M_{sys}$ , the mass of the system and therefore,  $\beta = \lim_{\Delta m \rightarrow 0} \frac{\Delta M_{sys}}{\Delta m} = 1$ . And by conservation of mass  $\frac{dM_{sys}}{dt} = 0$ . Substituting this we get final form of *Continuity Equation*.

$$\frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} = 0$$

Where  $a$  is the Pressure Wave speed

### 3.2.2 Momentum Equation

In the equation (1) put  $B_{sys} = P_{sys}$ , the momentum of the system, therefore,  $\beta = \lim_{\Delta m \rightarrow 0} \frac{\Delta P_{sys}}{\Delta m} = V$ . By Newton's 2nd law,  $\frac{dP_{sys}}{dt} = \sum F$ , the sum of external forces. Substituting this we get final form of *Momentum Equation*.

$$\frac{\partial H}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + \frac{f}{2gD} V|V| = 0$$

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Where  $f$  is the Darcy-Weisbach friction factor

## IV. METHOD OF CHARACTERISTICS

### 4.1 Method Formulation

As stated earlier, the method of characteristics, fundamentally converts the governing *Partial Differential Equations* into *Ordinary Differential Equations*, which are easier to solve. The governing equations of *continuity* and *momentum* in their suitable forms are:

$$g \frac{\partial H}{\partial x} + \frac{\partial V}{\partial t} + \frac{f}{2gD} V|V| + \lambda \left( \frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} \right) = 0$$

$$\frac{\partial H}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + \frac{f}{2gD} V|V| = 0$$

Consider the operation (6) +  $\lambda$ (9), ( $\lambda$ , arbitrary constant) then the composite equation becomes:

Rearrange the equation as shown:

Now, consider the total derivatives of  $H$  and  $V$

Compare these with the terms in circular-brackets of previous equation. The expressions are very similar, only

$$\lambda \left( \frac{\partial H}{\partial t} + \frac{g}{\lambda a^2} \frac{\partial H}{\partial x} \right) + \left( \frac{\partial V}{\partial t} + \frac{\lambda a^2}{g} \frac{\partial V}{\partial x} \right) + \frac{f}{2gD} V|V| = 0$$

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{dx}{dt} \frac{\partial H}{\partial x} \text{ and } \frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{dx}{dt} \frac{\partial V}{\partial x}$$

the value of  $\frac{dx}{dt}$  is different. If we take such a value of  $\lambda$  such that both expressions are same, i.e. =  $\pm \frac{g}{a}$ , then we get  $\frac{dx}{dt} = \pm a$ . These, define two straight lines in  $x$ - $t$  plane. And, these straight lines are called **Characteristic lines**. The  $\frac{dx}{dt} = a$  line is called  $C^+$  line and  $\frac{dx}{dt} = -a$  line is called  $C^-$  line. Therefore, we get two *Ordinary Differential Equations* along these two lines.

$$\frac{dH}{dt} + \frac{a}{g} \frac{dV}{dt} + \frac{af}{2gD} V|V| = 0 : C^+$$

$$\frac{dH}{dt} - \frac{a}{g} \frac{dV}{dt} - \frac{af}{2gD} V|V| = 0 : C^-$$

### Inclusion of Unsteady friction component

To incorporate the dynamic characteristics of the friction factor, the Zarzycki [2] unsteady friction model is used. It is based on the Zielke's model, according to which the unsteady component ( $J_u$ ) depends on *partial* time-derivative of flow velocity and a weighting function  $W(t)$  which depends on previous changes in velocity. The expression for  $J_u$  is:

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$$J_u = \frac{16\nu}{gD^2} \int_0^t \frac{\partial V}{\partial t}(u) W(t-u) du$$

For simplification the whole term can be split into two parts:  $J_u = j_u \frac{\partial V}{\partial t}(u)$ ,  $j_u = \frac{16\nu}{gD^2} \int_0^t W(t-u) du$

The partial derivative can be approximated as  $\frac{\partial V}{\partial t}(u) = \frac{V^t - V^{t-\Delta t}}{\Delta t}$  because of negative correlation between  $t$  and  $u$  [4].

## 4.2 Application of Numerical Scheme

The differential equations formed in the above section can be solved using *forward-difference* scheme along the characteristic lines. Let us consider a solution grid with  $n$  space-intervals (so,  $n + 1$  space points) and  $m$  time-intervals (so,  $m + 1$  time steps). Let us denote the space points as  $x_i$  and time steps as  $t_j$ . The space interval length is  $x$  and time-interval is of  $t$  seconds. Let us denote the pressure head and flow velocity at space point  $x_i$  and time step  $t_j$  as  $H_i^j$  and  $V_i^j$ . For this, consider a sub-grid of the whole solution grid, such that flow conditions for every space point  $x_i$  at time step  $t = t_j$  are known

Now, we have to solve for flow conditions for every space point at  $t = t_{j+1}$ . For this, we have

to manipulate the *Ordinary differential* equations, and convert them into linear equations in  $H_i^j$  and  $V_i^j$ .

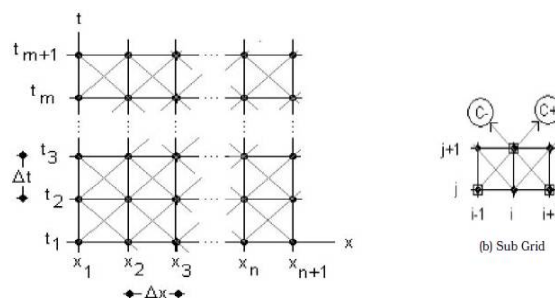
Now, revoke the equation (12) & (13) and multiply by  $dt$  throughout, and in last term of both equations substitute  $a = +\frac{dx}{dt}$  respectively, and define  $B = \frac{a}{g}$  and  $R = \frac{f}{2gD}$  then we get:

$$dH + BdV + RV|V|dx = 0 : C^+$$

$$dH - BdV + RV|V|dx = 0 : C^-$$

Now, integrate these equations along  $C^+$  and  $C^-$  lines respectively.

$$\int_{H_{i-1}^j}^{H_i^{j+1}} dH + B \int_{V_{i-1}^j}^{V_i^{j+1}} dV + R \int_{x_{i-1}^j}^{x_i^{j+1}} V|V|dx = 0$$



(a) Solution Grid



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$$\int_{H_{i+1}^j}^{H_i^{j+1}} dH - B \int_{V_{i+1}^j}^{V_i^{j+1}} dV + R \int_{x_{i+1}^j}^{x_i^{j+1}} V|V|dx = 0$$

For the third integral, assume that the  $V|V|$  term varies as [15]:

$$\int_{H_{i+1}^j}^{H_i^{j+1}} dH - B \int_{V_{i+1}^j}^{V_i^{j+1}} dV + R \int_{x_{i+1}^j}^{x_i^{j+1}} V|V|dx = 0$$

Here,  $\epsilon$  is a linearizing constant. For stable solutions  $|\epsilon| \leq 1$ . On computing the integrals we get:

$$(H_i^{j+1} - H_{i-1}^j) + B(V_i^{j+1} - V_{i-1}^j) + R[V_{i-1}^j + \epsilon(V_i^{j+1} - V_{i-1}^j)]|V_{i-1}^j|\Delta x = 0 : C^+$$

$$(H_i^{j+1} - H_{i+1}^j) - B(V_i^{j+1} - V_{i+1}^j) + R[V_{i+1}^j + \epsilon(V_i^{j+1} - V_{i+1}^j)]|V_{i+1}^j|\Delta x = 0 : C^-$$

Now, consider the unsteady component  $j_u$ . As defined earlier,  $J_u = j_u \left( \frac{V^t - V^{t+\Delta t}}{\Delta t} \right)$  where  $j_u = \frac{16\nu}{gD^2} \int_0^t W(t-u)du$ . Let  $j_u^j$  be the value of  $j_u$  at time step  $j$ , then at same time step the Value of whole component can be written as:

$$J_u^j = \frac{j_u^j V_{i-1}^j}{\Delta t} - \frac{j_u^j V_i^{j+1}}{\Delta t} : C^+$$

$$J_u^j = \frac{j_u^j V_{i+1}^j}{\Delta t} - \frac{j_u^j V_i^{j+1}}{\Delta t} : C^-$$

Now, if these values are incorporated in the equations (14) and (15), then following form of equations is achieved:

$$(H_i^{j+1} - H_{i-1}^j) + B(V_i^{j+1} - V_{i-1}^j) + R[V_{i-1}^j + \epsilon(V_i^{j+1} - V_{i-1}^j)]|V_{i-1}^j|\Delta x - j_u^j(V_i^{j+1} - V_{i-1}^j)a = 0 : C^+ \quad (10)$$

$$(H_i^{j+1} - H_{i+1}^j) - B(V_i^{j+1} - V_{i+1}^j) + R[V_{i+1}^j + \epsilon(V_i^{j+1} - V_{i+1}^j)]|V_{i+1}^j|\Delta x - j_u^j(V_i^{j+1} - V_{i+1}^j)a = 0 : C^- \quad (11)$$

Now, club the known quantities into constants, for simplification:

- $C_P^i = H_{i-1}^j + BV_{i-1}^j - R(1 - \epsilon)V_{i-1}^j|V_{i-1}^j|\Delta x - j_u^j V_{i-1}^j a$
- $C_M^i = H_{i+1}^j - BV_{i+1}^j + R(1 - \epsilon)V_{i+1}^j|V_{i+1}^j|\Delta x - j_u^j V_{i+1}^j a$
- $B_P^i = B + \epsilon R|V_{i-1}^j|\Delta x - j_u^j a$  and  $B_M^i = B + \epsilon R|V_{i+1}^j|\Delta x - j_u^j a$

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On simplification, following characteristic equations in the unknown states  $H_i^{j+1}$  and  $V_i^{j+1}$  are formed:

$$H_i^{j+1} = C_P^i - B_P^i V_i^{j+1} \quad ; \quad C^+$$

$$H_i^{j+1} = C_M^i + B_M^i V_i^{j+1} \quad ; \quad C^-$$

For the space points inside the boundaries, the flow characteristics can be solved, with solutions as:

$$H_i^{j+1} = \frac{C_P^i B_M^i + C_M^i B_P^i}{B_P^i + B_M^i} \quad \text{and} \quad V_i^{j+1} = \frac{C_P^i - C_M^i}{B_P^i + B_M^i}$$

**Note:** These equations can be solved simultaneously, only for space points which are *inside the boundaries*. For the boundary points individual characteristic equation along with appropriate **boundary condition** has to be used for calculating flow characteristics. Usually, steady-state is assumed as the **initial** condition.

## 4.3 Some Boundary Conditions

As stated earlier, to completely evaluate the flow conditions, calculation of flow state at boundaries is required. For the first space point  $x_1$ , we have only one characteristic equation, so another equation in the form of boundary condition is required. For example, if a constant head reservoir is present at pipe starting, then  $H_1^{j+1}$  can be assumed constant and then  $V_1^{j+1}$  can be calculated using characteristic equation. Some of the boundary conditions, and their corresponding application in *method of characteristics* are:

### Constant Head Reservoir at an End

If the Constant Head Reservoir is present at the *upstream* end of the pipe, then  $H_1^{j+1}$  can be assumed equal to the Reservoir head and then  $V_1^{j+1}$  can be calculated as  $V_1^{j+1} = \frac{H_1^{j+1} - C_M^1}{B_M^1}$

Otherwise, if the Constant Head Reservoir is present at the *downstream* end of the pipe, then  $H_{n+1}^{j+1}$  can be assumed equal to the Reservoir head and then  $V_{n+1}^{j+1}$  can be calculated as  $V_{n+1}^{j+1} = \frac{H_{n+1}^{j+1} - C_M^{n+1}}{B_M^{n+1}}$

### Free Discharge Valve at an End

If the valve is present at Downstream end of the conduit, then  $H_{n+1}^{j+1} = \frac{K_v}{2g} V_{n+1}^{j+1} |V_{n+1}^{j+1}|$ , where  $K_v$  is valve-loss coefficient. Further,  $K_v$  depends on the *Coefficient of Discharge* ( $C_d$ ) as  $K_v = C_d$  varies with valve-opening and the type of valve. Also,

$$H_{n+1}^{j+1} = C_P^{n+1} - B_P^{n+1} V_{n+1}^{j+1} \quad \text{for } H_{n+1}^{j+1}, \text{ we get:}$$

$$C_2 V_{n+1}^{j+1} |V_{n+1}^{j+1}| + B_P^{n+1} V_{n+1}^{j+1} - C_P^{n+1} = 0 \quad ; \quad C_2 = \frac{K_v}{2g}$$

This quadratic equation can be used to solve for  $V_{n+1}^{j+1}$  which can further give  $H_{n+1}^{j+1}$ . A similar evaluation method can be used when the Valve is at *downstream* end.

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## In-line Valve

When the Valve is present between two conduits as shown in the figure below. For such a case, firstly, we have to use equation of continuity at the junction and write characteristic equations for  $(n + 1)^{\text{th}}$  space point of first conduit and  $1^{\text{st}}$  space point for second conduit. Also, a head loss equation can be formed for the situation. Consider the mathematical form of all the above-said equations [7]:

1.  $V_{n+1}^{j+1}(1) = V_1^{j+1}(2) = V_C^{j+1}$  Assuming same area of cross-section
2.  $H_{n+1}^{j+1}(1) = C_P^{n+1}(1) - B_P^{n+1}(1)V_{n+1}^{j+1}(1)$
3.  $H_1^{j+1}(2) = C_M^1(2) + B_M^1(2)V_1^{j+1}(2)$
4.  $H_{n+1}^{j+1}(1) - H_1^{j+1}(2) = \frac{K_v}{2g} V_C^{j+1} |V_C^{j+1}|$

(a) In-line Valve

Combining all of them we get a quadratic equation in  $V_C^{j+1}$ :

$$C_2 V_C^{j+1} |V_C^{j+1}| + (B_P^{n+1}(1) + B_M^1(2)) V_{n+1}^{j+1} - C_P^{n+1}(1) + C_M^1(2) = 0 ; C_2 = \frac{K_v}{2g}$$

This equation can be solved to give values of  $V_C^{j+1} = V_1^{j+1}(2) = V_{n+1}^{j+1}(1)$ , which can be used further for getting  $H_1^{j+1}(2)$  &  $H_{n+1}^{j+1}(1)$ .

## In-line Centrifugal Pump

The evaluation strategy used for this boundary condition is similar to that for the in-line valves, only difference is between the modeling of head loss through the pump and its evaluation. To evaluate the head loss across a pump, pump characteristic curves have to be used. For understand the characteristic curves, define following non-dimensional quantities:

$$h_P = \frac{H_P}{H_R} \quad \nu_P = \frac{V_P}{V_R} \quad \alpha_P = \frac{N_P}{N_R} \quad \beta_P = \frac{T_P}{T_R}$$

The pump characteristics are given as a graph between  $\frac{h_P}{\sqrt{\frac{1}{\alpha_P^2} + \nu_P^2}}$  vs  $\theta = \tan^{-1} \frac{\alpha_P}{\nu_P}$  and  $\frac{\beta_P}{\alpha_P^2 + \nu_P^2}$  vs  $\phi = \tan^{-1} \frac{\sigma_P}{\nu_P}$  as suggested by M. Marchal et al. [17]. The data for some specific speeds

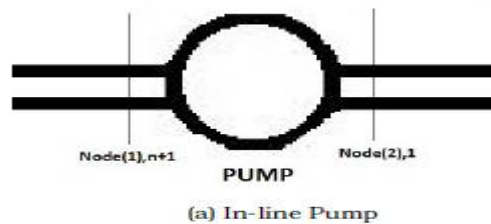
is given by G.O. Thomas [26] and also tabulated in *Applied Hydraulic transients* [7] book (see Appendix B). Let  $y_h(\theta)$  and  $y_{b\beta}(\phi)$  represent pressure and torque characteristics respectively.

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Similar to in-line valve case, firstly, we have to use equation of continuity at the junction and write characteristic equations for  $(n+1)^{th}$  space point of first conduit and  $1^{st}$  space point for second conduit. For the head loss equation, an iterative procedure has to be followed, which has been described as follows [7]:

Suppose that the values of  $p^j$  and  $v^j$  are known. Now, to evaluate the values of  $p^{j+1}$  and  $v^{j+1}$ , assume an estimate values for both  $\alpha_e = \alpha_p^j + \Delta\alpha$  and  $v_e = v_p^j + \Delta v$  ( $\Delta\alpha$  and  $\Delta v$  are changes in previous values). Now, to evaluate  $\phi = \tan^{-1} \frac{\alpha_e}{v_e}$  (if  $\phi < 0$  add  $360^\circ$  to it, to adjust it according to data) and find known grid-points between whom  $\phi$  lies, let them be  $\phi_1$  and  $\phi_2$ . Therefore, following equations can be written using linear interpolation of data between  $\phi_1$  and  $\phi_2$ , characteristic equations at space points of pipe-1 and pipe-2 and pump failure scenario:

$$H_R h_P = C_M^1(2) - C_P^{n+1}(1) + (B_P^{n+1}(1) + B_M^1(2)) V_R v_P$$

$$\alpha_P + C_3 \beta_P = \alpha - c_3 \beta$$

$$\frac{h_P}{\alpha_P^2 + v_P^2} = a_1 + a_2 \tan^{-1} \frac{\alpha_P}{v_P}$$

$$\frac{\beta_P}{\alpha_P^2 + v_P^2} = a_3 + a_4 \tan^{-1} \frac{\alpha_P}{v_P}$$

Here,  $\alpha_P$ ,  $v_P$ ,  $\beta_P$  and  $h_P$  are non-dimensional properties at  $j+1^{th}$  time step.  $C_3 = \frac{157 R \Delta t}{l_p \pi N_R}$  and  $\alpha = \alpha_p^j$ ,  $\beta = [(\alpha_p^j)^2 + (v_p^j)^2] y_\beta (\tan^{-1} \frac{\alpha_p^j}{v_p^j})$  are average values during the  $j^{th}$  time-interval.

$$a_1 = \frac{y_h(\theta_1)\theta_2 - y_h(\theta_2)\theta_1}{\theta_2 - \theta_1} \quad \& \quad a_2 = \frac{y_h(\theta_2) - y_h(\theta_1)}{\theta_2 - \theta_1}$$

$$a_3 = \frac{y_\beta(\theta_1)\theta_2 - y_\beta(\theta_2)\theta_1}{\theta_2 - \theta_1} \quad \& \quad a_4 = \frac{y_\beta(\theta_2) - y_\beta(\theta_1)}{\theta_2 - \theta_1}$$

Combining all the equations, following non-linear equations are formed in  $\alpha_P$  and  $v_P$ :

$$E_1 : a_1(\alpha_P^2 + v_P^2) + a_2(\alpha_P^2 + v_P^2) \tan^{-1} \frac{\alpha_P}{v_P} - \frac{[B_P^{n+1}(1) + B_M^1(2)]}{H_R} v_P V_R - \frac{[C_M^1(2) - C_P^{n+1}(1)]}{H_R} = 0$$

$$E_2 : \alpha_P + C_3[a_3(\alpha_P^2 + v_P^2) + a_4(\alpha_P^2 + v_P^2) \tan^{-1} \frac{\alpha_P}{v_P}] - \alpha + C_3 \beta = 0$$

For solving this pair of non-linear equation we can use 2D Newton-Raphson method.

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Define following iteration procedure:

$$\alpha_p(k) = \alpha_p(k-1) + \delta\alpha_p, v_p(k) = v_p(k-1) + \delta v_p$$

Where,

$$\delta\alpha_p = \frac{E_2 \frac{\delta E_1}{\delta v_p} - E_1 \frac{\delta E_2}{\delta v_p}}{\frac{\delta E_1}{\delta \alpha_p} \frac{\delta E_2}{\delta v_p} - \frac{\delta E_1}{\delta v_p} \frac{\delta E_2}{\delta \alpha_p}}, \delta v_p = \frac{E_2 \frac{\delta E_1}{\delta \alpha_p} - E_1 \frac{\delta E_2}{\delta \alpha_p}}{\frac{\delta E_1}{\delta v_p} \frac{\delta E_2}{\delta \alpha_p} - \frac{\delta E_1}{\delta \alpha_p} \frac{\delta E_2}{\delta v_p}}$$

Stop the iterations when  $|\delta\alpha_p|$  &  $|\delta v_p|$  become less than a tolerance value. Also,

Check whether the final solution obtained  $\theta_s = \tan^{-1} \frac{\alpha_p^{j+1}}{v_p^{j+1}}$  is in the range of  $\theta_1$  &  $\theta_2$

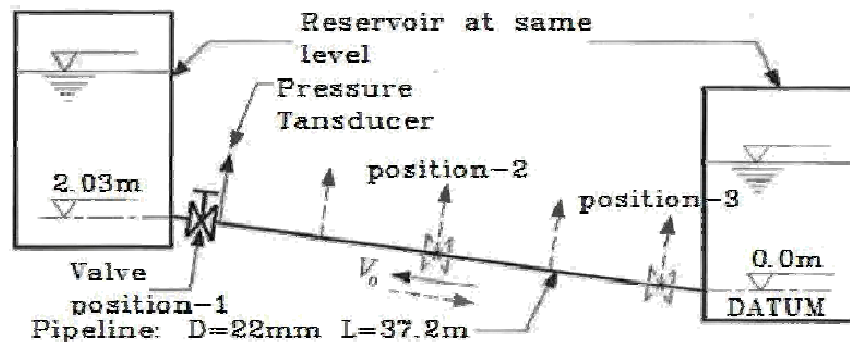
Using the solution, the flow conditions i.e.  $H_{n+1}^{j+1}(1)$ ,  $H_1^{j+1}(2)$ ,  $V_{n+1}^{j+1}(1)$  and  $V_1^{j+1}(2)$  can be obtained.

**Note:** For the case of centrifugal pump at an end, same method of evaluation can be used; there is only a change in the head loss equation.

## V. MODEL VALIDATION

For the verification of *method of characteristics* model the numerical results for a specific case are compared with experimental data as given by *M.S. Ghidaoui et al.*[8] (2002). The experiment setup used by them can be explained as follows.

### 5.1 Experimental Setup



It consists of two constant head reservoirs, joined by a 37.2 m long single pipe of 22 mm diameter, having same head of 32 m. However, they are kept at different elevations, with a difference of 2.03 m hence a resulting flow occurs in the conduit. A fast closing valve is also attached which induces the transient flow, when it is suddenly closed. The experiment is repeated with the valve kept at upstream end, middle of pipe and downstream end of the pipe, as shown in the figure. However, the experimental data used for verification is for the valve kept at downstream end of the pipe, i.e. at position-3. The setup is firstly allowed to achieve a steady state, with the valve at fully opened position. Then the valve is closed in 0.09 s. The wave speed is assumed to be 1319 m/s.

Experimental Setup

### 5.2 Equivalent Model

The results produced for the above setup are also valid for a setup in which the reservoirs are at same elevation but the difference in their pressure heads is 2.71 m ( $2.71 = 2.03/\sin\theta$ , where  $\theta$  is depression angle of pipe) and a



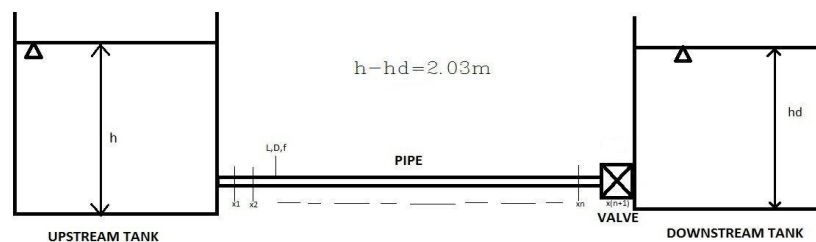
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valve is placed at downstream end of pipe. It must be noted that the change in inclination does not effect the method's formulation because the difference in gravitational head in the experimental case can be used as difference in pressure head as in numerical model. The setup utilized for verification is shown in the figure below.



Equivalent Model

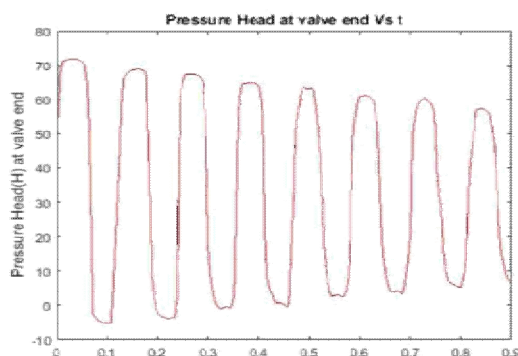
## 5.3 Comparison

The numerical results are able to predict the experimental data with an average absolute error of less than 0.2017 m or average relative error of 1.5 % of maximum pressure head. Therefore, the model, used for simulating the physical flow occurring in the experimental setup, is validated. Following graphs showcase the variation of pressure head at valve versus time, for the experimental setup and the numerical result with comparison to the experimental data.

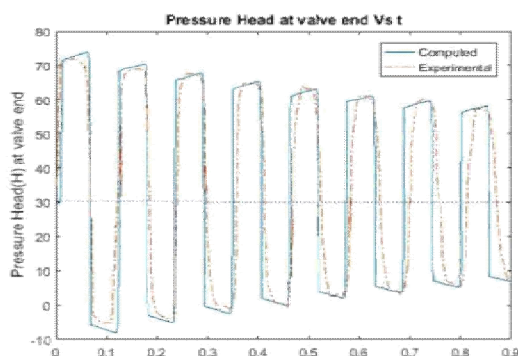
## 5.4 Discussion

The numerical result is very accurately able to predict the peak values of the experimental data. However, the experimental data indicates a change in the frequency of the pressure wave, which is not accommodated in the present model, which assumes a constant time period of  $\frac{4L}{a}$ . Therefore, there is a phase difference between the experimental data and the numerical result. The variable frequency in the actual data could possibly be a result of superposition between progressing and reflected pressure waves, but as they have similar amplitude, there is negligible change in the overall amplitude of the pressure wave.

The next section puts forward various numerical procedures, applying the hence verified method of characteristics model to produce numerical results for flow characteristics.



Experimental Data



Numerical Result

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## VI. CONCLUSION

Estimation of transient state flow conditions for different flow parameters has been a matter of interest for researchers in the field of fluid dynamics, worldwide. Their interest has been backed by the ever-growing need for *resource optimization* by the industry. The industry asks for design of pipelines which are not only safe (from pressure surges), but made from least amount of resources with minimum capital requirement. Thus, there is a need for accurate theoretical models for transient state flows. Researchers have been working on such models for over a century. Latest studies indicate that at present, the **method of characteristics** is the most widely used procedure for numerically solving transient state problems. The method fundamentally, involves converting of complex governing equations (*hyperbolic Partial Differential Equations* of 1<sup>st</sup> order, to be more accurate) into simpler ones (which is 1<sup>st</sup> order *Ordinary Differential Equations*) and then solving them using an appropriate numerical scheme.

The current report presents a prompt formulation of the method of characteristics, employing many of the advancements that have been devised by researchers after the second of 20<sup>th</sup> century. After validating the formulated model by comparison with experimental data by *M.S. Ghidaoui et al.* [8], a set of transient problems are also solved using this method to demonstrate the various boundary conditions and ways of incorporating different components used in day-to-day pipeline networks like valves, pumps and reservoirs. The results obtained by the model are also discussed to obtain a link between computed result and real phenomena.

Although many of the real scenarios are covered by the *method of characteristics*, it is still a work under progress. The formulation by method of characteristics has a very limited *range of applicability*. This limitation is mainly caused by the stability criterion for the numerical scheme due to which only a small number of space intervals are possible. This condition becomes more troublesome in the scenarios with low pressure wave *celerity* as the number of space intervals is further reduced, which makes the numerical solution inaccurate. Other than this, the dynamic natures of many fluid characteristics and flow parameters have not been implemented in the method because of high complexity. To overcome this, commercial software packages utilize different *method of characteristics* models for different range of flow parameters [9]. This is achieved by changing friction models, forms of differential equation and numerical schemes. The more the number of models the better is the software package.

Many methods which are more accurate, like *2D Quasi* model, have also developed in recent years, but they gain accuracy at the cost of efficiency. For example, a typical *2D Quasi* model takes at least 30 times more computational time than *1D method of characteristics* model, with an increase in accuracy of around 10 *percentage points*. Attempts have also been made on getting results more efficiently, for instance *Impulse Response method* which produces results faster but loses accuracy due to assumption of linear friction factor. Therefore, studies have been re-directed towards making the method of characteristics more accurate and versatile.

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