

Optimizing Octagonal Fuzzy Number EOQ Model Using Nearest Interval Approximation Method

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ABSTRACT: In this paper, an EOQ model is discussed. There are several approaches by different authors to solve an EOQ model. We introduce octagonal fuzzy numbers in EOQ by which we develop a new model to solve the problem. Here, an EOQ model with shortage, where the setup cost, holding cost, shortage cost and the demand are consider as octagonal fuzzy numbers. The fuzzy parameters are transformed into corresponding interval numbers and to determine the optimum order quantity to minimize the average total cost. Finally, this model is proposed with a numerical example.

KEYWORDS: Inventory, Interval number, EOQ, Fuzzy sets, Fuzzy optimization technique, Multi-objective programming, octagonal numbers.

I. INTRODUCTION

In the past few decades, inventory problems have been widely studied by researchers. Inventory control management is one of the most important fields in supply chain management. It becomes more and more important for enterprises in the real-life situations. A proper control and analyzing of inventory systems can significantly enhance a company's profit. [8]. In the crisp inventory models, all the parameters in the total cost are known and have definite values. But in the practical situation it is not possible. In order to deal with such situations fuzzy model is used [2], [9]. In such cases, fuzzy set theory, introduced by Zadeh [4] is acceptable. There are several studies on fuzzy EOQ model.

In 1981, Sommer[6] used fuzzy dynamic programming to solve a real-world inventory and production scheduling problem. Lin et al. [3] have developed a fuzzy model for production inventory problem. The model developed by S.U Malini et.al [7] introduced octagonal fuzzy numbers for transportation problems.

The model developed by S.Gajalakshmi et.al[5] that they introduced octagonal fuzzy numbers for inventory and solved using signed defuzzification. In our paper, we use octagonal fuzzy numbers in fuzzy EOQ. The problem is solved by fuzzy optimization technique. Finally using Lindo software we find the optimized value which minimizes the cost.

II. RELATED WORK

Our paper is to develop an EOQ model including octagonal fuzzy numbers. The model is solved by multi-objective EOQ problem. So many research works were done on EOQ problem. The major assumption in the classical EOQ model is all the costs are deterministic in nature. This paper deals with fuzzy EOQ model to determine the optimum order quantity so as to minimize the average cost.

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III. PRELIMINARIES

Definition:

A fuzzy set is characterized by a membership function mapping elements of domain, space, or universe of discourse X to the unit interval $[0, 1]$. (i.e) $A = \{x, \mu_A(x); x \in X\}$, here $\mu_A: X \rightarrow [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of $x \in X$ in the fuzzy set A .

Definition:

Let R be the set of all real numbers. An interval, may be expressed as

$$\bar{u} = [u_L, u_R] = \{x: u_L \leq x \leq u_R, u_L \in R, u_R \in R\} \rightarrow (1)$$

Where u_L and u_R are called the lower and upper limits of the interval $\bar{u}$, respectively.

If $u_L = u_R$ then $\bar{u} = [u_L, u_R]$ is reduced to a real number u , where $u = u_L = u_R$ alternatively an interval $\bar{u}$ can be expressed in mean-width or centre-radius form as $\bar{u} = \langle m(\bar{u}), w(\bar{u}) \rangle$, where $m(\bar{u}) = \frac{1}{2}(u_L + u_R)$ and $w(\bar{u}) = \frac{1}{2}(u_R - u_L)$ are respectively the mid-point and half-width of the interval $\bar{u}$ the set of all interval numbers in R is denoted by $I(R)$.

Optimization in interval environment

Now we define a general non-linear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize } \bar{Z} = \frac{\sum_{i=1}^n [u_{Li}, u_{Ri}] \prod_{j=1}^k x_j^{p_j}}{\sum_{i=1}^l [v_{Li}, v_{Ri}] \prod_{j=1}^n x_j^{q_j}} \rightarrow (2)$$

Subject to $x_j > 0, j=1, 2, \dots, n$ and $x \in S \subset R$ where S is a feasible region of x , $0 < u_{Li} < u_{Ri}, 0 < v_{Li} < v_{Ri}$ and p_i, q_j are positive numbers. Now we exhibit the formulation of the original problem (2) as a multi-objective non-linear problem.

Now $\bar{Z}(x)$ can be written in the form $\bar{Z}(x) = [Z_L(x), Z_R(x)]$

$$\text{Where } Z_L(x) = \frac{\sum_{i=1}^n u_{Li} \prod_{j=1}^k x_j^{p_j}}{\sum_{i=1}^l v_{Ri} \prod_{j=1}^n x_j^{q_j}} \rightarrow (3)$$

$$Z_R(x) = \frac{\sum_{i=1}^n u_{Ri} \prod_{j=1}^k x_j^{p_j}}{\sum_{i=1}^l v_{Li} \prod_{j=1}^n x_j^{q_j}} \rightarrow (4)$$

The centre of the objective function

$$Z_C(x) = \frac{1}{2} [Z_L(x) + Z_R(x)] \rightarrow (5)$$

Thus the problem (2) is transformed into

$$\text{Minimize } \{Z_C(x), Z_R(x); x \in S\} \rightarrow (6)$$

According to Gregorzewski we determine the interval approximation of a fuzzy number as: Let $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ be an arbitrary octagonal fuzzy number with a α -cuts $[A_L(\alpha), A_R(\alpha)]$ and with the following membership function

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$$\mu_A(x) = \begin{cases} 0 & , & x < a_1 \\ k \left(\frac{x-a_1}{a_2-a_1} \right) & , & a_1 \leq x \leq a_2 \\ k & , & a_2 \leq x \leq a_3 \\ k + (1-k) \left(\frac{x-a_3}{a_4-a_3} \right) & , & a_3 \leq x \leq a_4 \\ 1 & , & a_4 \leq x \leq a_5 \\ k + (1-k) \left(\frac{a_6-x}{a_6-a_5} \right) & , & a_5 \leq x \leq a_6 \\ k & , & a_6 \leq x \leq a_7 \\ k \left(\frac{a_8-x}{a_8-a_7} \right) & , & a_7 \leq x \leq a_8 \\ 0 & , & 0 \geq a_8 \end{cases}$$

Then by nearest interval approximation method for each case of octagonal fuzzy number the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\alpha) d\alpha$$

$$C_R = \int_0^1 A_R(\alpha) d\alpha$$

Case (i): $k=0$

$$[C_L, C_R] = \left[\frac{a_3 + a_4}{2}, \frac{a_6 + a_5}{2} \right]$$

Case (ii): $k=1$

$$[C_L, C_R] = \left[\frac{a_1 + a_2}{2}, \frac{a_8 + a_7}{2} \right]$$

Case (iii): $0 < k < 1$

$$[C_L, C_R] = \left[\frac{(a_1 + a_2)k}{2} + \frac{(a_3 + a_4)}{2}(1-k), a_6k - \frac{k^2}{2(1-k)}(a_6 - a_5) + a_8(1-k) + \frac{k}{2}(a_8 - a_7) + \frac{1}{2k}(a_7 - a_8) \right]$$

Model formulation:

In this model, an inventory with shortage is taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory item by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

Notations:

- C_1 : Holding cost per unit time per unit quantity.
- C_2 : Shortage cost per unit time per unit quantity.
- C_3 : Setup cost per period.
- D : The total number of units produced per time period.
- Q_1 : The amount which goes into inventory.
- Q_2 : The unfilled demand.
- Q : The lot size in each production run.

Assumptions:

- (i) Demand is known and uniform.
- (ii) Production or supply of commodity is instantaneous.

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- (iii) Shortages are allowed.
- (iv) Lead time is Zero.

Let the amount of stock for the item be Q_1 at time $t=0$ in the interval $(0, t (= t_1+t_2))$, the inventory level gradually decrease to meet the demands. By this process the inventory level reaches zero level at time t_1 and then shortages are allowed to occur in the interval (t_1, t) . The cycle repeats itself.

The order level $Q > 0$ which minimizes the average total cost (Q) per unit time is given by

$$\text{Min } C(Q) = \frac{1}{2} C_1 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} C_2 \left(\frac{Q_2^2}{Q} \right) + C_3 \left(\frac{D}{Q} \right)$$

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc; as real numbers (i.e) of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we approach with fuzzy variables, where demand and other cost components are considered as octagonal fuzzy numbers.

Let us assume the fuzzy demand $\tilde{D}$, fuzzy holding cost $\tilde{C}_1$, fuzzy shortage cost $\tilde{C}_2$, and fuzzy ordering cost $\tilde{C}_3$. Replacing the real valued variables D, C_1, C_2, C_3 by the octagonal fuzzy variables we get,

$$\tilde{C}(Q) = \frac{1}{2} \tilde{C}_1 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} \tilde{C}_2 \left(\frac{Q_2^2}{Q} \right) + \tilde{C}_3 \left(\frac{D}{Q} \right) \rightarrow (7)$$

Now we represent the fuzzy EOQ model for all the three cases to a deterministic form so that it can be easily tackled. Following Grzegorzewski [3], the fuzzy numbers are transformed into interval numbers as

$$\begin{aligned} \tilde{D} &= [D_L, D_R] \\ \tilde{C}_1 &= [C_{1L}, C_{1R}] \\ \tilde{C}_2 &= [C_{2L}, C_{2R}] \\ \tilde{C}_3 &= [C_{3L}, C_{3R}] \end{aligned}$$

Using the above expression (7) becomes

$$\tilde{C}(Q) = [g_L, g_R]$$

$$\text{Where, } g_L = \frac{1}{2} C_{1L} \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} C_{2L} \left(\frac{Q_2^2}{Q} \right) + C_{3L} \left(\frac{D_L}{Q} \right) \rightarrow (8)$$

$$g_R = \frac{1}{2} C_{1R} \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} C_{2R} \left(\frac{Q_2^2}{Q} \right) + C_{3R} \left(\frac{D_R}{Q} \right) \rightarrow (9)$$

The composition rules of intervals are used in these equations.

Hence, the proposed model can be stated as

$$\text{Minimize } \{g_L(Q), g_R(Q)\}, \rightarrow (10)$$

Generally, the multi-objective optimization problem (10), in case of minimization problem, can be formulated in a conservative sense from (3) as

$$\begin{aligned} &\text{Minimize } \{g_L(Q), g_R(Q)\} \\ &\text{Subject to } Q \geq 0 \\ &\text{Where } g_C = \frac{g_L + g_R}{2} \end{aligned}$$

Here, the interval valued problem (10) is represented as

$$\text{Minimize } \{g_L(Q), g_C(Q), g_R(Q)\} \rightarrow (11)$$

Subject to $Q \geq 0$

The expression (11) gives a better approximation than those obtained from (10)

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Fuzzy programming technique for solution:

To solve multi-objective minimization problem given by (11), we have used the following fuzzy programming technique.

For each of the objective functions $g_L(Q)$, $g_C(Q)$, $g_R(Q)$, we first find the lower bounds L_L , L_C , L_R (best values) and the upper bounds U_L , U_C , U_R (worst values), where L_L , L_C , L_R are the aspired level achievement and U_L , U_C , U_R are the highest acceptable level achievement for the objectives $g_L(Q)$, $g_C(Q)$, $g_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for the objective $g_k(Q)$, $k = L, C, R$. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of fuzzy programming technique are given below:

Step 1: Solve the multi-objective cost function as a single objective cost function using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k value corresponding to the set of solutions. The initial fuzzy model of (7) can then be stated as, in terms of the aspiration levels for each objective, as follows: find Q satisfying $g_k \leq L_k$, $k = L, C, R$ subject to the non negativity conditions.

Step 4: Define fuzzy linear membership function (μ_{f_k} ; $k = L, C, R$) for each objective function is defined by

$$\mu_{g_k} = \begin{cases} 1 & ; g_k \leq L_k \\ 1 - \frac{g_k - L_k}{d_k} & ; L_k \leq f_k \leq U_k \\ 0 & ; g_k \geq U_k \end{cases} \rightarrow (12)$$

Step 5: After determining the linear membership function defined in (12) for each objective functions following the problem (11) can be formulated an equivalent crisp model

Max α ,

$\alpha \leq \mu_{f_k}(x)$; $k = L, C, R$.

IV. NUMERICAL EXAMPLE

In this section, the above mentioned algorithm is illustrated by a numerical example. Here the parameters namely demand, ordering cost, holding cost and shortage cost are consider as octagonal fuzzy numbers. After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation method.

For different cases, the octagonal fuzzy numbers $\tilde{D}$, $\tilde{C}_1$, $\tilde{C}_2$, $\tilde{C}_3$ are transformed into interval numbers as,

Case (i): $k=0$

$$[C_{1L}, C_{1R}] = [6.5, 8.5], [C_{2L}, C_{2R}] = [25.5, 27.5], [C_{3L}, C_{3R}] = [87.5, 91.0] \\ [D_L, D_R] = [5250, 5750]$$

Case (ii): $k=1$

$$[C_{1L}, C_{1R}] = [4, 10.5], [C_{2L}, C_{2R}] = [21.5, 30], [C_{3L}, C_{3R}] = [85.5, 93.5] \\ [D_L, D_R] = [4100, 6100]$$

Case (iii): $0 < k < 1$

$$[C_{1L}, C_{1R}] = [4k + \frac{13}{2}(1-k), 9k - \frac{k^2}{2(1-k)} + 11(1-k) + \frac{k}{2} - \frac{1}{2k}] \\ [C_{2L}, C_{2R}] = [\frac{43}{2}k + \frac{51}{2}(1-k), 28k - \frac{k^2}{2(1-k)} + 31(1-k) + k - \frac{1}{k}]$$

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$$[C_{3L}, C_{3R}] = \left[\frac{171}{2}k + \frac{175}{2}(1-k), 92k - \frac{k^2}{2(1-k)}(2) + 94(1-k) + \frac{k}{2} - \frac{1}{2k} \right]$$

$$[D_L, D_R] = \left[4100k + 5250(1-k), 5800k + \frac{k^2}{(1-k)}(50) + 6200(1-k) + k(100) - \frac{100}{k} \right]$$

Individual minimum and maximum of objective functions g_L, g_C, g_R for three different cases namely $k=0, k<0<1$ and $k=1$ are given in the following table:

Table: 4.1

Cases	Objective functions	Optimized values	Q	Q_1
K=0	g_L	2181.4761	421.15979	335.61170
K=1	g_C	3274.0717	600.24648	436.54289
0<k<1	g_R	2783.4822	391.52846	292.99812

Finding optimum values:

Case (i): k=0

Max α

$$\text{Subject to } \frac{1}{2}(6.5) \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2}(25.5) \left(\frac{Q_2^2}{Q} \right) + (87.5) \left(\frac{5250}{Q} \right) + (184.905193) \alpha \leq 2366.381293$$

$$\frac{1}{2}(7.5) \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2}(26.5) \left(\frac{Q_2^2}{Q} \right) + (89.25) \left(\frac{5500}{Q} \right) + (949.018758) \alpha \leq 3274.0717$$

$$\frac{1}{2}(8.5) \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2}(27.5) \left(\frac{Q_2^2}{Q} \right) + (91) \left(\frac{5750}{Q} \right) + (216.963654) \alpha \leq 2834.928081 \quad Q, Q_1, Q_2, \alpha > 0$$

Using Lindo's software the above model is solved and the optimum value of α , Q and Q_1 for the case $k=0$ are tabulated in the following table:

Table: 4.2

Max α	Q^*	Q_1^*
0.92566164	409.81357	319.41357

Case (ii): k=1

Max α

$$\text{Subject to } (2) \left(\frac{Q_1^2}{Q} \right) + (10.75) \left(\frac{Q_2^2}{Q} \right) + \left(\frac{350550}{Q} \right) + 1440.91722\alpha \leq 2978.608922$$

$$\frac{1}{2}[(7.25) \left(\frac{Q_1^2}{Q} \right) + (25.75) \left(\frac{Q_2^2}{Q} \right) + \left(\frac{920900}{Q} \right) + 932.439588\alpha \leq 3227.9135]$$

$$(5.25) \left(\frac{Q_1^2}{Q} \right) + (15) \left(\frac{Q_2^2}{Q} \right) + \left(\frac{570350}{Q} \right) + 509.341602\alpha \leq 3487.950502,$$

$$Q, Q_1, Q_2, \alpha > 0$$

Using Lindo's software the above model is solved and the optimum value of α , Q and Q_1 for the case $k=1$ are tabulated in the following table:

Table: 4.3

Max α	Q^*	Q_1^*
0.99999996	382.96401	283.67704

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Case (iii): $0 < k < 1$

Max α

Subject to $(2.6) \left(\frac{Q_1^2}{Q} \right) + (11.75) \left(\frac{Q_2^2}{Q} \right) + \left(\frac{402225}{Q} \right) + 87.794619\alpha \leq 1938.529419$

$\frac{1}{2} (7.35) \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} (25.875) \left(\frac{Q_2^2}{Q} \right) + \left(\frac{473565.625}{Q} \right) + 947.757102\alpha \leq 3292.8461$

$(4.75) \left(\frac{Q_1^2}{Q} \right) + (14.125) \left(\frac{Q_2^2}{Q} \right) + \left(\frac{544906.25}{Q} \right) + 218.662681\alpha \leq 3002.144881,$

$Q, Q_1, Q_2, \alpha > 0$

Using Lindo's software the above model is solved and the optimum value of α , Q and Q_1 for the case $0 < k < 1$ are tabulated in the following table:

Table: 4.4

Max α	Q^*	Q_1^*
0.91844705	413.63460	326.82582

The minimum total cost of the objectives functions g_L^* , g_C^* and g_R^* for three cases of k values are tabulated as follows:

Table: 4.5

Cases	g_L^*	g_C^*	g_R^*
K=0	2184.2893	2331.8604	2609.0487
K=1	1612.3391	2295.4742	2978.6092
$0 < k < 1$	1857.8944	2329.6045	2801.3145

Following table shows the comparison between ordering costs, holding costs, shortage costs, economic order quantities and the minimum total costs for the different cases of k values.

Table: 4.6

Cases	C_1	C_2	C_3	Q^*	Q_1^*	$C^*(Q)$	α
K=0	[6.5, 8.5]	[25.5, 27.5]	[87.5, 91.0]	409.8136	319.412	[2184.289, 2609.049]	0.9257
K=1	[4, 10]	[21.5, 30]	[85.5, 93.5]	382.964	283.677	[1612.339, 2978.609]	0.9999
$0 < k < 1$	[5.2, 9.5]	[23.5, 28.5]	[86.5, 92.75]	413.635	326.826	[1857.894, 2801.315]	0.9184

V. CONCLUSION

In this paper, for more realistic, instead of crisp or probabilistic inventory model, we have presented an inventory model with shortage, where carrying cost, shortage cost, setup cost and demand are taken as octagonal fuzzy numbers. At first, expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantity is changed

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into interval numbers. After that, the problem of minimizing the cost function is transformed into a multi-objective inventory problem. Fuzzy optimization technique is then used to found out the optimal results. A numerical example illustrates the proposed method.

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