

MHD Mixed Convection Flow of a Casson Fluid With Convective Boundary Condition and Effect of Partial Slip In The Presence of Joule Heating Over a Vertically Stretching Sheet

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ABSTRACT: An analysis is carried out to study the mixed convection partial slip flow of a Casson fluid over a vertically stretching sheet with convective boundary condition. In particular, we investigate the effects of different parameters such as magnetic, mixed convection, Casson, convective, slip, Prandtl number in the presence of joule heating. The governing partial differential equations are transformed into ordinary differential equations by applying similarity transformations. These equations are then solved numerically by using finite difference scheme known as the Keller Box method. The effects of various parameters on velocity and temperature profiles are presented graphically.

KEYWORDS: Casson fluid, MHD, Mixed convection, Joule heating, Slip effect.

I. INTRODUCTION

Magneto hydrodynamics (MHD) is the study of the flow of electrically conducting fluids, which has important applications in polymer industry and engineering fields (Garnier [1]). Chakrabarathi and Gupta [2] discussed heat transfer of hydromagnetic flow over a stretching sheet. The study of non-Newtonian fluids has many applications in industrial and manufacturing process. Some examples are plastic polymers, optical fibres, paper production etc. In the past, so many researchers discussed various non-Newtonian models. Casson fluid model is one of the non-Newtonian models. Casson fluid was originally introduced by Casson and which explains that the rate of shear stress and strain relationships is nonlinear. Examples of such are sauce, honey, soup, jelly etc. For the analysis of boundary layer flows, this model is one of the best rheological models. MHD Casson Fluid Flow, Heat and Mass Transfer in a Vertical Channel with Stretching Wall is discussed by Sarojamma et al [3]. S Pramanik [4] discussed Casson fluid flow and heat transfer over an exponentially stretching surface. S. Nadeem et al [5] discussed MHD flow of a Casson fluid over an exponentially shrinking sheet. Krishnendu Bhattacharyya [6] studied MHD Stagnation-Point Flow of Casson Fluid over a Stretching Sheet with the effect of thermal Radiation.

In the present problem we consider the mixed convection flow and convective boundary conditions also. Mixed convection flow over a stretching sheet or surface has numerous applications in geophysics areas and engineering. Heat transfer under the convective boundary conditions has vital role in gas turbines, energy store, nuclear plant processes etc. Mixed convection MHD stagnation point flow on vertical, linearly stretching sheet is explained by Ishak [7-9]. Mixed convection flow of Casson nanofluid with convective boundary condition over a stretching sheet is explained by T.Hayat et al [10]. Ch. Ram Reddy et al [11] studied effects of joule heating and Hall effects on free convection in a Casson fluid over a vertical channel. Effect of convective boundary condition on mixed Convection stagnation point flow of Casson fluid is investigated by Mahanta and Shaw [12].

Also, in this problem we extended to joule heating and slip effects. Joule heating describes the process where the energy of an electric current is converted into heat as it flows through a resistance. Joule heating is pertinent to an electrical devices design. It is particularly relevant in terms of electrical system components such as conductors in

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electronics, electric heaters, power lines and fuses etc. Heat transfer of the mixed hydro dynamic/thermal slip and MHD flow over a vertically stretching sheet is discussed by Subhas Abel and Monayya Mareppa [13]. Mitiku Daba et al [14] described the impact of partial slip on mixed convection flow over a vertically stretching sheet with convective boundary condition. Swati Mukhopadhyay [15] investigated slip effects on MHD boundary layer flow over an exponentially stretching sheet. MHD mixed convection slip flow near a stagnation-point over a non-linearly vertical stretching sheet is discussed by Shen et al [16]. Impact of slip and convective boundary condition on boundary layer flow over a stretching sheet is studied by Nor Ishikin Abu Bakar [17]. Nooren Sher Akbar [18] described the effects of slip and convective boundary conditions on stagnation-point flow of suspended nanofluid over a stretching sheet. Free convection flow of Casson fluid on oscillating vertical plate with the effect of slip is discussed by Imran et al [19].

In this paper we described the solution to the problem of heat transfer of MHD mixed convection flow of a Casson fluid over a linearly stretching sheet under convective boundary condition and slip effects in the presence of joule heating by applying the Keller box method.

II. MATHEMATICAL FORMULATION

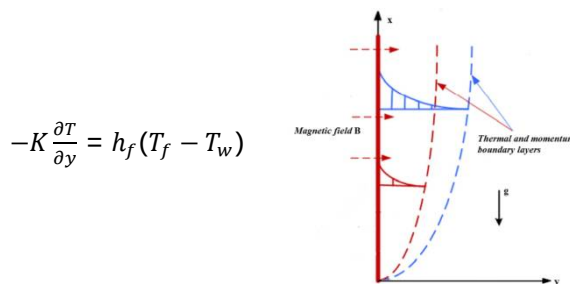


Fig 1: physical sketch of the flow

Let us consider a steady, two dimensional, laminar, incompressible MHD flow of a Casson fluid over a vertically stretching sheet of uniform surface temperature T_w . The x-axis is taken along the sheet in the upward direction and the y-axis is measured normal to the sheet as shown in the above Fig1. It is assumed that the sheet is stretched with linear velocity $u_w(x) = ax$ where 'a' is positive constant. A magnetic field B is applied in the direction normal to the sheet. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as

$$\tau_{ij} = \left(\mu_b + \frac{P_y}{\sqrt{2\pi}} \right) 2e_{ij} \quad \text{When } \pi > \pi_c \quad (1)$$

$$\tau_{ij} = \left(\mu_b + \frac{P_y}{\sqrt{2\pi_c}} \right) 2e_{ij} \quad \text{When } \pi < \pi_c \quad (2)$$

P_y is known as yield stress of the fluid, mathematically expressed as $P_y = \frac{\mu_b \sqrt{2\pi}}{\beta}$, μ_b is called as plastic dynamic viscosity of the non newtonian fluid, π is the product of the component of deformation rate with itself (i.e., $\pi = e_{ij}e_{ij}$) and (i,j)th component of the stress tensor is τ_{ij}

Considering u, v as velocity components in the directions of x and y respectively in the flow field. The governing equations of continuity, momentum, energy and concentration are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) - \frac{\sigma B^2}{\rho} u \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B^2}{\rho c_p} u^2 \quad (5)$$

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where ρ is the density of the fluid, σ is the electrical conductivity, ν is the kinematic viscosity, B is the magnetic induction, $\alpha (= \frac{K}{\rho C_p})$ is the thermal diffusivity, μ is the dynamic co-efficient of viscosity, C_p is the specific heat capacity at constant pressure, T_∞ is the free stream temperature. We consider that the left surface of the stretching sheet is heated by convection.

The boundary conditions considered are defined as

$$u = u_w + L \frac{\partial u}{\partial y}, \quad v = 0, \quad -K \frac{\partial T}{\partial y}(x, 0) = h_f(T_f - T_w) \quad \text{at } y = 0.$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty.$$

Where T_f is the temperature of fluid, $h_f = \frac{c}{\sqrt{x}}$ is the heat transfer coefficient, L = proportionality constant of the velocity slip.

Introducing the similarity transformations

$$\eta = \sqrt{\frac{a}{\nu}} y, \quad \psi(x) = \sqrt{ua} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$u = axf'(\eta)$, $v = -\sqrt{av}[f(\eta)]$ in to the equations (4) & (5) we get

$$(1 + \frac{1}{\beta})f''' + ff'' + \lambda\theta - Mf' = 0, \tag{6}$$

$$\frac{1}{Pr} \theta'' + f\theta' + Hf'^2 = 0. \tag{7}$$

Where $M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $\lambda = \frac{g\beta T(T_f - T_\infty)}{a^2 x}$ is the mixed convection parameter, $Ec = \frac{u_w x}{c_p(T_f - T_\infty)}$ is the Eckert number, $H = M.Ec$ is the Joule heating parameter.

The boundary conditions are reduced to

$$\eta = 0: f(0) = 0, f'(0) = 1 + \lambda_v f''(0), \theta'(0) = -\gamma(1 - \theta(0)).$$

$$\eta \rightarrow \infty: f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0.$$

Here $\lambda_v = L \left(\frac{a}{\nu}\right)$ is the slip parameter and $\gamma = \left\{\frac{c}{K} \left(\sqrt{\frac{\nu}{u_\infty}}\right)\right\}$ is the convective parameter.

The surface shear stress and heat transfer coefficient with constant density are given by $\tau_w = (\mu_b + \frac{P_y}{\sqrt{2\pi}}) \left(\frac{\partial u}{\partial y}\right)_{y=0}$,

$q_w = -K \left(\frac{\partial T}{\partial y}\right)_{y=0}$. We are defining the physical quantities of interest are the local skin friction $Cf_x = \frac{\sqrt{2x}}{\sqrt{Re_x}} (1 + \frac{1}{\beta}) f''(0)$ and the local Nusselt number $Nu_x = -\sqrt{Re_x} \theta'(0)$. Here $Re_x = \frac{Ux}{\nu}$ is the Reynolds number.

III. METHOD OF SOLUTION

The Keller box method is an effective method to solve ordinary and partial differential equations, which is described by Cebeci and Bradshaw [20]. This method involves the following steps.

Step1: Reducing higher order ODEs (systems of ODES) in to systems of first order ODEs.

Step2: Writing the systems of first order ODEs into difference equations using central difference scheme.

Step3: Linearizing the difference equations using Newton's method and writing it in vector form.

Step4: Solving the system of equations using block elimination method.

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IV. NUMERICAL DISCUSSION

The nonlinear ordinary differential equations 6 and 7 subjected to boundary conditions were solved numerically using the Keller box method. In this study, the step size we have used is $\Delta\eta = 0.01$. The following table1 shows that the values calculated by this method and of Magyari and Keller [21].

Table 1: Comparison of the values of $-\theta'(0)$ for different values of Pr with $\lambda=\gamma=\lambda_v=M=H=0$, $\beta \rightarrow \infty$ with previous work.

Pr	Present	Magyari and Keller[21]
0.5	-0.5946	-0.59434
1.0	-0.9548	-0.95478
3.0	-1.8691	-1.86908
5.0	-2.5002	-2.50014

V. RESULTS AND DISCUSSION

In this we discussed graphical results with/without the effect of joule heating parameter. Fig 2 explains the velocity field for different values of the magnetic parameter M. The boundary layer thickness decreases with higher values of the M because of the Lorentz force. Influence of mixed convection parameter on the velocity field is represented in Fig3. With the effect of buoyancy the fluid flow accelerates. Therefore the velocity increases.

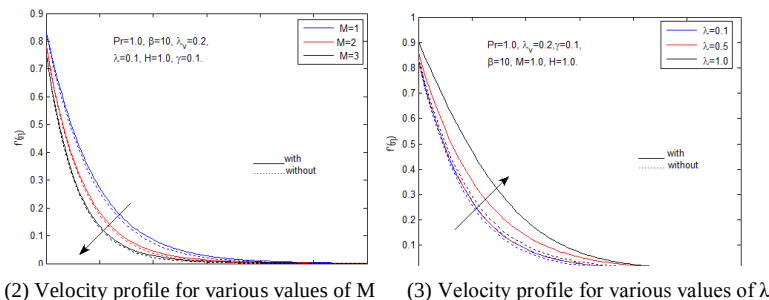
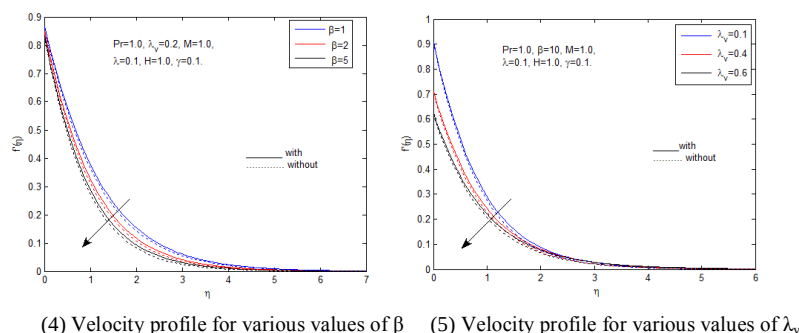


Fig 4 shows that the profile of velocity for the values of Casson parameter β . For this case, the boundary layer thickness decreases with the increase of β . Fig5 depicts that the velocity field for the values of slip parameter λ_v . The velocity is found to decrease with the increasing values of λ_v .



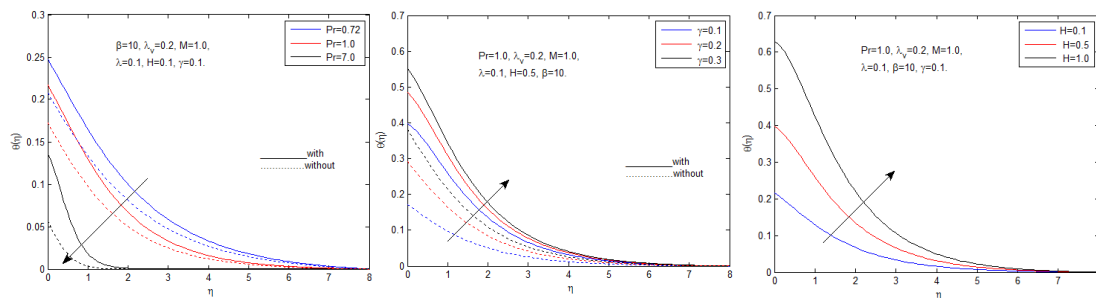
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Fig 6 demonstrates that effect of Prandtl number Pr on the field of temperature. Prandtl number can be defined as momentum diffusivity to thermal diffusivity. Therefore temperature profile decreases. Fig 7 indicates that impact of convective parameter on the profile of temperature. Therefore the boundary layer thickness increases. Fig 8 demonstrates that influence of the H on the field of temperature. It is observed that the temperature profile increases as the value of the H increases. The impact of the joule heating parameter in the velocity profile is very less as we can observe from the figures 2, 3, 4, 5. Similarly on the temperature profile is absurd from the figures 6 and 7. There is some change in the boundary layer thickness as well as in the boundary layer region in the presence/absence of joule heating parameter.



(6) Temperature profile for the values of Pr (7) Temperature profile for various values of γ (8) Temperature profile for various values of H

VI. CONCLUSION

- An increase in the magnetic parameter M and Casson parameter β decreases the velocity field.
- The velocity field increases with increasing values of mixed convection parameter λ and decreases for slip parameter.
- An increase in the value of Pr reduces the temperature field.
- With increasing values of convective parameter γ and joule heating parameter H , the temperature field increases.

REFERENCES

- [1] Garnier M, Magneto hydrodynamics in materials processing, Philos Trans R Soc Phys Sci Eng vol. 344, pp.249–263, 1962.
- [2] Chakrabarti A, Gupta AS, Hydromagnetic flow and heat transfer over a stretching sheet, Quart Appl Math, vol. 37, pp 73-78, 1979.
- [3] G Sarojamma, B Vasundhara, K Vendabai, MHD Casson fluid flow, Heat and mass transfer in a vertical channel with stretching walls, International Journal of Scientific and Innovative Mathematical Research, vol. 2, no. 10, pp. 800-810, 2014.
- [4] S Pramanik, Casson fluid flow and heat transfer past an exponentially stretching surface in presence of thermal radiation, vol. 5, no. 1, pp. 205-212, 2014.
- [5] S Nadeem, Rizwana Ul Haq, C Lee, MHD flow of a Casson fluid over an exponentially shrinking sheet, vol. 19, no. 6, pp. 1550-1553, 2012.
- [6] Krishnendu Bhattacharya, MHD stagnation point flow of Casson fluid and heat transfer over a stretching sheet with thermal radiation, Journal of Thermodynamics, 2013, Article Id 169674, 2013.
- [7] Ishak A, Nazar R, Pop I, Mixed convection on the stagnation point flow toward a vertical, continuously stretching sheet, J. Heat Transfer, vol. 129, no. 8, pp. 1087-1090, 2007.
- [8] Ishak A, Nazar R, Pop I, Post stagnation point boundary layer flow and mixed convection heat transfer over a vertical, linearly stretching sheet. Arch. Mech, vol. 60, pp. 303-322, 2008.
- [9] Ishak A, Jafar K, Nazar R, Pop, MHD stagnation point flow towards a stretching sheet, Physics A: Statistical Mechanics and its Applications, vol. 388, no. 17, pp. 3377-3383, 2009.
- [10] T Hayat, M Bilal Ashraf, S A Shehzad and A Alsaedi, Mixed convection flow of Casson nanofluid over a stretching sheet with convectively heated chemical reaction and heat source/sink, Journal of Applied Fluid Mechanics, vol. 8, no. 4, pp. 803-813, 2015.
- [11] Ch Ram Reddy, Ch Venkata Rao, O Surendar, Soret, Joule heating and hall effects on free convection in a Casson fluid saturated porous medium in a vertical channel in the presence of viscous dissipation, International conference on computational heat and mass transfer, Procedia Engineering, vol. 127, pp. 1219-1226, 2015.

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- [12] G Mahanta, S Shaw, Mixed convection stagnation point flow of Casson fluid with convective boundary conditions, International journal of Innovative Research in Science, Engineering and Technology, vol. 4, no. 2, pp. 370-379, 2015.
- [13] M Subhas Abel and Monayya Mareppa, MHD flow and heat transfer of the mixed hydrodynamic/thermal slip over a linear vertically stretching sheet, International Journal of Mathematical Archive, vol. 4, no. 5, pp. 156-163, 2013.
- [14] Ming Shen, Fei Wang and Hui Chen, MHD mixed convection slip flow near a stagnation point on a non-linearly vertical stretching sheet, Boundary value problems, 2015:78, 2015.
- [15] Mitiku Daba, P Devaraj, S V Subhasini, Mixed convection boundary layer flow over a vertically stretching sheet with convective boundary condition and effect of partial slip, Frontiers in Heat and Mass Transfer, vol. 6, Issue 3, 2015.
- [16] Swati Mukhopadhyay, Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation, Ain Shams Engineering Journal, vol. 4, no. 3, pp. 485-491, 2013.
- [17] Nor Ashikin Abu Bakar, Wan Mohd Khairy Adly Wan Zaimi, Rohana Abdul Hamid, Biliana Bidin and Anuar Ishak, Boundary layer flow over a stretching sheet with a convective boundary condition and slip effect, World Applied Sciences Journal, vol. 17, pp. 49-53, 2012.
- [18] Noreen Sher Akbar, Zafar, Hayat Khan, Sohail Nadeem, The combined effects of slip and convective boundary conditions on stagnation point flow of CNT suspended nanofluid over a stretching sheet, Journal of Molecular Liquids, vol. 196, pp. 21-25, 2014.
- [19] Muhammad A Imran, Shakila Sarwar and Muhammad Imran, Effects of slip on free convection flow of Casson fluid over an oscillating vertical plate, Boundary value problems, 2016:30, 2016.
- [20] T Cebeci, P Bradshaw, Physical and computational aspects of convective heat transfer, Springer-Verlag, 1984.
- [21] E Magyari, B Keller, Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, Journal of Physics D vol. 32, no. 5, pp. 577-585, 1999.