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Solution of Unsteady Flow Equations in High Pressure Pipe

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ABSTRACT: In hydro-electric project, when turbine valves are closed, the pressure rise in the conduit may cause a high hammer pressure. This study is concerned with the prediction of water pressure in pipe. The governing equations of continuity and momentum are non-linear. Numerical methods used are the 'Method of Characteristics' and the 'Lax Diffusive Method'. A model solution has been developed to solve for pressure head and discharge at different pipe sections. Assessment of friction factor is made by Fang's and Romeo's equations. Damping of discharge and dynamic pressure with time are shown. Identical results are obtained by both the methods. Use of Fang's and Romeo's equation is a new approach in finding friction factor, which suggests the validity of the equations. From relationship between friction and time, it is observed that average friction increases with time and shooting its value at regular intervals. Comparisons of the solutions are made with data of other investigators.

KEYWORDS: friction factor, unsteady flow, water hammer pressure, surge tank.

I. INTRODUCTION

This study work is the condition and situation with the occurrence of water hammer pressure in a long pipe line without a surge tank at the downstream of hydro power plant. The pipe is connected at the upstream to an infinite reservoir and in downstream, just close to the power house, a closing valve is fitted without a surge tank. Due to sudden closure of this valve, water hammer pressure in the pipe may be dangerous and its analysis is also very complex. The magnitude of this pressure rise depends on various factors such as elastic properties of pipe materials, length and diameter of pipe, frictional resistance, discharge, valve closure time etc. The friction factor plays an important role in the calculation of the pressure. Again in this situation, the unsteady flow equations of continuity and momentum are nonlinear. Thus numerical solution in this situation is essential. Correct assessment has also been made by finding friction factor value at every time step of solution.

II. RELATED WORK

Review of the literare on the solution of the basic equations shows that Schnyder (1932) and Bergeron (1936) developed graphical methods independently. Due to non-linearity, continuity and momentum equations in unsteady flow situation could not be solved analytically or numerically. Schnyder - Bergeron diagram for pressure surge analysis aided the approximate solution of the surge problems of the above situation. Allievi (1925) gave classical solution of the governing equations and the solution was for pressure head (H) only. Angus (1937) advocated that the Schnyder — Bergeron diagram was indispensable for design engineers. Jaeger (1956) also gave a similar type of classical solution as Allievi. In their classical solutions friction was neglected which is the main parameter of damping of pressure and



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discharge. Pickford (1969) compared the graphical and Allievi's classical solution and he suggested that graphical methods are better than Allievi's solution, as it gives solutions for both pressure (H) and discharge (Q) at different points of the pipe line at different time. The foregoing graphical and analytical method has been replaced by digital computer methods since the beginning of 1960's. Streeter (1969) used the Method of Characteristic to solve the equations, but he did not carry the solution to further increase of time to observe the damping. Shimada and Okushima (1984) proposed a new numerical model solution technique for water hammer pressure by series solution methods and Newton-Raphson methods. Chudhury and Hussaini (1985) solved the water hammer equations by MacCormack, Lambda, and Gabutti explicit FD schemes. Simpson and Wylie (1991) investigated the variation, magnitude and shape of short duration pressure pulses for a simple reservoir valve system connected by a hypothetical frictionless pipe. Sibetheros et al. [11] investigated the method of characteristics (MoC) with spline polynomials for interpolations required in numerical water hammer analysis for a frictionless horizontal pipe. Chatterjee (1995) solved the equations by Method of Characteristics. Silva-Arya and Choudhury (1997) developed a new model for computation of unsteady friction loss in transient flow and solved the hyperbolic part of the governing equation by MoC in 1D form and the parabolic part of the equation by quasi in 2D form. Pezzinga (1999) developed a quasi 2-D stress model for unsteady flow analysis in pipe networks using finite difference implicit scheme. Head oscillations were solved with constant friction for 4 seconds only. Ghidaoui and Kolyshikim (2001) worked on laminar and turbulent water-hammer flows and found that Reynolds numbers and dimensionless timescale are the main parameters for stability of transient flows and the results obtained were plotted in Reynolds number versus timescale space. Ghidaoui et al. (2000) analyzed two layer and five layer eddy viscosity models of water hammer. Bergant et al. (2001), studied on water hammer analysis with Method of Charecteristic (MoC). Zhao and Ghidaoui (2004) solved a quasi 2D model for turbulent flow in water hammer. They used turbulent shear stress as resistance instead of friction factor. Zhao and Ghidaoui (2004) formulated, applied and analyzed first and second order explicit finite volume (FV) Godunov-type schemes for water hammer problems. They have compared both the FV schemes with MoC. Das Saikia and Sarma (2006) developed a model using MoC with Barr's resistance equation in a particular reservoir and pipe length of 12000ft. They compared discharge solution with available solution of Streeter. Some discrepancy with Streeter solution is observed. Thus the review on previous work shows that a significant amount of research on this problem has been undertaken using graphical, classical and numerical methods. Therefore in this study, two numerical methods, method of characteristics (MoC) and Lax Diffusive Finite Difference method (Lax F D) are developed for solution of the basic equations. Fang's (2011) and Romeo's (2008) explicit C-W equation are used for friction factor calculation at every time step. Medhi Das and Sarma (2016) and Medhi Das et al. (2016) used these two resistance equations successfully in surge height analysis in unsteady situation. Both MoC and Lax FD show almost same result. The results obtained are also compared with Streeter's solution and Das Saikia's solution for verification of the proposed model solution.

III. BASIC EQUATIONS OF UNSTEADY FLOW



(a) static position of flow when the valve is closed initially



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(b) steady state of flow when the valve is open



(c) unsteady state of flow when the valve is closed

Fig.1 Schematic illustration of different flow state in static, steady and unsteady situations

Figures 1 (a), (b) and (c) show the different flow situations, while flowing from reservoir to powerhouse. Figure 1(a) shows the static position of flow, before opening the valve. Figure 1(b) shows the steady state flow with head loss ($h_{j_{a}}$, when the valve is opened. Figure 1(c) shows the unsteady flow situation just after valve closure. Pressure rises even more than the static head of the reservoir and pressure wave moves upstream. The whole pipe length is divided into four pipe sections and therefore in above figures, points 1, 2, 3, 4 and 5 represents the points of the pipe sections from reservoir end.

The continuity and momentum equations in unsteady flow, due to closure of valve at x=L is given respectively as:

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0$$
(1)
$$\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{iQ[Q]}{2gDA^2} = 0$$
(2)

where H = pressure head, A = area of pipe, a = velocity of pressure wave, Q = discharge, g = acceleration due to gravity, t = time, f = friction factor, D = diameter of pipe or conduit, x = distance along the pipe. Equations (1) and (2) are non-linear partial differential equations and solution for pressure head (H) and discharge (Q) cannot be obtained analytically. The above two equations are applicable to both horizontal and slopping pipe lines.

The different numerical methods of solution of the continuity and momentum equations in pressure conduit, without a surge tank are: 1.Central difference scheme, 2.Lambda scheme, 3.Gabutti scheme, 4.Lax diffusive scheme, and 5. Mac Cormack scheme.



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IV. METHOD OF CHARACTERISTIC (MoC)

In Method of Characteristic, the partial differential equations (1) and (2) transforms into ordinary differential equations (3) and (4) along characteristic line for solution of pressure head (H) and discharge (Q),

$$= \frac{1}{2} (H_{A} + H_{B}) + \frac{1}{2} \frac{a}{gA} (Q_{A} - Q_{B}) + \frac{f\Delta t}{DA^{2}} (Q_{B} |Q_{B}| - Q_{A} |Q_{A}|)$$
(3)
$$= \frac{1}{2} (Q_{A} + Q_{B}) + \frac{1}{2} \frac{gA}{a} (H_{A} - H_{B}) - \frac{f\Delta t}{4\Omega A} (Q_{A} |Q_{A}| + Q_{B} |Q_{B}|)$$
(4)

From the equations (3) and (4) for known values of H and Q at points (A and B in figure 4) for initial conditions at t = 0, the values of H and Q can be calculated for the next time step (say at P in figure 4).



Fig.2 Finite Difference Grid in x-t Plane by MoC

Figure (2) shows the finite difference grid in x-t plane, along positive and negative characteristics, (nd c^-) by Method of Characteristic, where x = distance along the direction of flow and t = time. The finite difference form of the above equations (3) and (4) can be derived by putting,

$$H_{p} = H_{k}^{j+1}, \ H_{A} = H_{k-1}^{j}, \ H_{B} = H_{k+1}^{j}, \ Q_{p} = Q_{k}^{j+1}, \ Q_{A} = Q_{k-1}^{j}, \ Q_{B} = Q_{k+1}^{j} \text{ and }$$
written for H and P as:
$$H_{k}^{j+1} = \frac{1}{2} \left(H_{k-1}^{j} + H_{k+1}^{j} \right) + \frac{1}{2} \frac{a}{gA} \left(Q_{k-1}^{j} - Q_{k+1}^{j} \right) + \frac{1}{2} \frac{af\Delta t}{2gDA^{2}} \left[Q_{k+1}^{j} |Q_{k+1}^{j}| - Q_{k-1}^{j} |Q_{k-1}^{j}| \right]$$
And
$$I_{k}^{j+1} = \frac{1}{2} \left(I_{k-1}^{j} - I_{k+1}^{j} \right) + \frac{1}{2} \frac{gA}{gA} \left(Q_{k-1}^{j} - Q_{k+1}^{j} \right) + \frac{gA}{2} \frac{af\Delta t}{2gDA^{2}} \left[Q_{k+1}^{j} |Q_{k+1}^{j}| - Q_{k-1}^{j} |Q_{k-1}^{j}| \right]$$
(5)

 $\mathbf{Q}_{k}^{j+1} = \frac{1}{2} \left(\mathbf{Q}_{k-1}^{j} + \mathbf{Q}_{k+1}^{j} \right) + \frac{1}{2} \frac{\mathbf{g}\mathbf{A}}{\mathbf{a}} \left[\left(\mathbf{H}_{k-1}^{j} - \mathbf{H}_{k+1}^{j} \right) - \frac{\mathbf{a}\mathbf{L}\mathbf{M}}{4\mathbf{g}\mathbf{D}\mathbf{A}^{2}} \left(\mathbf{Q}_{k-1}^{j} \left| \mathbf{Q}_{k-1}^{j} \right| + \mathbf{Q}_{k+1}^{j} \left| \mathbf{Q}_{k+1}^{j} \right| \right) \right]$ (6) Thus the equations (5) and (6) are the finite difference forms of equations (3) and (4) and are used to calculate the

transient pressure head (H) and discharge (Q) at fixed and equally spaced interior sections throughout the pipe from known initial values at $t = t_{Q}$, along the abscissa axis. After having calculated Q and H at all interior points at time $t = t_1$, boundary values at upstream and downstream points are to be imposed. Then values at $t = t_1$ becomes the initial values to calculate Q and H for the next time $t_2 = t_1 + \Delta t$ and solutions thus are advanced in time until the time t_{final} will be reached.



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V. LAX DIFFUSIVE FINITE DIFFERENCE EXPLICIT METHOD

In finite difference form equation (1) and (2) is written as:

$$H_{k}^{j+1} == \frac{1}{2} (H_{k-1}^{j} + H_{k+1}^{j}) - \frac{\Delta t a^{2}}{gA} \cdot \frac{1}{2\Delta x} (Q_{k+1}^{j} - Q_{k-1}^{j})$$
And

$$Q_{k}^{j+1} = \frac{1}{2} (Q_{k-1}^{j} + Q_{k+2}^{j}) - \frac{gA\Delta t}{2\Delta x} (H_{k-1}^{j} + H_{k+2}^{j}) - \frac{f\Delta t}{gDA} (Q_{k-1}^{j} + Q_{k+1}^{j}) |Q_{k-1}^{j} + Q_{k+2}^{j}|$$
(8)

The method of calculation of Q and H at interior points is same as that of MoC. Thus equations (7) and (8) are used to calculate the transient pressure head and discharge in different cross-sections of the pipe with increasing time step.

VI. BOUNDARY CONDITIONS

The above equations are not applicable at the boundary points since there are no grid points outside the flow domain.

VII. DOWNSTREAM BOUNDARY CONDITIONS (at X = L)

At downstream (i.e. at x = L), the values of P and Q after first step are Q_L^{j+1} and H_L^{j+1} and to evaluate these values, principle of orifice during value closure time are used for discharge Q_L^{j+1} under the head of H_L^{j+1} as:

$$Q_{L}^{j+1} = (C_{d}A_{v}) \sqrt{2g H_{L}^{j+1}}$$
(9)

Where C_{d} = co-efficient of discharge and A_{u} = area of the value at the time j+1.

$$\mathbf{H}_{\mathbf{L}}^{j+1} = \mathbf{H}^{j}_{\mathbf{L}-\Delta \mathbf{X}} + \frac{a}{gA} \mathbf{Q}^{j}_{\mathbf{L}-\Delta \mathbf{X}} - \frac{a}{gA} \mathbf{Q}_{\mathbf{L}}^{j+1} - \frac{at\Delta t}{gA^{2}(2D)} \mathbf{Q}^{j}_{\mathbf{L}-\Delta \mathbf{X}} |\mathbf{Q}^{j}_{\mathbf{L}-\Delta \mathbf{X}}|$$
(10)

The above two equations (9) and (10) have two unknowns Q_L^{j+1} and H_L^{j+1} and solving these simultaneous equations,

$$\mathbf{Q}_{\mathrm{L}}^{j+1} = \frac{(\mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{v}})^{2}\mathbf{a}}{\mathbf{A}} \left[1 + \frac{\frac{2\left(\mathbf{H}^{j}_{\mathrm{L}-2X} + \frac{\mathbf{a}}{g\mathbf{A}}\mathbf{Q}^{j}_{\mathrm{L}-2X} - \frac{\mathbf{a}T\mathbf{D}\mathbf{X}}{2g\mathbf{D}\mathbf{A}^{2}}\mathbf{Q}^{j}_{\mathrm{L}-2X} |\mathbf{Q}^{j}_{\mathrm{L}-2X}|\right)}{\frac{\mathbf{C}_{\mathrm{d}}^{2}\mathbf{A}_{\mathrm{v}}^{2}\mathbf{a}^{2}}{g\mathbf{A}^{2}}} \right]$$
(11)

And

$$H_{L}^{j+1} = H^{j}_{L-\Delta X} + \frac{a}{gA} Q^{j}_{L-\Delta X} - \frac{af\Delta t}{2gDA^{2}} Q^{j}_{L-\Delta X} |Q^{j}_{L-\Delta X}| - \frac{a}{gA} Q_{L}^{j+1}$$
(12)

Equations (11) and (12) are used to calculate boundary values at x=L upto previous time step of complete value closure.

When the valve is completely closed,

$$Q_L^{j+1} = 0 \text{ and } A_v = 0$$
 (13)

$$H_{L}^{j+1} = H^{j}_{L-\Delta X} + \frac{a}{gA} Q^{j}_{L-\Delta x} - \frac{ar\Delta t}{2gDA^{2}} (Q^{j}_{L-\Delta x} | Q^{j}_{L-\Delta x} |)$$
(14)

Thus equations (13) and (14) are used to calculate the boundary values at downstream.

VIII. UPSTREAM BOUNDARY CONDITIONS (at X = 0)

At upstream (i.e. at x = 0), reservoir is assumed to be infinitely large reservoir and hence there is no effect of backward and forward movement of flows through conduit and the pressure head remains constant,



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 $\begin{aligned} H_{\mathcal{H}=0}^{j+1} &= H_{0} \end{aligned} \tag{15} \\ \text{Where } H_{0} &= \text{constant initial head at reservoir and discharge is computed along the negative characteristic line (c⁻) as: \\ Q_{\mathcal{H}=0}^{j+1} &= \frac{gA}{a} \left[H_{0} - H_{0+\Delta \pi}^{j} + \frac{a}{gA} Q_{0+\Delta \pi}^{j} - \frac{af\Delta \tau}{2gDA^{2}} Q_{0+\Delta \pi}^{j} |Q_{0+\Delta \pi}^{j}| \right] \end{aligned} \tag{16} \\ \text{Thus boundary values at upstream reservoir are calculated by equations (15) and (16)} \end{aligned}$

IX. COMPUTER PROGRAM FOR NUMERICAL SOLUTION

A computer program has been developed both for MoC and Lax F D to solve for discharge (Q) and pressure heads (H) at different cross-sections of the pipe at various time intervals during unsteady flow situations. The explicit Colebrook-White equations of Barr's, Fang's and Romeo's are used to calculate the friction factor at the desired time steps. A particular problem is chosen to illustrate the solution of the equations. The numerical values are : length L = 3658m, diameter d = 0.65m, steady Discharge Q =0.56634 m³/se, steady heads at pipe sections 1, 2, 3, 4 and 5 are 182.88m, 177.55m, 172.21m, 166.88m and 161.54m respectively, time interval Δt = 1sec, valve closing time = 4sec, roughness size k = 0.002162, Length of a pipe section (Δx) = 914.5m (as pipe length is 3658m and is divided into 4 sections).

X. STEPS FOR CALCULATIONS IN THE PROGRAM

- The pressure heads (H) and discharges (Q) at the interior grid points, represented by the figure 4, are calculated by equations (5) and (6) for MoC and equations (7) and (8) for Lax F D method.
- The resistance equation to evaluate friction factor at every time step is taken as explicit Colebrook-White equation of

Barr:
$$\frac{1}{\sqrt{t}} = -2\log_{10}\left[\frac{k}{2.710} + \frac{5.1296}{R^{0.89}}\right]$$
 (17)

Fang:
$$f = 1.613 \left[\ln \left(.234 \left(\frac{k}{D} \right)^{1.1007} - \frac{60.535}{R^{1.1105}} + \frac{50.291}{R^{1.0712}} \right) \right]^{-2}$$
 (18)

Romeo:
$$\frac{1}{\sqrt{t}} = -2\log_{10}\left(\frac{k}{2.7065 \text{ D}} + \frac{5.0272\text{ B}}{\text{R}}\right)$$
 (19)

Where

$$A = \log_{10} \left[\left(\frac{k}{7.791 \text{ sD}} \right)^{0.9924} + \left(\frac{5.3326}{208.815 + \text{R}} \right)^{0.9245} \right]$$
(20)

$$B = \log_{10} \left(\frac{k}{8.827D} + \frac{4.507A}{R} \right)$$
(21)

- After calculations at interior points, boundary conditions are imposed for discharges and pressure heads by equations (9) to (14).
- Thus values of H and Q at time $t_1 = t_0 + \Delta t$ at different cross-sections are obtained. These values are then converted to initial values for next time $t_2 = t_1 + \Delta t$ and computations of discharges, pressure heads and friction factors at interior and boundary points are made by the same process. Again these values are initialized for the next time step $t_2 = t_2 + \Delta t$ and solution is advanced in time upto the required time step of t_{final} .



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XI. RESULTS AND DISCUSSIONS

The results obtained from calculations are then stored in phased manner, for producing them in tabulated and • graphical format using computers. The values can be presented for discharge, pressure head and friction factor at different cross-sections against corresponding time steps.



Fig.3 Discharge (Q) vs. time (t) at all pipe sections by MoC with Fang's resistance equation

Figure 3 shows the variation of discharge (Q) with increasing time (t) up to 200 seconds at all cross sections of the pipe by MoC method using Fang's resistance equation. Damping effect is clear with increasing time.



Fig.4 Pressure head (H) vs. time (t) at all pipe sections by MoCwith Fang's resistance equation

Figure 4 shows the variation of pressure heads (H) with increasing time (t) up to 200 seconds at all cross sections of the pipe by MoC using Fang's resistance equation.



Fig.5 Discharge (Q) vs. time (t) at all pipe sections by MoC with Romeo's resistance equation



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Similarly figure 5 show the variation of discharge (Q) against time (t) up to 120 seconds at all the cross sections of the pipe by the method MoC using Romeo's resistance equation. It is seen that even in this small time 120 seconds, damping of discharge (Q) in all the pipe sections are quite significant.



Fig.6 Pressure head (H) vs. time (t) at all pipe sections by MoC withRomeo's resistance equation

In figure 6, similar plot of pressure head (H) is presented for increasing time up to 300 seconds and damping of the parameter in the figure is then very much pronounced. Solutions for discharge and pressure head both by Fang's and Romeo's resistance equation at all the cross-sections shows almost the same result. Peak of discharge by Romeo's resistance equation are slightly greater and pressure peaks by Romeo's equation are slightly less than by Fang's equation.



Fig.7 Discharge (Q) vs. time (t) at all pipe sections by Lax F D with Fang'sresistance equation

Figure7 shows the positive and negative flow within the pipe with time at all the cross-sections by Lax F D method using Fang's resistance equation. Damping effect is also clear as time increases. MoC and Lax F.D. method both have given approximately the similar plot.



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Fig.8 Pressure head (H) vs. time (t) at all pipe sections by Lax F D with Fang's resistance equation

Figure8 shows the plot of pressure head (H) against time upto 120 seconds by Lax F D method with Fang's resistance equation. It shows the rise and fall of pressure head above and below the reservoir level and damping effect with time.



Fig.9 Discharge (Q) and Pressure head (H) against time (t), at all pipe sections by Lax F D with Romeo's resistance equation

In figure 9 (a) and (b), similar plots of discharge and pressure head are presented for increasing time by Lax F D method with Romeo's resistance equation respectively.



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Fig.10 Friction factor (f) vs. time (t) at pipe section 2 by MoC with Fang'sresistance equation

Figure 10 shows how friction factor (f) varies with increasing time of solution at different cross-sections using Fang's resistance equation. Abrupt increase in friction factor values is displayed in the figure when flow direction begins to change.



Fig.11 Comparative plot of friction factor (f) vs. time (t) at pipe section 2 by both MoC and Lax F D for Romeo's resistance equation

In figure 11, a comparative plot of friction factor against time between MoC and Lax F.D. method using Romeo's friction factor at cross section 2 is presented. In this plot both the methods give the same pattern of variation of friction factor. In the comparison of solutions by the MoC and Lax F D methods, almost similar results are observed. Lax F D method gives slightly greater peaks in case of discharge and lesser peaks for pressure. However from the plots it appears that MoC shows better result than Lax F D method.

XII. COMPARISON OF AUTHORS SOLUTION WITH STREETER'S SOLUTION AND DAS SAIKIA'S SOLUTION

Streeter solved the same nonlinear equations by different methods. He presented his solutions up to only 50 seconds. The results presented by Streeter have been adopted to assess the author's results in a comparative plot. Das Saikia also solved these equations by MoC method with Barr's resistance equations. Thus comparisons are also made with Das Saikia's solution.



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Fig.12 Comparative plot of discharge (Q) vs. time (t) at section 3, between authors solution (by MoC and Lax F D) and Streeter's solution

Figure12 shows a comparative plot of discharge solution up to 50 seconds, by MoC and Lax F.D methods with Streeter's solution using Romeo's resistance equation.



Fig.13 Comparative plot of discharge (Q) vs. time (t) at reservoir section, between authors solution by Lax F D method and Streeter's solution using all the three resistance equations.

In Figure 13, discharge solution by Lax F.D method is compared with Streeter's solution up to 50 seconds using all the three resistance equations. With Streeter's solution comparison tends to match as the time increases though slight differences in frequency is observed only during initial few seconds (about 20 seconds). Streeter solution gives somewhat higher peaks. This is probably due to the use of constant friction factor adopted by Streeter and variable friction factor adopted by the author in the present study.



Fig.14 Comparative plot of discharge (Q) vs. time (t) at reservoir section, with Streeter's and Das Saikia's solution by MoC using Fang's and Romeo's resistance equation



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Figure 14 shows the comparison of discharge solutions with available Streeter's solution and Das Saikia's solution using MoC method. With Das Saikia's solution, comparison appears to be quite satisfactory with both the methods using Fang's and Romeo's friction factor equations. On the whole, these comparisons give a satisfactory verification of the proposed model solution.

XIII. CONCLUSION

The study is conducted by the developing of numerical solutions of the nonlinear equations (1) and (2). Positive and negative flows, rise and fall of pressure within the pipe line are satisfactorily presented and damping of these two parameters with increase of time is distinct in the plots. Higher undulations of discharges at peak values at cross-section number 1 i.e. at reservoir sections is observed which is practically quite expected as the backward and forward flows at this reservoir section meet the constant head infinite reservoir, such fluctuations are quite possible in this boundary point of the pipe. Fluctuations of discharges at interior sections are a bit smaller as compared to cross section no 1.

In case of pressure head plots, at valve ends, in both the methods fluctuation of pressure head at peak values are observed. It is not unexpected that these should be same unsteady peak values at valve ends due to sudden obstruction. Plots of pressure heads at the interior points are smooth. It appears from the plots that the Romeo's correlation is little better than Fang's correlation. Romeo's correlation gives slightly higher values of discharges at peak and slightly lower values of pressure at peak. Although both shows almost nearer values. In transient flow stage a quick change of velocity takes place. At the time of change of velocity direction, Reynolds number becomes very small. Friction factor value shoots up at this stage. Therefore, this increase of friction factor indicates the use of variable friction factor.The part of study can suggest the future investigators not to use constant friction factor values which may lead to wrong solutions. Resistance to flow is an important phase of any analysis and therefore, it is always advisable to assess the resistance at every stage of flow.

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