

# Effects of Heat Source/Sink on MHD Flow of Casson Fluid and Heat Transfer Over an Unsteady Stretching Sheet

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**ABSTRACT:** In this analysis, effects of heat source/sink on MHD flow of Casson fluid and heat transfer over an unsteady stretching sheet is studied. Similarity transformations are used to convert the governing time dependent non-linear boundary layer equations into non-linear ordinary differential equations and solved it numerically by Runge-Kutta fourth order method with a shooting technique. The numerical predictions have been compared with already published papers and good agreement is obtained. The governing non-dimensional parameters on velocity and temperature are presented through graphs while local skin friction coefficient and Nusselt number are represented numerically.

**KEYWORDS:** MHD, Casson fluid, unsteady stretching sheet, Heat source/sink.

## I. INTRODUCTION

The study of non-Newtonian fluids has a variety of applications in engineering and industry especially in extraction of crude oil from petroleum products. Casson fluid is a non-Newtonian fluid which exhibits yield stress. Human blood can also be treated as a Casson fluid due to the blood cells' chain structure and the substances contained like protein, fibrinogen, rouleaux etc. Hence the Casson fluid has its own importance in scientific as well as in engineering areas. Heat transfer characteristics of Casson fluid flow through an exponentially stretching sheet in presence of porous medium and thermal radiation was discussed by Pramanik [1] and concluded that an increase in the value of Casson parameter suppresses the velocity field.

Gnanaswara Reddy [2] investigated an unsteady two-dimensional flow of a non-Newtonian fluid past a stretching surface in presence of thermal radiation and variable thermal conductivity. Khalid et al. [3] discussed an unsteady free convection MHD flow of Casson fluid through an oscillating vertical plate embedded in a porous medium with constant wall temperature. An unsteady two-dimensional Casson fluid flow of non-Newtonian fluid past a stretching surface in presence of porous medium was illustrated by Kirubhashankar et al. [4]. Mukhopadhyay et al. [5] analysed Casson fluid flow over an unsteady stretching surface. Bhattacharyya et al. [6] analysed the effects of slip on unsteady boundary layer stagnation point flow past a stretching sheet. Hayat et al. [7] discussed the three-dimensional flow of Jeffery fluid past a stretching surface. Partha et al. [8] analysed the effect of viscous dissipation in the mixed convection flow from an exponentially stretching surface in a quiescent viscous fluid.

El-Aziz [9] investigated viscous dissipation effect on mixed convection flow of micropolar fluid passing through an exponentially stretching sheet. Pal [10] reported mixed convection flow passing through an exponentially stretching surface in presence of a magnetic field. Kuppala et al. [11] investigated unsteady MHD convective heat and

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mass transfer of a Casson fluid past a semi-infinite vertical permeable moving plate with heat source/sink. The study of unsteady magneto-hydrodynamic boundary layer flow over a stretching surface with viscous dissipation and Joule heating have been investigated by Jat and Chaudhary [12].

Nadeem et al. [13] studied the numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles. Nadeem et al. [14] investigated the MHD three dimensional Casson fluids towards past a porous linearly stretching sheet. Sakiadis [15] initiated the study of the boundary layer flow over a stretched surface moving with constant velocity and formulated boundary layer equations for two-dimensional and axisymmetric flows. Anjali Devi and Ganga [16] have considered the viscous dissipation effects on MHD flows past stretching porous surfaces in porous media. Working fluid heat generation or absorption effects are important in certain porous media applications.

Cortell [17] studied flow and heat transfer behaviour of a fluid through a porous medium over a stretching sheet with internal heat generation or absorption and suction or blowing. Elbashbeshy and Emam [18] studied the effects of thermal radiation and heat transfer over an unsteady stretching surface embedded in a porous medium in the presence of heat source or sink, when the surface is kept at the constant temperature. Recently, Chaudhary et al. [19] studied the effects of thermal radiation on hydromagnetic flow over an unsteady stretching sheet embedded in a porous medium in the presence of heat source or sink.

Motivated by the above mentioned investigations and applications, the main purpose of the present paper is to extend the problem of Chaudhary et al. [19] and include Casson fluid with heat transfer over an unsteady stretching sheet. The governing equations are solved numerically by Runge-Kutta fourth order method with shooting technique.

## II. MATHEMATICAL FORMULATION

Consider an unsteady, two-dimensional incompressible Casson fluid flow over an unsteady stretching sheet. The  $x$ -axis is taken along the continuous stretching surface embedded in the direction of motion with the slot as the origin, and  $y$ -axis is perpendicular to it and the flow is confined in half plane  $y > 0$ . A uniform magnetic field of strength  $B_0$  is assumed to be applied normal to the stretching surface as, shown in Fig. 1. The magnetic Reynolds number is taken to be very small so the induced magnetic field is negligible. The surface is assumed to be highly elastic and is stretched in the  $x$ -direction with surface velocity  $U_w = \frac{bx}{1-\gamma t}$  and surface temperature

$$T_w = T_\infty + \frac{b}{2\nu x^2} \frac{1}{(1-\gamma t)^{3/2}}$$

where  $b$  is the positive contact,  $x$  is the coordinate measured along the stretching

surface,  $\gamma$  is the rate of stretching constant,  $t$  is the time,  $T_\infty$  is the free stream temperature and  $\nu$  is the kinematic viscosity.

The rheological equation of state for the Cauchy stress tensor of Casson fluid can be written as:

$$\tau_{ij} = \begin{cases} 2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c \end{cases}$$

Where  $\pi = e_{ij}e_{ij}$  and  $e_{ij}$  is the  $(i, j)^{th}$  component of the deformation rate itself,  $\pi$  is the product of the component of the deformation rate with itself,  $\pi_c$  is a critical value of this product based on the non-Newtonian fluid,  $\mu_B$  is plastic dynamic viscosity of the non-Newtonian fluid and  $p_y$  is yield stress of fluid.

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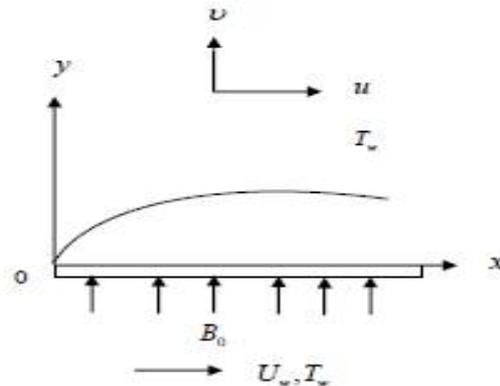


Fig.1. Physical model of the problem

The governing equations that describe the case are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - \frac{\sigma_e B_0^2}{\rho} u \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\mu}{\rho C_p} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_e B_0^2}{\rho C_p} u^2 \quad (3)$$

The boundary conditions for the velocity and temperature fields are

$$\left. \begin{aligned} y = 0 : u = U_w(x, t), v = 0, T = T_w(x, t) \\ y \rightarrow \infty : u \rightarrow 0, T \rightarrow T_\infty \end{aligned} \right\} \quad (4)$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions, respectively,  $K$  is the permeability,  $\sigma_e$  is the electrical conductivity,  $\rho$  is the fluid density,  $T$  is the temperature of the fluid,  $k$  is the thermal conductivity,  $C_p$  is the specific heat at constant pressure,  $q_r$  is the radiative heat flux,  $Q$  is the heat source when  $Q > 0$  or heat sink when  $Q < 0$ ,  $\mu$  is the coefficient of fluid viscosity and  $\beta$  is the Casson parameter.

Using Rosseland approximation for radiation (Brewster [20]), the radiative heat flux is simplified as:

$$q_r = - \frac{4 \sigma^*}{3 k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

Where  $\sigma^*$  and  $k^*$  are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively.

Assuming that the temperature differences within the flow is such that the term  $T^4$  may be expressed as a linear function of temperature. Hence, expanding  $T^4$  in a Taylor series about  $T_\infty$  and neglecting higher-order terms we obtain

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

In view of Eqs. (5) and (6), Eq. (3) becomes

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( 1 + \frac{16 \sigma^* T_\infty^3}{3 K^* k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_e B_0^2}{\rho C_p} u^2 \quad (7)$$

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### III. SIMILARITY ANALYSIS

The continuity Eq. (1) is satisfied by introducing a stream function  $\psi(x, y)$  such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (8)$$

The momentum and energy Eqs. (2) and (7) can be transformed into the corresponding ordinary differential equations by introducing the following similarity transformations (Chaudhary [21]):

$$\psi(x, y) = \sqrt{\frac{\nu b}{(1-\gamma t)}} x f(\eta) \quad (9)$$

$$\eta = \sqrt{\frac{b}{\nu(1-\gamma t)}} y \quad (10)$$

$$T = T_\infty + \frac{b}{2\nu x^2} \frac{1}{(1-\gamma t)^{3/2}} \theta(\eta) \quad (11)$$

where  $f(\eta)$  is the dimensionless stream function,  $\eta$  is the similarity variable,  $y$  is the coordinate measured along normal to the stretching surface and  $\theta(\eta)$  is the dimensionless temperature. The transformed ordinary differential equations are:

$$\left(1 + \frac{1}{\beta}\right) f'' + f'' f - f'^2 - A \left(\frac{1}{2} \eta f'' + f'\right) - \lambda f' - M f' = 0 \quad (12)$$

$$\left(1 + \frac{4}{3R}\right) \theta'' + \text{Pr} \left\{ f \theta' - \frac{A}{2} (\eta \theta' + 3\theta) + 2 f' \theta + \delta \theta \right. \\ \left. + \left(1 + \frac{1}{\beta}\right) \text{Ec} f'^2 + M \text{Ec} f'^2 \right\} = 0 \quad (13)$$

with the boundary conditions:

$$\eta = 0: f = 0, f' = 1, \theta = 1 \quad (14)$$

$$\eta \rightarrow \infty: f' \rightarrow 0, \theta \rightarrow 0$$

where primes denote differentiation with respect to  $\eta$ .

$A = \frac{\gamma}{b}$  is the unsteadiness parameter,  $\lambda = \frac{\nu^2 \text{Re}_x}{K U_w^2}$  is the porous parameter,  $\text{Re}_x = \frac{U_w x}{\nu}$  is the local Reynolds

number,  $R = \frac{k k^*}{4 \sigma^* T_\infty^3}$  is the thermal radiation parameter,  $M = \frac{\sigma_e B_0^2 \nu \text{Re}_x}{\rho U_w^2}$  is the magnetic parameter,  $\text{Pr} = \frac{\mu C_p}{k}$  is

the Prandtl number,  $\delta = \frac{Q \nu^2 \text{Re}_x}{\mu C_p U_w^2}$  is the heat source or sink parameter and  $\text{Ec} = \frac{U_w^2}{C_p (T_w - T_\infty)}$  is the Eckert number.

The physical quantities of interest are the local skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$ , which are defined as:

$$C_f = \frac{\mu \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}} \quad (15)$$

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$$Nu_x = \frac{-x \left( \frac{\partial T}{\partial y} \right)_{y=0}}{(T_w - T_\infty)} \quad (16)$$

Which in the present case, can be expressed in the following forms

$$C_f = \frac{2}{\sqrt{Re_x}} \left( 1 + \frac{1}{\beta} \right) f''(0) \quad (17)$$

$$Nu_x = -\sqrt{Re_x} \theta'(0) \quad (18)$$

Numerical values of the function  $f''(0)$  and  $\theta'(0)$  which represent the wall shear stress and the heat transfer rate at the surface respectively for various values of the parameter are presented in Tables 1 and 2.

### IV. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs. 2-13, to illustrate the influence of physical parameters viz., the magnetic parameter  $M$ , the unsteadiness parameter  $A$ , the porous parameter  $\lambda$ , the thermal radiation parameter  $R$ , the Prandtl number  $Pr$ , the Casson parameter  $\beta$ , the heat source or sink parameter  $\delta$ , the Eckert number  $Ec$ , on the velocity  $f'(\eta)$ , temperature  $\theta(\eta)$ , shear stress  $f''(\eta)$  and temperature gradient  $\theta'(\eta)$ .

In order to test the accuracy of the present method, our results are compared with those of Elbashbeshy and Emam [18] and Santosh Chaudhary [19] (for reduced cases) and found that there is an excellent agreement, as shown in Tables 1 and 2. Throughout the calculations, the parametric values are fixed to be  $A = 0.8, \lambda = 0.1, M = 0.01, \beta = \infty, R = 0.3, Pr = 10, Ec = 0.01, \delta = -0.5$  unless otherwise indicated.

Figs. 2 and 3 instance the effects of the velocity and temperature dependent unsteadiness parameter  $A$  on velocity and temperature profiles respectively. We compare the velocity and temperature profiles for both Newtonian and non-Newtonian cases. It is discerned that the velocity and temperature profiles decreases for both cases with the increase in unsteadiness parameter  $A$ . Consequently momentum boundary layer thickness and thermal boundary layer thickness decreases.

The influence of the porous parameter  $\lambda$  on the velocity and temperature profiles for both Newtonian and non-Newtonian cases. Fig. 4 shows the variation in velocity field for several values of  $\lambda$ . We observe that for both cases velocity decreases with the increase of porous parameter  $\lambda$  and consequently the momentum boundary layer thickness decreases. While from Fig. 5 shows the variation in temperature profiles for several values of  $\lambda$ . It is noticed that for both cases temperature decreases with the increase of porous parameter  $\lambda$  and consequently the thermal boundary layer thickness decreases.

The variation on velocity and temperature profiles for several values of magnetic parameter  $M$  is shown in Fig. 6 and Fig. 7. It is shown that the velocity and temperature profiles decreases for both cases with the increase of magnetic parameter  $M$ . Moreover the effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive force called Lorentz force. This force has the tendency to slow down the fluid motion.

Fig. 8 and Fig. 9 demonstrate the effect of the Casson parameter  $\beta$  on velocity and temperature distributions. It is noticed that the velocity and temperature profiles decreases and consequently the momentum boundary layer thickness decrease with the increasing values of Casson parameter  $\beta$  under both Newtonian and non-Newtonian cases.

Figs. 10-13 depicts that the temperature profiles  $\theta(\eta)$  for different values of the thermal radiation parameter  $R$ , the Prandtl number  $Pr$ , the heat source or sink parameter  $\delta$  and the Eckert number  $Ec$  respectively. From these

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figures it is noticed that the temperature decreases with the increasing values of the thermal radiation parameter  $R$  and the Prandtl number  $Pr$ , while the reverse phenomenon occurs for the heat source or sink parameter  $\delta$  and the Eckert number  $Ec$  under both Newtonian and non-Newtonian cases.

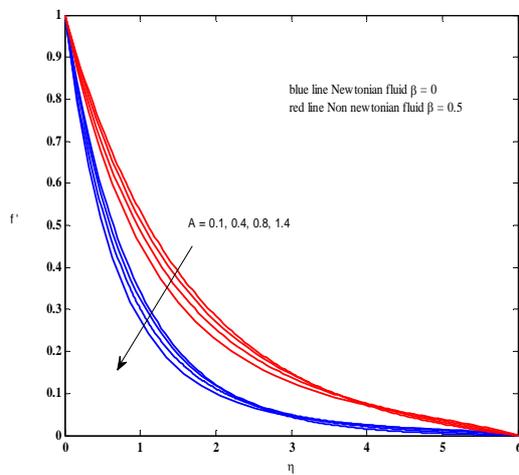


Fig 2. Velocity profiles against  $\eta$  for various values of  $A$  with  $Pr=10, M=0.01, R=0.3, \delta = -0.5, Ec=0.01, \lambda = 0.1$

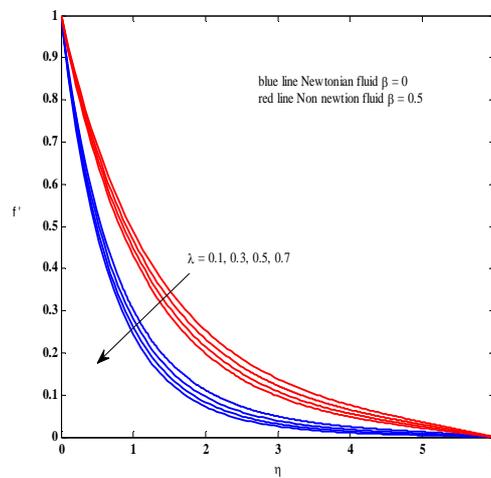


Fig 4. Velocity profiles against  $\eta$  for various values of  $\lambda$  with  $Pr=10, A=0.8, M=0.01, R=0.3, \delta = -0.5, Ec=0.01$

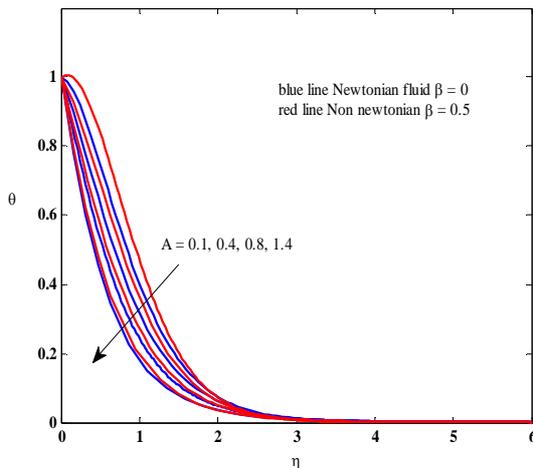


Fig 3. Temperature profiles against  $\eta$  for various values of  $A$  with  $Pr=10, M=0.01, \lambda = 0.1, R=0.3, \delta = -0.5, Ec=0.01$

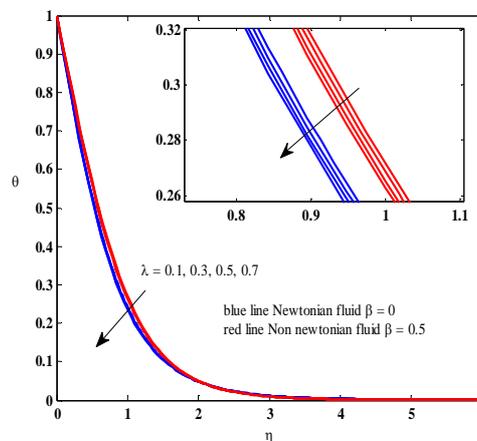


Fig 5. Temperature profiles against  $\eta$  for various values of  $\lambda$  with  $Pr=10, M=0.01, R=0.3, \delta = -0.5, Ec=0.01, A=0.8$

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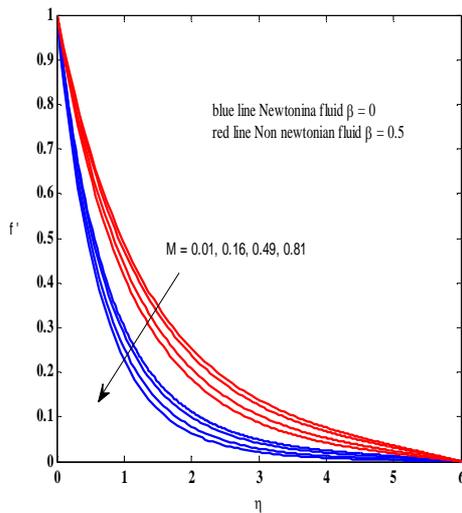


Fig 6. Velocity profiles against  $\eta$  for various values of  $M$  with  $Pr=10, A=0.8, \lambda=0.1, R=0.3, \delta=-0.5, Ec=0.01$

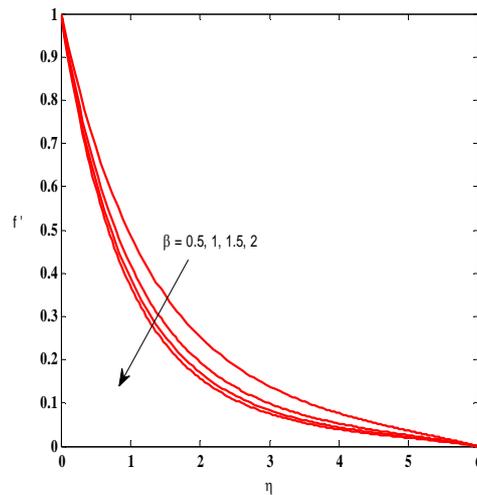


Fig 8. Velocity profiles against  $\eta$  for various values of  $\beta$  with  $R=0.3, Ec=0.01, \lambda=0.1, M=0.01, Pr=10, \delta=-0.5, A=0.8$

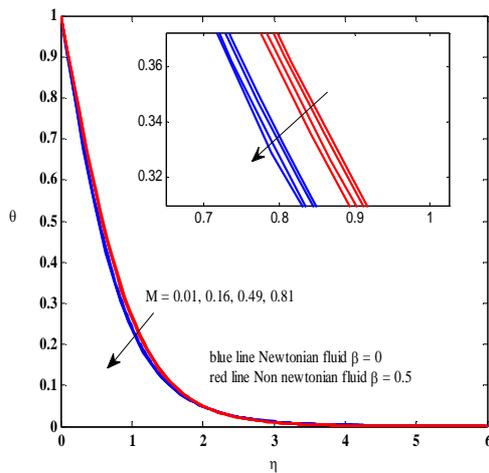


Fig 7. Temperature profiles against  $\eta$  for various values of  $M$  with  $Pr=10, \lambda=0.1, R=0.3, \delta=-0.5, Ec=0.01, A=0.8$

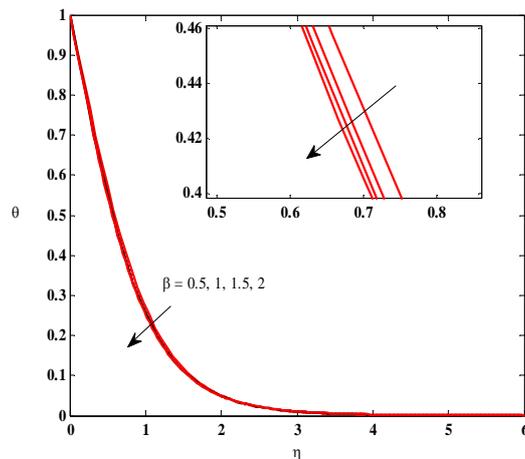


Fig 9. Temperature profiles against  $\eta$  for various values of  $\beta$  with  $R=0.3, Ec=0.01, \lambda=0.1, M=0.01, Pr=10, \delta=-0.5, A=0.8$

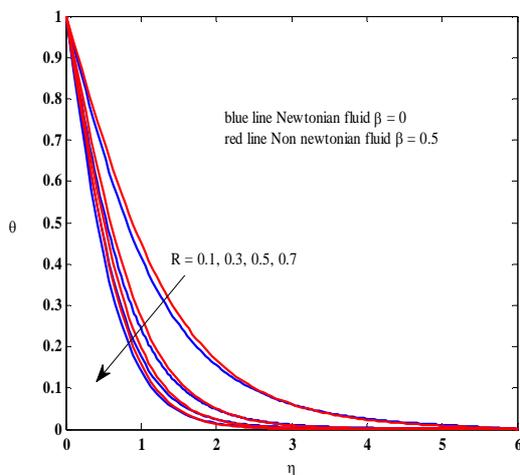


Fig 10. Temperature profiles against  $\eta$  for various values of R with  $Pr=10, \lambda=0.1, M=0.01, \delta=-0.5, Ec=0.01, A=0.8$

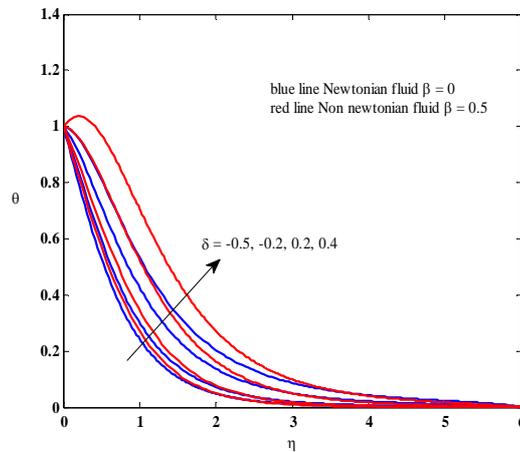


Fig 12. Temperature profiles against  $\eta$  for various values of  $\delta$  with  $R=0.3, \lambda=0.1, M=0.01, Pr=10, Ec=0.01, A=0.8$

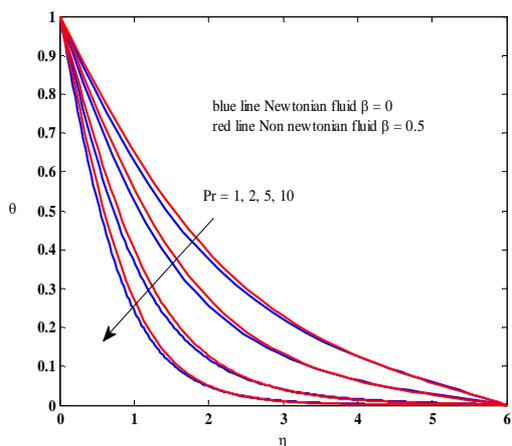


Fig 11. Temperature profiles against  $\eta$  for various values of Pr with  $R=0.3, \lambda=0.1, M=0.01, \delta=-0.5, Ec=0.01, A=0.8$

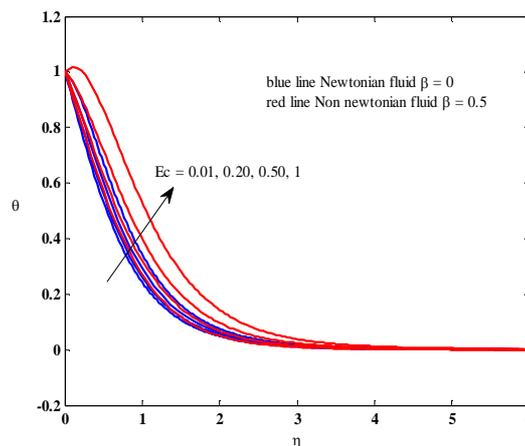


Fig 13. Temperature profiles against  $\eta$  for various values of Ec with  $R=0.3, \lambda=0.1, M=0.01, Pr=10, \delta=-0.5, A=0.8$

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Table 1. Comparison of  $-(1+1/\beta)f''(0)$  for various values of  $A$ ,  $\lambda$  and  $M$  with  $\beta = \infty$ ,  $R = 0.3$ ,  $Ec = 0.01$ ,  $Pr = 10$  and  $\delta = -0.5$ .

$A$	$\lambda$	$M$	Elbashbeshy and Emam [18]	Santhosh Chaudhary [19]	Present study
0.4	0.1	0	1.17853	1.17862910	1.178581
0.8	0.1	0	1.30035	1.30044740	1.300354
0.8	0.3	0	1.37550	1.37559607	1.375502
0.8	0.5	0	1.44668	1.44676744	1.446673
0.8	0.7	0	1.51445	1.51454348	1.514448

Table 2. Comparison of  $-\theta''(0)$  for various values of  $A$ ,  $\lambda$ ,  $M$ ,  $R$ ,  $Pr$ ,  $\delta$  and  $Ec$  with  $\beta = \infty$ .

$A$	$\lambda$	$M$	$R$	$Pr$	$\delta$	$Ec$	Elbashbeshy and Emam [18]	Santhosh Chaudhary [19]	Present study
0.4	0.1	0	0.3	10	-0.5	0	0.53765	0.53781852	0.537698
0.8	0.1	0	0.3	10	-0.5	0	0.98601	0.98623175	0.986015
0.8	0.3	0	0.3	10	-0.5	0	1.00056	1.00077724	1.000562
0.8	0.5	0	0.3	10	-0.5	0	1.01364	1.01385453	1.013640
0.8	0.7	0	0.3	10	-0.5	0	1.02552	1.02573274	1.025519
0.8	0.1	0	0.3	10	-0.2	0	0.73898	0.73920476	0.738980
0.8	0.1	0	0.3	10	0.2	0	0.31343	0.31368315	0.313438
0.8	0.1	0	0.3	10	0.4	0	0.02044	0.02072673	0.020456

### V. CONCLUSION

- Effects of unsteadiness parameter  $A$ , porous parameter  $\lambda$ , Magnetic parameter  $M$  and Casson parameter  $\beta$  on velocity  $f'(\eta)$  and temperature  $\theta(\eta)$  are similar.
- The influence of the thermal radiation parameter  $R$  and the Prandtl number  $Pr$  on temperature distribution  $\theta(\eta)$ . We observe that the temperature  $\theta(\eta)$  decreases. While the opposite behaviour on temperature  $\theta(\eta)$  for several values of heat source or sink parameter  $\delta$  and Eckert number  $Ec$  respectively.
- From Table 1 the Skin-friction for various values of unsteadiness parameter  $A$ , porous parameter  $\lambda$  are demonstrated. It is shows that the Skin-friction coefficient decreases with the increasing values of unsteadiness parameter  $A$ , porous parameter  $\lambda$ . Further it is observe that the values of local Skin-friction coefficient are always negative for all the values of physical parameters considered. Physically positive sign of Skin-friction coefficient implies that the fluid exerts a drag force on the sheet and negative sign implies the opposite meaning.
- From Table 2 illustrate the effects of the unsteadiness parameter  $A$ , porous parameter  $\lambda$  and heat source or sink parameter  $\delta$  on the heat flux  $-\theta'(0)$  at the surface. We observe that the local Nusselt number  $-\theta'(0)$

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decreases with the increasing values of the unsteadiness parameter  $A$ , porous parameter  $\lambda$ , while the opposite phenomenon occurs for the heat source or sink parameter  $\delta$ .

- The Skin-friction coefficient and Nusselt number are found that present results have an excellent agreement with the existed studies under some special cases.

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