

Symmetric Reverse Bi-Derivations in Semiprime Rings

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ABSTRACT: Let R be a 2-torsion and 3-torsion free semiprime ring. Let $D: (.,.): R \times R \rightarrow R$ and $B(.,.): R \times R \rightarrow R$ be a symmetric reverse bi-derivation and symmetric bi-additive mapping. If $D(d(x), x) = 0$ and $d(d(x)) = f(x)$ holds for all x in R , where d be a trace of D and f be a trace of B . In this case $D = 0$.

KEYWORDS: Semiprime ring, Symmetric mapping, Trace, Derivation, Reverse derivation, Symmetric bi-derivation, Symmetric bi-additive mapping, Symmetric reverse bi-derivation

I. INTRODUCTION

The concept of a symmetric bi-derivation has been introduced by Maksa in [6], [7]. A classical result in the theory of centralizing mappings is a theorem first proved by Posner [9]. Vukman [10], [11] studied some results concerning symmetric bi-derivations on prime and semi prime rings. In this paper we proved some results symmetric reverse bi-derivations in semiprime rings. Jaya Subba Reddy et al. [1], [2], [3], [4], [5] has studied some results concerning reverse derivations and symmetric reverse bi-derivations on prime and semiprime rings.

Throughout this paper R will be associative. We shall denote by $Z(R)$ the center of a ring R . Recall that a ring R is prime if $aRb = (0)$ implies that $a = 0$ or $b = 0$. A ring R is semiprime if $aRa = 0$ implies that $a = 0$. We shall write $[x, y]$ for $xy - yx$ and use the identities $[xy, z] = [x, z]y + x[y, z]$, $[x, yz] = [x, y]z + y[x, z]$. An additive map $d: R \rightarrow R$ is called derivation if $d(xy) = d(x)y + xd(y)$, holds for all $x, y \in R$. A mapping $B(.,.): R \times R \rightarrow R$ is said to be symmetric if $B(x, y) = B(y, x)$, holds for all $x, y \in R$. A mapping $f: R \rightarrow R$ defined by $f(x) = B(x, x)$, where $B(.,.): R \times R \rightarrow R$ is a symmetric mapping, is called a trace of B . It is obvious that, in case $B(.,.): R \times R \rightarrow R$ is symmetric mapping which is also bi-additive (i. e. additive in both arguments) the trace of B satisfies the relation $f(x + y) = f(x) + f(y) + 2B(x, y)$, for all $x, y \in R$. We shall use also the fact that the trace of a symmetric bi-additive mapping is an even function. A symmetric bi-additive mapping $D(.,.): R \times R \rightarrow R$ is called a symmetric bi-derivation if $D(xy, z) = D(x, z)y + xD(y, z)$ is fulfilled for all $x, y, z \in R$. Obviously, in this case also the relation $D(x, yz) = D(x, y)z + yD(x, z)$ for all $x, y, z \in R$. A symmetric bi-additive mapping $D(.,.): R \times R \rightarrow R$ is called a symmetric reverse bi-derivation if $D(xy, z) = D(y, z)x + yD(x, z)$ for all $x, y, z \in R$. Obviously, in this case also the relation $D(x, yz) = D(x, z)y + zD(x, y)$ for all $x, y, z \in R$. A mapping $f: R \rightarrow R$ is said to be commuting on R if $[f(x), x] = 0$, holds for all $x \in R$. A mapping $f: R \rightarrow R$ is said to be centralizing on R if $[f(x), x] \in Z(R)$ is fulfilled for all $x \in R$. A ring R is said to be n -torsion free if whenever $na = 0$, with $a \in R$, then $a = 0$, where n is nonzero integer.

II. RELATED WORK

Theorem1: Let R be a 2 torsion free semiprime ring suppose there exists such a symmetric reverse bi-derivation $D(.,.): R \times R \rightarrow R$ such that $D(d(x), x) = 0$ holds for all $x \in R$ where d denotes the trace of D . In this case we have $D = 0$.

Proof: We have $D(d(x), x) = 0$ for all $x \in R$ (1)

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By linearizing (1), we get

$$D(d(x+y), x+y) = 0$$

$$D(d(x) + d(y), x+y) = 0$$

$$D(d(x), x) + D(d(x), y) + D(d(y), x) + D(d(y), y) + 2(D(x, y), x) + 2(D(x, y), y) = 0$$

Using (1), we get

$$D(d(x), y) + D(d(y), x) + 2(D(x, y), x) + 2(D(x, y), y) = 0 \text{ for all } x \in R \quad (2)$$

Replace x by -x in (2), we get

$$D(d(-x), y) + D(d(y), -x) + 2(D(-x, y), -x) + 2(D(-x, y), y) = 0$$

$$D(d(x), y) - D(d(y), x) + 2(D(x, y), x) - 2(D(x, y), y) = 0 \text{ for all } x \in R \quad (3)$$

Adding (2) and (3), we get

$$D(d(x), y) + 2(D(x, y), x) = 0 \text{ for all } x \in R \quad (4)$$

We replace y by yx (4), we get

$$D(d(x), yx) + 2(D(x, yx), x) = 0$$

$$xD(d(x), y) + D(d(x), x)y + 2D(xD(x, y) + (D(x, x)y, x) = 0$$

$$xD(d(x), y) + D(d(x), x)y + 2D(xD(x, y) + (d(x))y, x) = 0$$

$$xD(d(x), y) + 2D(xD(x, y), x) + 2D(d(x))y, x) = 0$$

$$xD(d(x), y) + 2D(x, y)d(x) + 2D(D(x, y), x)x + 2yD(d(x), x) + 2D(y, x)d(x) = 0$$

$$xD(d(x), y) + 2D(x, y)d(x) + 2D(D(x, y), x)x + 2D(y, x)d(x) = 0 \text{ for all } x, y \in R \quad (5)$$

We multiply (4) by x on right hand side, we get

$$D(d(x), y)x + 2(D(x, y), x)x = 0, \text{ we get} \quad (6)$$

We subtract (6) from (5), we get

$$xD(d(x), y) - D(d(x), y)x + 2(D(x, y)d(x) + 2D(y, x)d(x) = 0$$

$$[x, D(d(x), y)] + 2(D(x, y)d(x) + 2D(y, x)d(x) = 0, \text{ for all } x, y \in R \quad (7)$$

Replace y by yx in (7), we get

$$[x, D(d(x), yx)] + 2(D(x, yx)d(x) + 2D(yx, x)d(x) = 0$$

$$[x, xD(d(x), y)] + D(d(x), x)y] + 2x(D(x, y)d(x) + 2(D(x, x)y, x)d(x) + 2xD(y, x)d(x) + 2D(x, x)y, x)d(x) = 0$$

$$x[x, D(d(x), y)] + [x, x]D(d(x), y) + [x, D(d(x), x)]y + D(d(x), x)[x, y] + 2xD(x, y)d(x) + 2D(x, x)y, x)d(x) + 2xD(y, x)d(x) + 2D(x, x)y, x)d(x) = 0$$

by using (1), we get

$$x[x, D(d(x), y)] + 2xD(x, y)d(x) + 2d(x)y, x)d(x) + 2xD(y, x)d(x) + 2d(x)y, x)d(x) = 0$$

$$x\{[x, D(d(x), y)] + 2D(x, y)d(x) + 2D(y, x)d(x)\} + 2d(x)y, x)d(x) + 2d(x)y, x)d(x) = 0$$

by using (7), we get

$$d(x)y, x)d(x) + d(x)y, x)d(x) = 0 \text{ for all } x, y \in R$$

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$$2d(x)y d(x) = 0 \text{ for all } x, y \in R$$

Since R is 2-torsion free and semiprime ring, we get

Which implies that $d(x) = 0$, for all $x \in R$.

Theorem2: Let R be a semi prime ring which is 2- torsion and 3- torsion free. Let $D(\cdot, \cdot): R \times R \rightarrow R$ and $B(\cdot, \cdot): R \times R \rightarrow R$ be a symmetric bi-derivation and a symmetric bi-additive mapping, respectively. Suppose that $d(d(x)) = f(x)$ holds for all $x \in R$ where d is the trace of D and f is the trace of B . In this case we have $D = 0$.

Proof:

We have $d(d(x)) = f(x)$, for all $x \in R$. (8)

Linearization of the relation (8), we get

$$\begin{aligned} d(d(x+y)) &= f(x+y) \\ d(d(x) + d(y) + 2D(x,y)) &= f(x) + f(y) + 2B(x,y) \\ d(d(x) + d(y)) + d(2D(x,y)) + 2D((d(x) + d(y)), 2D(x,y)) &= f(x) + f(y) + 2B(x,y) \\ d(d(x)) + d(d(y)) + 2D(d(x), d(y)) + 4d(D(x,y)) + 4D(d(x), D(x,y)) + 4D(d(y), D(x,y)) &= f(x) + f(y) + 2B(x,y) \end{aligned}$$

By using (8) in the above equation, we get

$$\begin{aligned} 4d(D(x,y)) + 2D((d(x), d(y)) + 4D(d(x), D(x,y)) + 4D(d(y), D(x,y)) &= 2B(x,y) \\ 2d(D(x,y)) + D((d(x), d(y)) + 2D(d(x), D(x,y)) + 2D(d(y), D(x,y)) &= B(x,y), \text{ for all } x, y \in R. \end{aligned} \quad (9)$$

we replace x by $-x$ in (9), we get

$$\begin{aligned} 2d(D(-x,y)) + D((d(-x), d(y)) + 2D(d(-x), D(-x,y)) + 2D(d(y), D(-x,y)) &= B(-x,y) \\ 2d_1(D(x,y)) + D((d(x), d(y)) - 2D(d(x), D(x,y)) - 2D(d(y), D(x,y)) &= -B(x,y) \text{ for all } x, y \in R. \end{aligned} \quad (10)$$

Subtract (10) from (9), we get

$$\begin{aligned} 4D(d(x), D(x,y)) + 4D(d(y), D(x,y)) &= 2B(x,y) \\ 2D(d(x), D(x,y)) + 2D(d(y), D(x,y)) &= B(x,y) \text{ for all } x, y \in R. \end{aligned} \quad (11)$$

We replace x by $2x$ in (11), we get

$$\begin{aligned} 2D(d(2x), D(2x,y)) + 2D(d(y), D(2x,y)) &= B(2x,y) \\ 16D(d(x), D(x,y)) + 4D(d(y), D(x,y)) &= 2B(2x,y) \text{ for all } x, y \in R. \end{aligned} \quad (12)$$

Subtract two times of (11) from (12), we get

$$12D(d(x), D(x,y)) = 0, \text{ for all } x, y \in R. \quad (13)$$

Since we have assumed that R is not only of characteristic different from two but also of characteristic different from three, which leads to

$$D(d(x), D(x,y)) = 0, \text{ for all } x, y \in R. \quad (14)$$

and $d(d(x)) = 0, x \in R$.

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We replace x by y and y by x in (14), we get

$$D(d(y), D(x, y)) = 0, \text{ for all } x, y \in R. \quad (15)$$

Using (14), (15) in (11), we get

$$B(x, y) = 0, \text{ for all } x, y \in R. \quad (16)$$

We replace y by zy in (14), we get

$$\begin{aligned} D(d(x), D(x, zy)) &= 0 \\ D(d(x), yD(x, z) + D(x, y)z) &= 0 \\ D(d(x), yD(x, z)) + D(d(x), D(x, y)z) &= 0 \\ D(x, z)D(d(x), y) + D(d(x), D(x, z))y + D(d(x), z)D(x, y) + zD(d(x), D(x, y)) &= 0 \\ D(x, z)D(d(x), y) + D(d(x), z)D(x, y) &= 0 \end{aligned}$$

In particular, for $z = d(x)$, we obtain

$$\begin{aligned} D(x, d(x))D(d(x), y) + D(d(x), d(x))D(x, y) &= 0 \\ D(x, d(x))D(d(x), y) + d(d(x))D(x, y) &= 0 \end{aligned}$$

Therefore

$$D(x, d(x))D(d(x), y) = 0, \text{ for all } x, y \in R. \quad (17)$$

Replace y by xy in (17), we get

$$\begin{aligned} D(x, d(x))D(d(x), xy) &= 0 \\ D(x, d(x))yD(d(x), x) + D(x, d(x))D(d(x), y)x &= 0 \end{aligned}$$

Using (17), we get

$$\begin{aligned} D(x, d(x))yD(d(x), x) &= 0 \\ D(x, d(x))yD(x, d(x),) &= 0 \end{aligned}$$

Since R is a semiprime, we get

$$D(x, d(x)) = 0.$$

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