

An Analytical Approach for Unsteady MHD Dusty Viscoelastic Fluid Couette Flow in a Vertical Wavy Channel with Varying Mass Diffusion

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ABSTRACT: The present work finds the effects of Forchheimer and MHD on unsteady dusty viscoelastic fluid couette flow with heat absorbing chemical reaction, viscous dissipation, ohmic effect, Joule effect with varying mass diffusion over a vertical wavy channel. The governing equations of the physical problem are solved using double perturbation technique. The numerical computation for the solution of velocity, temperature & concentration are carried out for various non-dimensional parameters of the problem. For the industrial applications, the expressions of skin friction, rate of heat and mass transfer at the boundary of the vertical wavy channel are obtained and found a good agreement with earlier works for the particular cases of the problem for fluid flow.

KEYWORDS: MHD, wavy channel, varying mass diffusion, couette flow.

I. INTRODUCTION

Transition from the study of Newtonian to non-Newtonian fluid shows an tremendous growth for the importance of many practical or industrial applications like food preservation techniques, power engineering, petroleum production, polymer sheet extrusion from a dye and drawing of plastic firm, biomechanical problems, chemical industries. This is mainly because of the process in which high temperature need cooling by the movement of the fluid flow which transfers the temperature to lower magnitudes. On the other hand, using different non-Newtonian fluids exhibits the reduction of temperature of the medium or reducing the boundary layers thickness for the physical model in terms of temperature. Such a study first come up with Beard and Walters[1] by introducing the boundary layers treatment for an idealized one of the nonlinear Newtonian fluid as viscoelastic fluid. This leads to continuous interest in the investigation of natural convection, forced convection or mixed convection of heat transfer for a non-Newtonian fluid by many authors in the literature (see Rajagopal and Na [2], Chang and Lundgren [3], Syed-Ahmed [4], Attia [5], Mekheimer and Kot [6]) because of the above applications. By introducing the additional forces on dusty viscoelastic fluid have a wide range of applications in the recent development due to the electromagnetic field are relevant in geophysical, astrophysical, metallurgy, filaments drawn through a quiescent fluid and cooling of continuous strips (see examples Tao [7] Hayat and Kara [8]). The combined effects of heat and mass transfer under the influence of external constraints like magnetic field, porous media have attracted many researchers because of its importance in many branches of science and technology. Some of these variety of practical situations are seen in solid-liquid extract ion, coating process, biochemical system in dialyzer, oxygenator and other biomechanical membrane process were investigated by many authors (see Pal and Mondal [9], Srinivas, Muthuraj [10]). In these applications, we come across a couette flow fashion in irregular channels under the influence of many external forces which drives the flow, temperature and species or concentration with varying mass diffusion, MHD, drag force, viscous dissipation. In literature to the best of our knowledge the mathematical modeling for such a physical system involving additional forces for a plane channel studied by many authors (Patil, and Kulkarni [11], Nalinakshi et al [12]) for a wavy channel with the effect of Soret and Dufour effect (Mahdy [13], Vajravelu, Sastri [14]). But in these models to our knowledge the drag force in terms of contraction or expansion called Forchheimer effect experienced by the velocity as been not

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considered in the earlier studies. The main objective of this work is to study an analytical solution for unsteady MHD dusty viscoelastic fluid couette flow in a vertical wavy channel with varying mass diffusion for the Forchheimer model due to its importance in many above applications. In this model, we examine the influence of MHD, heat absorption, radiation, viscous dissipation, joule effect, chemical reaction and thermal diffusion on couette flow of a viscoelastic fluid in vertical irregular porous channel with convective varying mass diffusion.

II. MATHEMATICAL FORMULATION

A unsteady two-dimensional flow of an incompressible, electrically conducting, dusty viscoelastic fluid (Walter's liquid-B model) Couette flow in an vertical irregular porous channel with convective cooling and varying mass diffusion for Forchheimer model is shown in Fig. 1

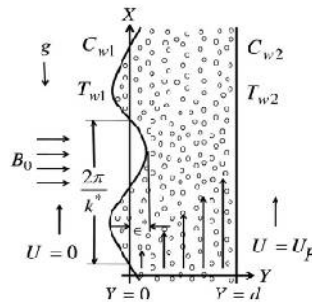


Fig. 1: Flow geometry of the problem.

The X-axis is taken in the direction along the channel which is set to motion and the Y-axis is taken perpendicular to it. The flow field is exposed the influence of an external, transversely applied, uniform magnetic field of strength B_0 and buoyancy effect. The magnetic Reynolds number is assumed to be very small so that both the Hall effects and the induced magnetic field are negligible. The heat equation includes the terms involving heat absorption, radiative heat flux, Joule heating and viscous dissipation whereas the mass transfer equation includes the effects of first-order chemically reactive species and thermal diffusion. The wavy wall $Y = \varepsilon^* \cos(K_2 X)$ maintain a temperature T_{w1} which represent the convective boundary $-k \frac{\partial T}{\partial Y} = h_f (T_1 - T + (T_2 - T_1)At^*)$ and concentration $C_{w1} = C_1 + (C_2 - C_1)At^*$ at $Y = \varepsilon^* \cos(K_2 X)$ lower than the flat wall ($Y = d$) temperature T_{w2} which represent the convective boundary condition $-k \frac{\partial T}{\partial Y} = h_f (T_2 - T + (T_2 - T_1)At^*)$ and concentration $C_{w2} = C_2 + (C_2 - C_1)At^*$ respectively. It is assumed that the convective heat exchange with the wall temperatures at the channel surface follows Newton's law of cooling. The viscosity, density, thermal conductivity, molecular diffusivity of the fluid and the pressure with negligible body forces are considered as constant. Taking into consideration of these assumptions, the equations that describe the physical situation can be written in Cartesian frame of references, as follows.

$$\frac{\partial U}{\partial t^*} = v \left(1 - K_0 \frac{\partial}{\partial t^*} \right) \frac{\partial^2 U}{\partial Y^2} + \frac{K_1 N_0}{\rho} (V - U) - \frac{v}{K^*} U - \frac{\sigma_e B_0^2}{\rho} U + g\beta_T (T - T_1) + g\beta_c (C - C_1) - \frac{C_b U^2}{\sqrt{K^*}} \quad (1)$$

$$m \frac{\partial V}{\partial t^*} = K_1 (U - V) \quad (2) ; \quad \frac{\partial T}{\partial t^*} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial Y^2} - \frac{Q_T}{\rho C_p} (T - T_1) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial Y} + \frac{v}{C_p} \left(\frac{\partial U}{\partial Y} \right)^2 + \frac{\sigma_e B_0^2}{\rho C_p} U^2 \quad (3)$$

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$$\frac{\partial C}{\partial t^*} = D \frac{\partial^2 C}{\partial Y^2} - K_R(C - C_1) + \frac{DK_T}{T} \frac{\partial^2 T}{\partial Y^2} \quad (4)$$

The appropriate boundary conditions of the problem are

$$t^* = 0 : U=0=V, T=T_1, C = C_1 \text{ for } Y \in (\varepsilon^* \cos(K_2 X), d) \quad (5)$$

$$t^* > 0 : U=0=V, -k \frac{\partial T}{\partial Y} = h_f(T_1 - T + (T_2 - T_1)At^*) \quad C = C_1 + (C_2 - C_1)At^* \text{ at } Y = \varepsilon^* \cos(K_2 X) \quad (6)$$

$$U=U_p=V, -k \frac{\partial T}{\partial Y} = h_f(T_2 - T + (T_2 - T_1)At^*) \quad C = C_2 + (C_2 - C_1)At^* \text{ at } Y = d \quad (7)$$

The radiative heat flux is given by

$$\frac{\partial q_r}{\partial Y} = 4(T - T_1)I', I' = \int_0^\infty K_{\lambda W} \frac{\partial e_{b\lambda}}{\partial T} d\lambda_1 \quad (8)$$

Introducing the following non-dimensional quantities

$$x = \frac{X}{d}, \quad y = \frac{Y}{d}, \quad v = \frac{V}{U_0}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad \phi = \frac{C - C_1}{C_2 - C_1}, \quad t = \frac{vt^*}{d^2}, \quad E = \frac{K_0 v}{d^2}, \quad \delta = \frac{mN_0}{\rho}, \quad W = \frac{mv}{K_1 d^2},$$

$$G_r = \frac{g\beta_T(T_2 - T_1)d^2}{\nu U_0}, \quad G_c = \frac{g\beta_c(C_2 - C_1)d^2}{\nu U_0}, \quad M^2 = \frac{\sigma_e B_0^2 d^2}{\mu}, \quad \frac{1}{K} = \frac{d^2}{K^*}, \quad P_r = \frac{\mu C_p}{k}, \quad \alpha_T = \frac{Q_T d^2}{\mu C_p},$$

$$F = \frac{4I'd^2}{\mu C_p}, \quad E_c = \frac{U_0^2}{C_p((T_2 - T_1))}, \quad S_c = \frac{\nu}{d}, \quad K_r = \frac{K_R d^2}{\nu}, \quad S_r = \frac{DK_T d^2 (T_2 - T_1)}{\nu T (C_2 - C_1)}, \quad A = \frac{U_0^2}{\nu}, \quad B_i = \frac{h_f d}{k},$$

$$u_p = \frac{U_p}{U_0}, \quad \lambda = K_2 d, \quad \varepsilon = \frac{\varepsilon^*}{d}, \quad h = \varepsilon \cos(\lambda x), \quad (9)$$

In view of the equation (9) the basic field of equations (1)-(4) can be expressed in non dimensional form as

$$\frac{\partial u}{\partial t} = \left(1 - E \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial y^2} + \frac{\delta}{w}(v - u) - \left(M^2 + \frac{1}{k}\right)u + G_r \theta + G_c \phi - \Pi u^2 \quad (10) \quad w \frac{\partial v}{\partial t} = v - u \quad (11)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{p_r} \frac{\partial^2 \theta}{\partial y^2} - (\alpha_T + F)\theta + E_c M^2 u^2 \quad (12) \quad \frac{\partial \phi}{\partial t} = \frac{1}{S_c} \frac{\partial^2 \phi}{\partial y^2} - K_r \phi + S_r \frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

The corresponding boundary conditions (5)-(7) in dimensionless form are

$$t = 0 : u = 0 = v, \theta = 0, \phi = 0 \text{ for } y \in (h, 1) \quad (14)$$

$$t > 0 : u = 0 = v, \frac{\partial \theta}{\partial y} = B_i(\theta - t), \phi = t \text{ at } y = h \quad (15)$$

$$u = u_p = v, \frac{\partial \theta}{\partial y} = B_i(\theta - 1 - t), \phi = 1 + t \text{ at } y = 1 \quad (16)$$

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III. METHOD OF SOLUTION

To solve the eqs.(10)-(16), according to prakash et al [5], we assume

$$u(y,t) = u_0(y) + \Gamma e^{-nt} u_1(y) + o(\Gamma^2) \quad (17) \quad v(y,t) = v_0(y) + \Gamma e^{-nt} v_1(y) + o(\Gamma^2) \quad (18)$$

$$\theta(y,t) = \theta_0(y) + \Gamma e^{-nt} \theta_1(y) + o(\Gamma^2) \quad (19) \quad \phi(y,t) = \phi_0(y) + \Gamma e^{-nt} \phi_1(y) + o(\Gamma^2) \quad (20)$$

The shear stress for dusty fluid coefficient of rate of heat and mass transfer at any point in the fluid can be characterized in order by

$$\tau_f^* = -\mu U', \quad Nu^* = -kT', \quad Sh^* = -DC' \quad (21)$$

In dimensionless form

$$\tau_f = \frac{\tau_f^* d}{\mu U_0} = -u', \quad Nu = \frac{Nu^* d}{k(T_1 - T_0)} = -\theta', \quad Sh = \frac{Sh^* d}{D(C_1 - C_0)} = -\phi' \quad (22)$$

IV. RESULTS AND DISCUSSION

The analytical expressions for velocity, temperature and concentration for unsteady MHD dusty viscoelastic fluid couette flow in a vertical wavy channel with varying mass diffusion for forchheimer model has been derived which is a function of many non dimensional parameters involved in the physical model $\alpha T, Bi, E, F, t, M, K, Gr, Gc, Sc, Kr, up, Pr, W, Sr$.

A numerical computation has been carried out to study the effect of these non dimensional parameters on velocity, temperature and concentration by fixing some of the parameters as $\lambda = 2\pi, x=1, Gr=10, Gc=10, Pr=5, Sc=.96, Kr=1, W=3$. The graphical representations of the velocity profile for the various parameters are observed and due space constraints, only a few results are displayed. In Fig. 2 velocity enhances for the increase in effect of heat absorption and similar characteristics are observed for a thermal radiation parameter. Due to rate of heat transfer and radiative heat flux of the viscoelastic fluid. An opposite behavior of velocity characteristic is observed for the effect of Biot number, viscoelastic, time parameter and magnetic field parameter. Due to an adiabatic heat transfer maintained at the boundary, higher viscosity of the fluid and increase in time variation and enhancement with magnetic effect.

The effect of temperature of the fluid flow increases the magnitude or decrease in the negative direction for increasing effects of heat absorption. Similar characteristics are observed in Biot number, thermal radiation and Hartmann number. An opposite behavior of temperature characteristics is observed in the case of porous permeability and time parameter.

The concentration profiles increase with the enhancement of heat absorption and thermal radiation parameters, but for lower values behaves oppositely for the higher values of the parameters because concentration dominates over the temperature for lower values. Once the temperature overcomes the barrier of heat transfer, concentration diminishes. The concentration profile enhancement with increase in Biot number is displayed and similar behaviour is seen in Soret number also. The effect of concentration decreases with enhancement of porous permeability and time parameter which is shown in Fig. 7

All the above results on velocity temperature and concentration for various parameters are on par with earlier studies in the literature and as well as satisfy the practical situations of the physical problem. Due to limited space availability as per publisher guidelines only few results are displayed.

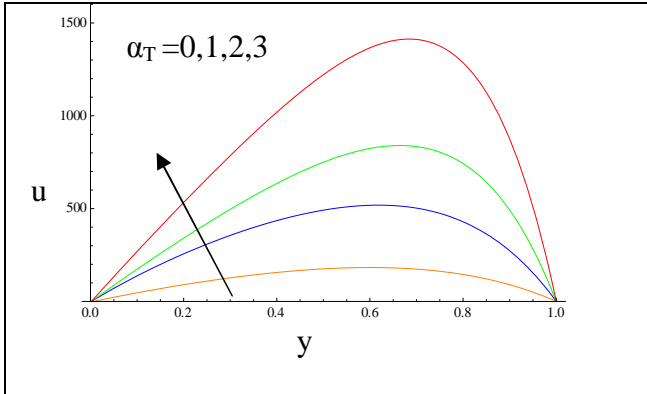


Fig. 2 : Effect of heat absorption parameter in velocity distribution

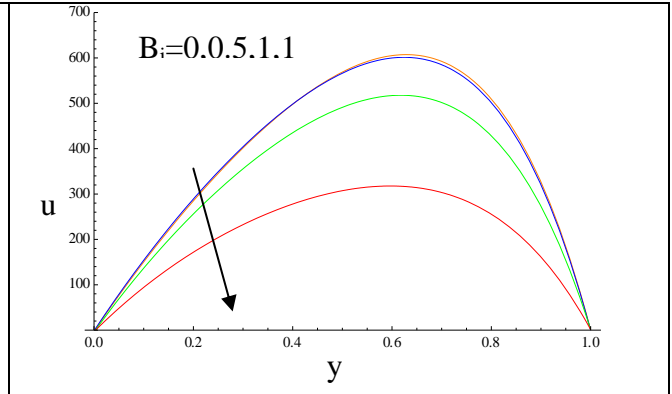


Fig. 3 : Effect of Biot number in velocity distribution

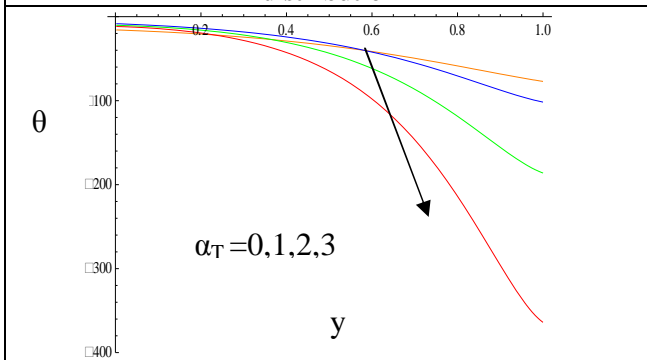


Fig. 4 : Effect of heat absorption parameter in temperature distribution

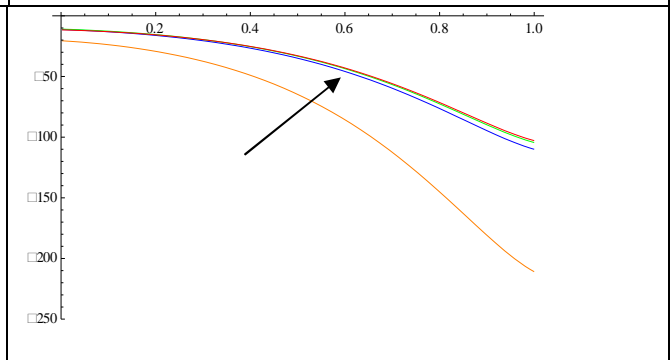


Fig. 5 : Effect of porous permeability parameter in temperature distribution

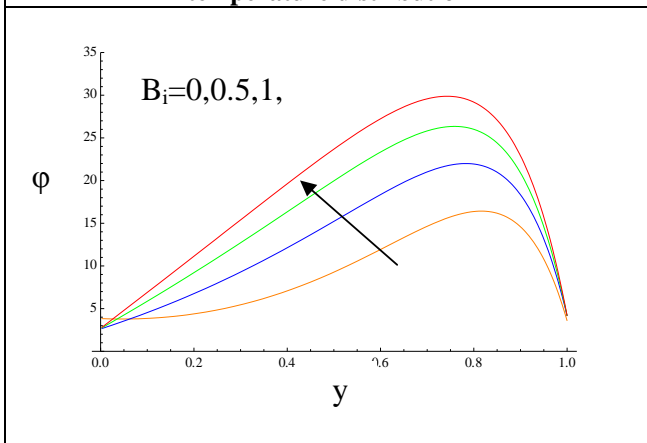


Fig. 6 : Effect of Biot number in concentration distribution

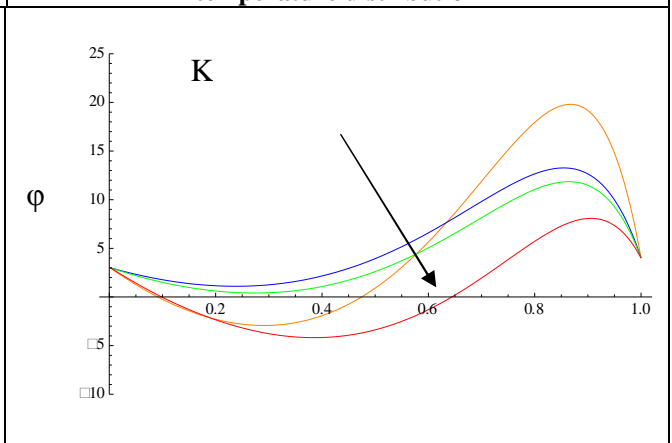


Fig. 7 : Effect of porous permeability parameter in concentration distribution

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