

Effects of Oscillatory Flow of a Couple stress Fluid over a Semi-Infinite Vertical Permeable Plate in a Porous Medium with Viscous Dissipation and Ohmic Effect

Suresh Babu R¹, Rushi Kumar B², Dinesh P.A¹Department of Mathematics, Ramaiah Institute of Technology, Bengaluru, India¹School of Advanced Sciences, Fluid Dynamics Division, VIT University, Vellore, India²

ABSTRACT: This problem deals with unsteady, two-dimensional, Boussinesq, laminar, oscillatory flow of an incompressible couple stress fluid over an infinite vertical permeable plate through the porous medium under the influence of viscous dissipation and Ohmic effects is analytically. The highly non-linear coupled partial differential equations are modelled for the boundary layer flow, heat and mass transfer using fully developed flow conditions and are transferred into a set of linear PDE's using non-dimensional parameters. The analytical solutions of the resulting boundary layer equations are assumed of oscillatory type and are solved using a two-term regular perturbation technique. Numerical computation are performed to analyze the effect of various non-dimensional parameters of the physical model like Grashof number Gr , Prandtl number Pr , Viscous ratio λ , Permeability parameter k , Material parameter β and Frequency number n on velocity, temperature and angular velocity profiles and are presented graphically. The present results are compared with earlier works in the absence of some parameters of the model and found to be in good arrangement.

KEYWORDS: Oscillatory flow, Couple stress fluid, Viscous Dissipation, Ohmic effect, Porous medium.

I. INTRODUCTION

In the last five decades, convective heat and mass transfer flow in a porous medium has been a subject of great interest because of its numerous industrial, thermal and chemical engineering applications in different disciplines like medicine, science and technology. For example, blood flow in a tube, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport, enhanced oil recovery system, thermal insulation, geothermal reservoirs, drying of porous solids, tidal waves etc. It is very much interesting to study the effects of the free convection currents on the oscillatory flow type of boundary layer flow and it is very important for technological point of view. Such a study was first initiated by Lighthill by considering an incompressible, 2-dimensional viscous fluid flow by assuming that the flow is a regular fluctuating superimposed flow in a boundary layer.

Soundalgekar [1] analysed a two-dimensional flow of an incompressible viscous fluid past an semi-infinite vertical porous plate in a porous medium with constant suction. Rapits et al. [2] studied free convection currents of mass transfer flow through a porous medium bounded by an infinite vertical limiting surface with constant suction analytically. An extensive study of free convection effects on the oscillatory flow of a couple stress fluid through a porous medium was discussed by Hiremath and Patil [3]. Raptis and Thakur [4] presented an analytical solutions for couple stress fluid which is heated from below in porous medium. Magyari and Pop [5] also performed analytical solutions for unsteady free convection flow over a vertical plate through a porous media.

Many transport processes occurring both in nature and in industries involves fluid flows with the combined heat and mass transfer and it plays an essential role in transport phenomena. Such flows are driven by multiple buoyancy effects which are arising from the density variations due to temperature and species concentration. Mixed convective

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flows in a porous medium have been examined during the last four decades due to its practical applications which can be modelled or approximated of a boundary layer flow. Such a study was illustrated by Rudraiah et al.[6] on mixed convective oscillatory flow in a vertical porous stream and Ogulu [9] On the oscillating plate temperature flow of a polar fluid past a vertical porous plate in the presence of couple stresses and radiation. Many authors like Youn Kim [7], Chamkha [8], Sahin Ahmed [10], Umamaheswari [11] and Raju et al. [12] have done their extensive research work by considering an additional MHD effect past a vertical plate in a porous medium. Comprehensive literature surveys, references regarding the subject of porous medium can be hand in recent books by Nield and Bejan [13].

The main objective of this work is, therefore, to study effects of oscillatory flow of a couple stress fluid over a semi-infinite vertical permeable plate in a porous medium along the viscous dissipation and Ohmic effects analytically with oscillatory suction at the vertical porous plate and oscillatory free stream velocity away from the plate in a porous medium. To achieve the objective of the present work, the plan of this work is as follows. We consider in the next section a vertical porous plate embedded in a fluid saturated porous medium with oscillatory suction velocity at the vertical plate. In this section we also give the conservation equations for momentum and energy with suitable boundary and inertia effect by considering Darcy-Lapwood-Brinkman equation. The relevant physical parameters are also discussed in this section. These basic equations involve variable coefficients which are solved analytically using regular perturbation technique. These solutions are numerically computed and the results are discussed in the last section.

II. MATHEMATICAL FORMULATION

We consider an unsteady, two-dimensional, laminar, incompressible, Boussinesq, couple stress fluid over a semi-infinite vertical plate in a porous medium. Here, the x -axis is taken along the porous plate with direction opposite to the direction of gravity and the y -axis is taken exactly perpendicular to the porous plate. We assume the following assumptions in order to find the analytical solution for the above problem. x varies between $-\infty$ to $+\infty$; all physical quantities are independent of x except pressure; density is constant throughout the momentum equation except for body force; density ρ is a function of temperature only, hence $\rho = \rho_0 [1 - \beta(T - T_\infty)]$; oscillatory suction velocity and the free stream velocity far away from the porous plate oscillates about a mean constant value in a direction parallel to the x -axis.

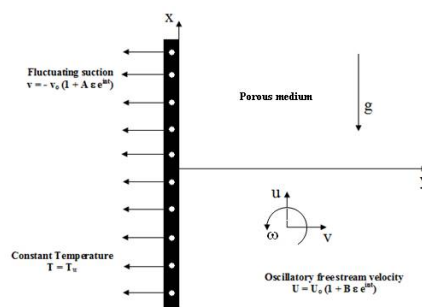


Fig.1: Physical model.

Under these assumptions, the basic governing equations of motion are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\bar{\nu}}{k} u + 2\nu \frac{\partial \omega}{\partial y} + \beta \bar{g} (T - T_\infty), \tag{2}$$

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$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\bar{\nu}}{k C_p} u^2, \quad (3)$$

$$\frac{\partial \omega}{\partial t} + v \frac{\partial \omega}{\partial y} = \frac{\alpha}{I} \frac{\partial^2 \omega}{\partial y^2}. \quad (4)$$

The associated boundary conditions for the physical model are

$$y = 0 : u = 0, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad T = T_w,$$

$$y \rightarrow \infty : u = U(t), \quad \omega \rightarrow 0, \quad T \rightarrow T_\infty. \quad (5)$$

Making these equations dimensionless using the known physical quantities as

$$y^* = \frac{v_0 y}{\nu}, \quad u^* = \frac{u}{U_0}, \quad t^* = \frac{v_0^2 t}{\nu}, \quad n^* = \frac{\nu n}{v_0^2}, \quad y^* = \frac{v}{v_0}, \quad k = \frac{\nu^2 k^*}{v_0^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\text{Pr} = \frac{\nu}{\kappa}, \quad \text{Gr} = \frac{\beta \nu g (T_w - T_\infty)}{U_0 v_0^2}, \quad E = \frac{U_0^2}{C_p (T_w - T_\infty)}, \quad \omega^* = \frac{\omega U_0 \nu_0}{\nu} \quad (6)$$

The fluctuating free-stream and suction velocities respectively defined as,

$$U(t) = U_0 (1 + A \varepsilon e^{\text{int}}) \quad \text{and} \quad v(t) = -v_0 (1 + B \varepsilon e^{\text{int}}). \quad (7)$$

Using Eq.(6) and (7), rewriting equations (2), (3) and (4) in the dimensionless form are,

$$\frac{\partial u}{\partial t} - v_0 (1 + B \varepsilon e^{\text{int}}) \frac{\partial u}{\partial y} = \frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + \frac{\lambda}{k} (U - u) + 2 \frac{\partial \omega}{\partial y} + \text{Gr} \theta, \quad (8)$$

$$\frac{\partial \theta}{\partial t} - v_0 (1 + B \varepsilon e^{\text{int}}) \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + E \left[\left(\frac{\partial \theta}{\partial y} \right)^2 + \frac{\lambda}{k} u^2 \right], \quad (9)$$

$$\frac{\partial \omega}{\partial t} - v_0 (1 + B \varepsilon e^{\text{int}}) \frac{\partial \omega}{\partial y} = \frac{1}{\beta} \frac{\partial^2 \omega}{\partial y^2}. \quad (10)$$

The corresponding dimensionless boundary conditions takes the form,

$$y = 0 : \quad u = 0, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad \theta = 1,$$

$$y \rightarrow \infty : u \rightarrow (1 + \varepsilon A e^{\text{int}}), \quad \omega \rightarrow 0, \quad \theta \rightarrow 0. \quad (11)$$

III. METHOD OF SOLUTION

In order to solve the above non-linear coupled PDE's (8), (9) and (10) along the boundary conditions (11), we use the double regular perturbation method for the amplitude of fluctuation in terms of $\varepsilon e^{\text{int}}$ and the Eckert number E in the form of

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$$u(y,t) = u_{00}(y) + E u_{01}(y) + \varepsilon (u_{11}(y) + E u_{12}(y)) e^{\text{int}} + O(E^2, \varepsilon^2), \quad (12)$$

$$\theta(y,t) = \theta_{00}(y) + E \theta_{01}(y) + \varepsilon (\theta_{11}(y) + E \theta_{12}(y)) e^{\text{int}} + O(E^2, \varepsilon^2), \quad (13)$$

$$\omega(y,t) = \omega_{00}(y) + E \omega_{01}(y) + \varepsilon (\omega_{11}(y) + E \omega_{12}(y)) e^{\text{int}} + O(E^2, \varepsilon^2). \quad (14)$$

Solving Eqs.(8)-(9) for velocity, temperature and angular velocity fields by substituting (12), (13) and (14), we get

$$\begin{aligned} u(y,t) = & (1 + A_1 e^{-m_2 y} + A_2 e^{-pr y} + A_3 e^{-\beta y}) + E (A_4 e^{-m_2 y} + A_5 e^{-\beta y} + A_6 e^{-pr y} + A_7 e^{-2m_2 y} + A_8 e^{-2pr y} \\ & + A_9 e^{-2\beta y} + A_{10} e^{-(pr+m_2)y} + A_{11} e^{-(\beta+m_2)y} + A_{12} e^{-(pr+\beta)y}) + \varepsilon \{ (A_{13} + A_{14} e^{-m_4 y} + A_{15} e^{-pr y} + A_{16} e^{-m_8 y} + \\ & A_{17} e^{-m_6 y}) + E (A_{18} e^{-m_8 y} + A_{19} + A_{20} e^{-m_4 y} + A_{21} e^{-pr y} + A_{22} e^{-m_2 y} + A_{23} e^{-\beta y} + A_{24} e^{-2pr y} + A_{25} e^{-m_8 y} \\ & + A_{26} e^{-m_6 y} + A_{27} e^{-2m_2 y} + A_{28} e^{-2\beta y} + A_{29} e^{-(m_2+pr)y} + A_{30} e^{-(\beta+m_2)y} + A_{31} e^{-(\beta+pr)y} + A_{32} e^{-(m_2+m_4)y} + \\ & A_{33} e^{-(m_2+m_8)y} + A_{34} e^{-(pr+m_6)y} + A_{35} e^{-(\beta+m_8)y} + A_{36} e^{-(\beta+m_6)y} + A_{37} e^{-(\beta+m_4)y} + A_{38} e^{-(pr+m_8)y} + A_{39} \\ & e^{-(m_2+m_6)y} + A_{40} e^{-(pr+m_4)y} + A_{41} e^{-m_6 y} + A_{42} e^{-m_8 y}) \} e^{\text{int}}. \end{aligned} \quad (15)$$

$$\begin{aligned} \theta(y,t) = & e^{-pr y} + E (B_1 e^{-pr y} + B_2 e^{-2m_2 y} + B_3 e^{-2pr y} + B_4 e^{-2\beta y} + B_5 e^{-m_2 y} + B_6 e^{-\beta y} + B_7 e^{-(pr+m_2)y} + B_8 e^{-(m_2+\beta)y} \\ & + B_9 e^{-(pr+\beta)y}) + \varepsilon \{ B_{10} (e^{-pr y} - e^{-m_4 y}) + E (B_{12} e^{-pr y} + B_{13} e^{-2pr y} + B_{14} e^{-\beta y} + B_{15} e^{-2\beta y} + B_{16} e^{-m_2 y} + B_{17} e^{-2m_2 y} \\ & + B_{18} e^{-(m_2+pr)y} + B_{19} e^{-(m_4+pr)y} + B_{20} e^{-(pr+m_6)y} + B_{21} e^{-(pr+m_8)y} + B_{22} e^{-(\beta+pr)y} + B_{23} e^{-(m_2+\beta)y} + B_{24} e^{-(m_4+\beta)y} + B_{25} \\ & e^{-(m_6+\beta)y} + B_{26} e^{-(m_8+\beta)y} + B_{27} e^{-(m_2+m_4)y} + B_{28} e^{-(m_2+m_6)y} + B_{29} e^{-(m_2+m_8)y} + B_{30} e^{-m_4 y} + B_{31} e^{-m_6 y} + B_{32} e^{-m_8 y}) \} e^{\text{int}}. \end{aligned} \quad (16)$$

$$\omega(y,t) = C_1 e^{-\beta y} + E C_2 e^{-\beta y} + \varepsilon \{ (C_3 e^{-m_6 y} + C_4 e^{-\beta y}) + E (C_5 e^{-m_6 y} + C_6 e^{-\beta y}) \} e^{\text{int}}. \quad (17)$$

Here, the constants $A_i (i = 1 \text{ to } 42)$, $B_j (j = 1 \text{ to } 32)$ and $C_k (k = 1 \text{ to } 6)$ are the functions of non-dimensional parameters which are involved in the problem. Knowing the velocity and temperature fields, the skin friction and the rate of heat transfer are obtained as

$$\tau = \left(\frac{du}{dy} \right)_{y=0} = \left(\frac{du_0}{dy} + \varepsilon e^{\text{int}} \frac{du_1}{dy} \right)_{y=0} = \left(\frac{du}{dy} \right)_{y=0} + \varepsilon |B| \cos(nt + \alpha), \quad (18)$$

$$q = \left(\frac{d\theta}{dy} \right)_{y=0} = \left(\frac{d\theta_0}{dy} + \varepsilon e^{\text{int}} \frac{d\theta_1}{dy} \right)_{y=0} = \left(\frac{d\theta_0}{dy} \right)_{y=0} + \varepsilon |Q| \cos(nt + \beta) \quad (19)$$

where $B = B_r + i B_i$, $Q = Q_r + i Q_i$ are coefficients of $\varepsilon e^{\text{int}}$, $|B|$ = amplitude of the skin friction, $|Q|$ = amplitude of Heat flux, $\tan \alpha = \left(\frac{B_i}{B_r} \right)$ is the phase of the skin friction and $\tan \beta = \left(\frac{Q_i}{Q_r} \right)$ is the phase of the heat flux.

IV. RESULTS AND DISCUSSION

A numerical computation is carried out for various non-dimensional parameters of the physical model on velocity, temperature and angular velocity through the graphs. The variations of velocity, temperature and angular velocity for different values of Gr both positive and negative values are shown in Figs.2-4. In the process of cooling of the plate

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by free convection currents ($Gr > 0$) are carried away from the plate to the free stream and as the free stream is in the upward direction. The free convection current induces the mean velocity and increases the velocity near the boundary in case of air. However, in the case of heating by the free convection currents of the plate ($Gr < 0$) the velocity decreases near the plate is observed from Fig.2. From Fig.3, we observe that as increasing Grashof number Gr , the temperature decreases due to fact that both heating and cooling of the vertical permeable plate away from the plate, the rate of temperature of the flow decreases gradually near the boundary.

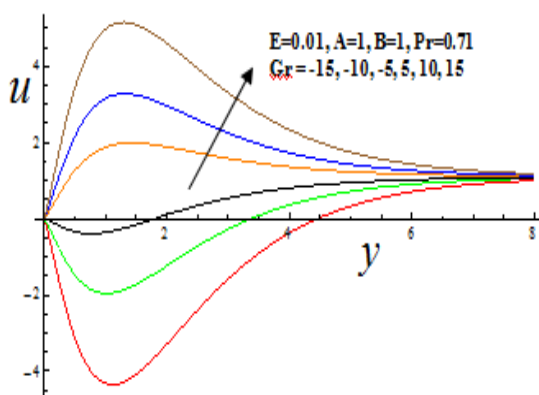


Fig.2: Velocity for different values of Gr .

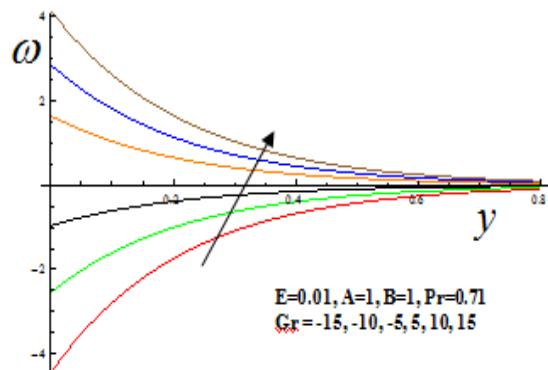


Fig.4: Angular velocity for different values of Gr .

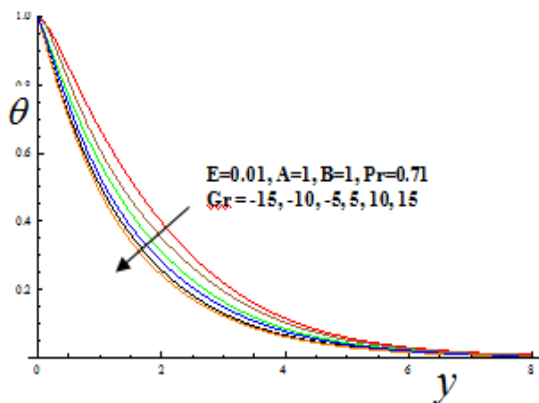


Fig.3: Temperature for different values of Gr .

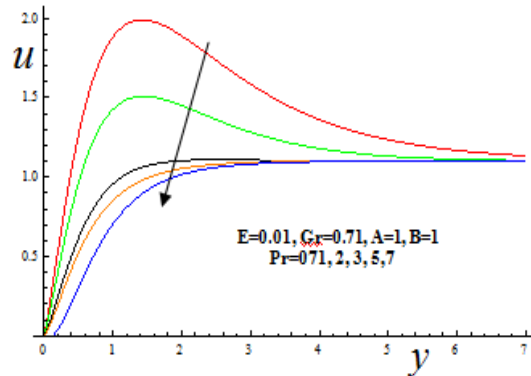


Fig.5: Velocity for different values of Pr .

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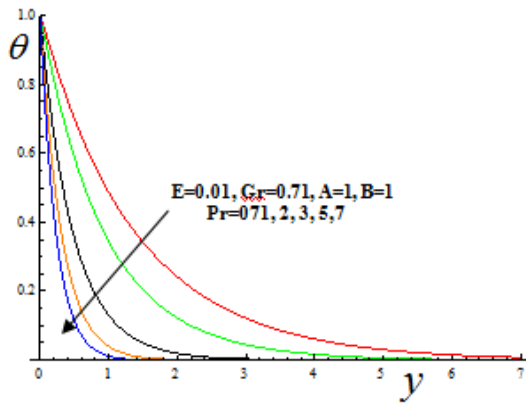


Fig.6: Temperature for different values of Pr .

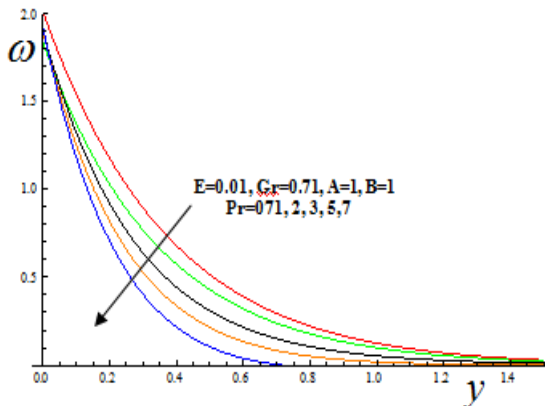


Fig.7: Angular velocity for different values of Pr .

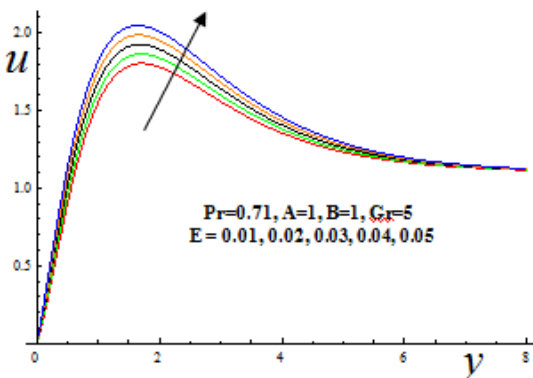


Fig.8: Velocity for different values of E .

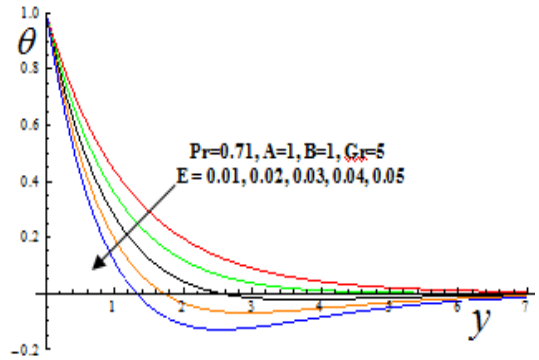


Fig.9: Temperature for different values of E .

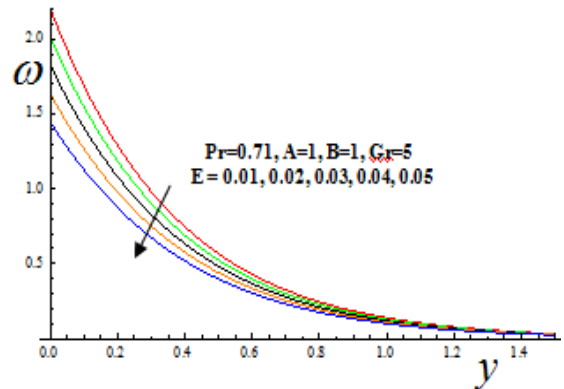


Fig.10: Angular velocity for different values of E .

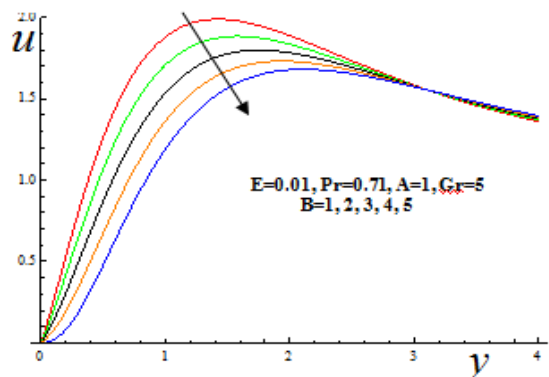


Fig.11: Velocity for different values of B .

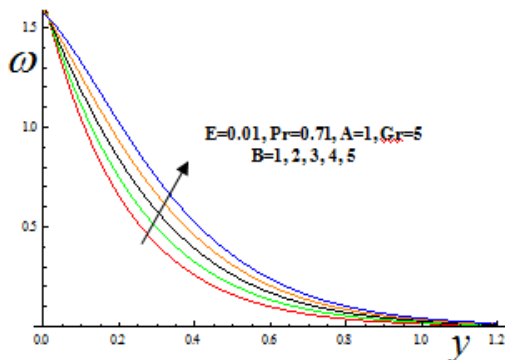


Fig.12: Angular velocity for different values of B .

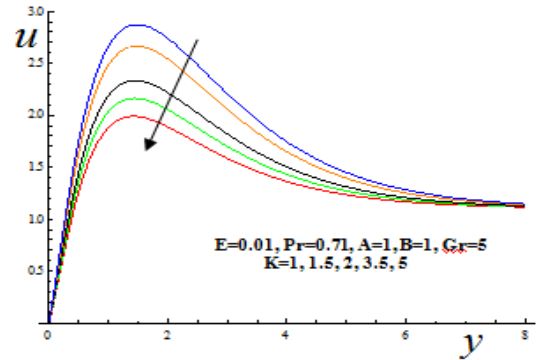


Fig.13: Velocity for different values of K .

Fig.4 shows that an opposite behaviour of angular velocity is that of velocity for both positive and negative values. The angular velocity decreases near the plate and attains a constant value for away from the plate. The characteristics of the fluid for different values of Prandtl number Pr are discussed from Figs.5-7. The velocity, temperature and angular velocity of the fluid decreases near the vertical plate as increases Pr because there is a increment in the viscous diffusion or decrement in the thermal diffusion of the fluid. Figs.8-10 shows the effect of velocity, temperature and angular velocity on Eckert number E respectively. From Fig.8, it is observed that with increasing the viscous dissipation the velocity of the fluid increases moderately nears the vertical permeable plate and becomes constant for away from the plate because of the boundary layer thickness. Fig.9 shows the temperature decreases with increasing of viscous dissipation or decreasing of thermal conductivity/convection. A similar behaviour is seen in the case of angular velocity on E from Fig.10. Figs.11-12 indicate the variations of velocity and angular velocity for different values of the amplitude of the fluctuating free stream velocity parameter B of the fluid respectively. From Fig.11 indicates that with the increasing in the amplitude of the fluctuating free stream velocity B velocity decreases near the plate by the increasing the boundary layer thickness. This is because of the effect of free stream velocity is higher than the suction velocity. But in the case of angular velocity an opposite behaviour is seen with the increasing of amplitude of the free stream velocity B from Fig.12. The effect of permeable parameter on velocity is observed from Fig.13. It is observed that with increasing permeable parameter K of the porous media increases the moment of the flow particle in practically and the same is depicted in the Fig.13. But in the case of temperature and angular velocity, there is no significant difference on the permeable parameter K .

V. CONCLUSION

A theoretical analysis is performed to investigate the effects of oscillatory flow of a couple stress fluid over a semi-infinite vertical permeable plate in a porous medium along the viscous dissipation and Ohmic effects analytically. The following conclusions are drawn based on the analysis. As increases the Grashof number Gr , the velocity and angular velocity increases, whereas the temperature decreases near the vertical plate respectively. As increases the Prandtl number Pr , the velocity, temperature and angular velocity decreases. As increases the Eckert number E , the velocity increases but an opposite behaviour is seen in the case of temperature and angular velocity. As increases the amplitude of the fluctuating free stream velocity parameter B , the velocity decreases, temperature and angular velocity increases respectively, but variation less in the temperature profile. As increases the permeable parameter K of the porous media, the velocity decreases and there is no variations in temperature and angular velocity distributions.

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