

Signed Roman Domination Number of Corona Graph $C_n \odot K_m$

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ABSTRACT: Domination in graphs is an emerging area of research in graph theory and at present it has been studied extensively. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [5, 6]. Dominating sets have applications in diverse areas such as logistics and Communication Networks design, mobile computing, resource allocation and telecommunication etc.

In Discrete Mathematics, Product of graphs occurs naturally as tools in combinatorial constructions. They give rise to important classes of graphs and deep structural problems. Frucht and Harary [4] introduced a new product on two graphs G_1 and G_2 , called corona product denoted by $G_1 \odot G_2$. In this paper we present some results on Signed Roman dominating functions of corona graph of a cycle with a complete graph.

KEYWORDS: Corona Product, Signed Roman dominating function, Signed Roman domination number, Total Signed Roman dominating function, Total Signed Roman domination number.

I. INTRODUCTION

In modern mathematics, Graph theory finds applications in various branches of Science & Technology. Domination in graphs has a wide range of applications to many fields like Engineering, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[2], Cockayne, E.J. and Hedetniemi, S.T. [3] have studied various domination parameters of graphs.

Recently, dominating functions in domination theory have received much attention. A purely graph – theoretic motivation is given by the fact that the dominating function problem can be seen as a proper generalization of the classical domination problem.

The Signed Roman dominating function of a graph G was introduced by Ahangar et al.[1].

Among the variations of domination, A Signed Roman dominating function of a graph $G(V, E)$ is a function $f : V \rightarrow \{-1, 1, 2\}$ such that $\sum_{u \in N_G[v]} f(u) \geq 1$, for each $v \in V$. Then the Signed Roman domination problem is to find

a Signed Roman dominating function of minimum weight for a graph.

Frucht and Harary [3] introduced a new product on two graphs G_1 and G_2 , called corona product denoted by $G_1 \odot G_2$. The automorphisms of a graph G form a permutation group and it is known as the group of the graph G . The object of corona product is to construct a new and simple operation on two graphs G_1 and G_2 with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2 .

In this paper, we present some results on Signed Roman domination and Total Signed Roman domination for the corona product graphs.

II. CORONA PRODUCT OF TWO GRAPHS

The corona product of two graphs G and H is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^{th} - vertex of G with all the vertices of the i^{th} - copy of H .

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III. SIGNED ROMAN DOMINATION NUMBER OF CORONA PRODUCT GRAPH $C_n \odot K_m$

In this section we present the Signed Roman dominating functions of corona product graph of a cycle with a complete graph. The Signed Roman domination number and total Signed Roman domination number of the graph is obtained.

Some properties of the corona product graph $G = C_n \odot K_m$ are presented in [8]. We require the following theorem whose proof can be found in Siva Parvathi, M. [7]. Authors also studied minimal signed Roman dominating functions of corona product graph a path with a star [9].

Theorem 2.1: The degree of a vertex v in $G = C_n \odot K_m$ is given by

$$d(v) = \begin{cases} m + 2, & \text{if } v \in C_n, \\ m, & \text{if } v \in K_m. \end{cases}$$

Definition: Let $G(V, E)$ be a graph. A function $f: V \rightarrow \{-1, 1, 2\}$ is called a **Signed Roman dominating function (SRDF)** of G if $f(N[v]) \geq 1$, for each $v \in V$ and satisfying the condition that every vertex

u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$.

Theorem 3.2:- The Signed Roman domination number of $C_n \odot K_m$ is $\begin{cases} 2n & \text{if } m \text{ is even} \\ n & \text{if } m \text{ is odd} \end{cases}$.

Proof: Let $G = C_n \odot K_m$ be a graph with vertex set V .
The degree of a vertex v in V is given by

$d(v) = \begin{cases} m + 2, & \text{if } v \in C_n, \\ m, & \text{if } v \in K_m. \end{cases}$

$$d(v) = \begin{cases} m + 2, & \text{if } v \in C_n, \\ m, & \text{if } v \in K_m. \end{cases}$$

There are two possible cases arise.

Case 1: Suppose m is even.

Define a function $f: V \rightarrow \{-1, 1, 2\}$ by $f(v) = \begin{cases} 2, & \text{if } v \in C_n, \\ -1, & \text{for } \frac{m}{2} \text{ vertices of } K_m, \\ 1, & \text{otherwise.} \end{cases}$

Now we check the condition for Signed Roman dominating function at every vertex of G . we classify the vertices according to its degree.

Subcase 1: Let $v \in C_n$ be such that $d(v) = m + 2$ in G .

Here v is adjacent to 2 vertices of C_n and m vertices of the corresponding copy of K_m .

Then $N[v]$ contains m vertices of K_m and three vertices of C_n in G .

Now $\sum_{u \in N[v]} f(u) = 2 + 2 + 2 + \frac{m}{2}(-1) + \frac{m}{2}(1) = 6 > 1$.

Subcase 2: Let $v \in K_m$ be such that $d(v) = m$ in G .

Here the vertex v is adjacent to $m - 1$ vertices of K_m and one vertex of C_n .

Then $N[v]$ contains m vertices of K_m and one vertex of C_n in G .

Now $\sum_{u \in N[v]} f(u) = 2 + \frac{m}{2}(-1) + \frac{m}{2}(1) = 2 > 1$.

Therefore for all possibilities, we get

$$\sum_{u \in N[v]} f(u) \geq 1 \quad \forall v \in V.$$

Let u be any vertex in G such that $f(u) = -1$ and $v \neq u$ be a vertex in G such that $f(v) = 2$.

Then $u \in K_m$ and $v \in C_n$.

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Obviously if $u \in K_m$ then u is adjacent to $v \in C_n$, since every vertex in C_n is adjacent to every vertex in the corresponding copy of K_m .

This implies that f is a SRDF of G .

Now

$$f(V) = \sum_{u \in V} f(u) = \underbrace{2 + 2 + \dots + 2}_{n\text{-times}} + n \left[\frac{m}{2}(-1) + \frac{m}{2}(1) \right] = 2n + 0 = 2n.$$

Hence by the definition of Signed Roman Domination number,

$$\gamma_{SR}(C_n \odot K_m) \leq 2n \text{ --- (1)}.$$

Since the degree of each vertex of G is either m or $m+2$, it follows that if ' f ' is a Signed Roman dominating function of G , then we can see that among ' m ' vertices of K_m at most $\frac{m}{2}$ vertices can have functional value -1 and at least one adjacent vertex v of a vertex u such that $f(u) = -1$ must have functional value 2. As there are ' n ' copies of K_m , we must assign the functional value '2' for a vertex which adjacent to the corresponding copy of K_m .

Therefore sum of the functional values of each copy of K_m is greater than or equal to 2.

Then $f(V) = \sum_{u \in V} f(u) \geq 2n$.

This is true for any Signed Roman dominating function of G .

Therefore $\min\{f(V) / 'f' \text{ is a Signed Roman dominating function of } G\} \geq 2n$.

$$\gamma_{SR}(C_n \odot K_m) \geq 2n \text{ --- (2)}.$$

Hence $\gamma_{SR}(C_n \odot K_m) = 2n$.

Case 2: Suppose m is odd.

Define a function $f: V \rightarrow \{-1, 1, 2\}$ by $f(v) = \begin{cases} 2, & \text{if } v \in C_n, \\ -1, & \text{for } \left\lfloor \frac{m}{2} \right\rfloor \text{ vertices of } K_m, \\ 1, & \text{otherwise.} \end{cases}$

If m is odd then we can assign the functional value -1 for $\left\lfloor \frac{m}{2} \right\rfloor = \frac{m-1}{2}$ vertices of K_m and the functional value 1 for $\frac{m-1}{2}$ vertices of K_m .

In similar lines to Case 1 we can see that the condition of signed Roman dominating function is satisfied at the vertices $v \in C_n$ and $v \in K_m$ of G .

Hence f is a SRDF of G .

Now

$$f(V) = \sum_{u \in V} f(u) = \underbrace{2 + 2 + \dots + 2}_{n\text{-times}} + n \left[\frac{m+1}{2}(-1) + \frac{m-1}{2}(1) \right] = 2n - n = n.$$

Hence by the definition of Signed Roman Domination number,

$$\gamma_{SR}(C_n \odot K_m) \leq n \text{ --- (3)}.$$

Since the degree of each vertex of G is either m or $m+2$, it follows that if ' f ' is a Signed Roman dominating function of G , then we can see that among ' m ' vertices of K_m at most $\frac{m+1}{2}$ vertices can have functional value -1 and at least one adjacent vertex v of a vertex u such that $f(u) = -1$ must have functional value 2. As there are ' n ' copies of K_m , we must assign the functional value '2' for a vertex which adjacent to the corresponding copy of K_m .

Therefore sum of the functional values of each copy of K_m is greater than or equal to 1.

Then $f(V) = \sum_{u \in V} f(u) \geq n$.

This is true for any Signed Roman dominating function of G .

Therefore $\min\{f(V) / 'f' \text{ is a Signed Roman dominating function of } G\} \geq n$.

$$\gamma_{SR}(C_n \odot K_m) \geq n \text{ --- (4)}.$$

Hence $\gamma_{SR}(C_n \odot K_m) = n$.

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Definition: Let $G (V, E)$ be a graph. A function $f : V \rightarrow \{-1, 1, 2\}$ is called a **Total Signed Roman dominating function** (TSRDF) of G if $f(N(v)) = \sum_{u \in N(v)} f(u) \geq 1$, for each $v \in V$ and satisfying the condition that every

vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$.

Theorem 3.3:- The Total Signed Roman domination number of $C_n \odot K_m$ is $\begin{cases} 2n & \text{if } m \text{ is even} \\ 3n & \text{if } m \text{ is odd} \end{cases}$.

Proof: Let $G = C_n \odot K_m$ be a graph with vertex set V . There are two possible cases arise.

Case 1: Suppose m is even.

Define a function $f: V \rightarrow \{-1, 1, 2\}$ by $f(v) = \begin{cases} 2, & \text{if } v \in C_n, \\ -1, & \text{for } \frac{m}{2} \text{ vertices of } K_m, \\ 1, & \text{otherwise.} \end{cases}$

In similar lines to Case 1 of Theorem 3.2, we have

$$\sum_{u \in N(v)} f(u) = 2 + 2 + \frac{m}{2}(-1) + \frac{m}{2}(1) = 4 > 1, \text{ if } v \in C_n.$$

$$\text{also } \sum_{u \in N(v)} f(u) = 2 + \frac{m}{2}(-1) + \left(\frac{m}{2} - 1\right)(1) = 2 - 1 = 1, \text{ if } v \in K_m \text{ and } f(v) = 1.$$

$$\text{and } \sum_{u \in N(v)} f(u) = 2 + \left(\frac{m}{2} - 1\right)(-1) + \left(\frac{m}{2}\right)(1) = 2 + 1 = 3, \text{ if } v \in K_m \text{ and } f(v) = -1.$$

Therefore for all possibilities, we get

$$\sum_{u \in N(v)} f(u) \geq 1 \quad \forall v \in V.$$

Also if a vertex $u \in K_m$ such that $f(u) = -1$ then u is adjacent to a vertex $v \in C_n$ such that $f(v) = 2$, since every vertex in C_n is adjacent to every vertex in the corresponding copy of K_m .

This implies that f is a TSRDF of G .

$$\text{Now } f(V) = \sum_{u \in V} f(u) = \underbrace{2 + 2 + \dots + 2}_{n\text{-times}} + n \left[\frac{m}{2}(-1) + \frac{m}{2}(1) \right] = 2n + 0 = 2n.$$

Hence by the definition of Total Signed Roman Domination number,

$$\gamma_{TSR}(C_n \odot K_m) \leq 2n \quad \dots (1).$$

If 'f' is a Total Signed Roman dominating function of G , then we can see that among 'm' vertices of K_m atmost $\frac{m}{2}$ vertices have functional value -1 and atleast one adjacent vertex v of a vertex u such that $f(u) = -1$ must have functional value 2. As there are 'n' copies of K_m , we must assign the functional value '2' for a vertex which adjacent to the corresponding copy of K_m .

Therefore sum of the functional values of each copy of K_m is greater than or equal to 2.

$$\text{Then } f(V) = \sum_{u \in V} f(u) \geq 2n.$$

This is true for any Total Signed Roman dominating function of G .

Therefore $\min\{f(V) / \text{'f' is a Total Signed Roman dominating function of } G\} \geq 2n$.

$$\gamma_{TSR}(C_n \odot K_m) \geq 2n \quad \dots (2).$$

Hence $\gamma_{TSR}(C_n \odot K_m) = 2n$.

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Case 2: Suppose m is odd.

Define a function $f: V \rightarrow \{-1, 1, 2\}$ by $f(v) = \begin{cases} 2, & \text{if } v \in C_n, \\ -1, & \text{for } \frac{m-1}{2} \text{ vertices of } K_m, \\ 1, & \text{otherwise.} \end{cases}$

If m is odd then we can assign the functional value 1 for $\frac{m+1}{2}$ vertices of K_m and the functional value -1 for $\frac{m-1}{2}$ vertices of K_m .

We can verify in similar lines as in Case 1 that f is a TSRDF of G .

Now

$$f(V) = \sum_{u \in V} f(u) = \underbrace{2 + 2 + \dots + 2}_{n\text{-times}} + n \left[\frac{m-1}{2}(-1) + \frac{m+1}{2}(1) \right] = 2n + n = 3n.$$

Hence by the definition of Total Signed Roman Domination number,

$$\gamma_{TSR}(C_n \odot K_m) \leq 3n \quad \text{--- (3)}.$$

In G , the degree of each vertex is either m or $m+2$. If ' f ' is a Total Signed Roman dominating function of G , then we can see that among ' m ' vertices of K_m atmost $\frac{m-1}{2}$ vertices have functional value -1, and $\frac{m+1}{2}$ vertices have functional value 1 and at least one adjacent vertex v of a vertex u such that $f(u) = -1$ must have functional value 2. As there are ' n ' copies of K_m , we must assign the functional value '2' for a vertex which adjacent to the corresponding copy of K_m .

Therefore sum of the functional values of each copy of K_m is greater than or equal to 3.

Then $f(V) = \sum_{u \in V} f(u) \geq 3n$.

This is true for any total Signed Roman dominating function of G .

Therefore $\min\{f(V) / 'f' \text{ is a Total Signed Roman dominating function of } G\} \geq n$.

$$\gamma_{TSR}(C_n \odot K_m) \geq 3n \quad \text{--- (4)}.$$

Hence $\gamma_{TSR}(C_n \odot K_m) = 3n$.

REFERENCES

- [1] Ahangar, H.A., Henning, M.A., Zhao, Y., Lowenstein, C., Samodivkin, V. – Signed Roman domination in graphs, J. Comb. Optim., (2012).
- [2] Allan, R.B. and Laskar, R.C. – On domination, independent domination numbers of a graph. Discrete Math., 23 (1978), 73 – 76.
- [3] Cockayne, E.J. and Hedetniemi, S.T. - Towards a theory of domination in graphs. Networks, 7 (1977), 247 – 261.
- [4] Frucht, R. and Harary, F. - On the corona of Two Graphs, Aequationes Mathematicae, Volume 4, Issue 3 (1970), 322 – 325.
- [5] Haynes, T.W., Hedetniemi, S.T. and Slater, P.J. - Domination in Graphs: Advanced Topics, Marcel Dekker, Inc., New York, (1998).
- [6] Haynes, T.W., Hedetniemi, S.T. and Slater, P.J. - Fundamentals of domination in graphs, Marcel Dekker, Inc., New York, (1998).
- [7] Siva Parvathi, M – Some studies on dominating functions of corona product graphs, Ph. D Thesis, Sri Padmavati Mahila Visvavidyalayam, Tirupati, Andhra Pradesh, India, (2013).
- [8] Siva Parvathi, M and Maheswari, B. - Minimal Dominating Functions of Corona Product Graph of a Cycle with a Complete Graph in International Journal of Computer Engineering & Technology, Volume 4, Issue 4, July – August (2013), pp.248 – 256.
- [9] Siva Parvathi, M and Maheswari, B – Minimal Signed Roman dominating functions of a corona product graph of a path with a star - International Journal of Mathematics and Computer Applications Research, Vol. 6, Issue 2, Apr 2016, 67-72.