

# Unsteady MHD Boundary Layer Flow and Mass Transfer for Maxwell Fluid over a Stretching Sheet Embedded in a Porous Medium with Chemical Reaction

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**ABSTRACT:** In this paper we analysed the unsteady flow and mass transfer of a Maxwell fluid past a stretching sheet embedded in porous medium subject to a transverse magnetic field in the presence of chemical reaction with velocity slip and convective mass boundary condition. The governing equations are reduced to nonlinear ordinary differential equations by means of similarity transformations. These equations are then solved numerically by applying Runge-Kutta-Fehlberg method along with shooting technique. The velocity and concentration distributions are discussed graphically for different parameters. The numerical results that obtained are in good agreement with the existing literature for the limited case.

**KEYWORDS:** Maxwell fluid, unsteady flow, Porous medium, Mass transfer, MHD flow, chemical reaction..

## I. INTRODUCTION

Now a day's the boundary layer flow of non-Newtonian fluids have been attracting the attention of many researchers because most of the industrial fluids are non-Newtonian in nature. It has large number of industrial applications and manufacturing processes which includes synthetic fibres, food stuffs to extrusion of molten plastic, printing ink and in some flows of polymer solutions. In literature the majority of non-Newtonian fluid models are concerned with the fluids of grade two or three. But these fluid models do not predict the effect of stress relaxation. A subclass of fluids namely Maxwell model can predict the stress relaxation and hence become more popular. Recently Tan et al. [1], Hayat et al. [2], Fetecau and Zierep [3] studied the flows of Maxwell fluid. Choi et al. [4] examined the two-dimensional, steady incompressible flow of Maxwell fluid. The boundary layer flow problem over a stretching surface was introduced by Sakiadis [5].

Mass transfer phenomenon is mainly used for various systems and mechanisms which involves molecular and convective transport of atoms and molecules. The driving force for mass transfer is the difference in concentration. The study of MHD flow has been a subject of great interest due to its widespread applications in MHD generators, accelerators, designing cooling systems with liquid metals. Mostly the research on convective flows in porous media has been directed to the problems of steady flows over heated bodies embedded in fluid saturated porous media. The first study on unsteady boundary layer flow on a flat surface in porous media was made by Johnson and Cheng [6]. Zhao et al. [7] studied unsteady boundary layer natural convection heat transfer of Maxwell viscoelastic fluid. Mukhopadhyay et al. [8] examined heat transfer flow of Maxwell fluid past an unsteady stretching permeable surface embedded in porous medium with thermal radiation. Recently Haritha et al. [9] discussed MHD heat and mass transfer of the unsteady flow of a Maxwell fluid over a stretching surface with Navier slip and convective boundary conditions.

The present paper has been taken to analyze the unsteady flow and mass transfer of a Maxwell fluid past a stretching sheet embedded in porous medium subject to a transverse magnetic field in the presence of chemical reaction

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

with velocity slip and convective mass boundary condition. The effects of different parameters like Biot number, unsteadiness parameter, slip parameter, porosity parameter and Schmidt number on velocity and concentration distributions are discussed.

## II. MATHEMATICAL FORMULATION

Consider the unsteady two dimensional boundary-layer MHD flows and mass transfer of a Maxwell fluid over a stretching surface which coincides with the plane  $y = 0$ . The flow is in the region  $y > 0$  and is subjected to transverse magnetic field of strength  $B(x) = B_0(1 - \alpha t)^{-1/2}$  applied normally to the surface,  $B_0$  is the initial strength of the magnetic field. Sheet moving with the velocity of  $u_w(x, t) = bx/(1 - \alpha t)$ , where  $b, \alpha$  are positive constants with dimensions. By applying the above assumptions the boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left( u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k_p^*} u - \frac{\sigma B^2}{\rho} \left( u + \lambda v \frac{\partial u}{\partial y} \right) \tag{2}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty) \tag{3}$$

Subject to the boundary conditions

$$u = u_w = u_w + Nv \frac{\partial u}{\partial y}, v = 0, -D \frac{\partial C}{\partial y} = h_s(C_f - C) \text{ at } y = 0 \tag{4}$$

$$u \rightarrow 0, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \tag{5}$$

where,  $u$  and  $v$  are fluid velocity components along  $x$  and  $y$ -axes, respectively,  $\nu$  is kinematic viscosity,  $\lambda = \lambda_0(1 - \alpha t)$ , is the relaxation time,  $\lambda_0$  is a constant,  $\rho$  is density of the fluid,  $\sigma$  is electrical conductivity,  $k_p^*$  is the permeability of the porous medium,  $D$  is the mass diffusivity,  $k_1(t) = k_0/(1 - \alpha t)$  is the time dependent reaction rate,  $C_\infty$  is the concentration far away from sheet,  $N = N_0(1 - \alpha t)^{1/2}$  is the velocity slip factor which changes with time,  $N_0$  is the initial value of the velocity slip factor,  $h_s$  is the concentration transfer coefficient and  $C_f$  is the ambient fluid concentration.

The surface concentration of the sheet vary with the square of distance  $x$  from the slot and time  $t$  in the form

$$C_f(x, t) = C_\infty + C_{ref} \frac{bx^2}{2\nu(1-\alpha t)^{3/2}} \tag{6}$$

Where  $C_{ref}$  is reference concentration, which can be taken as  $C_{ref} = C_\infty$  in the present study.

$$\psi(x, y, t) = \left( \frac{\nu b}{1-\alpha t} \right)^{1/2} x f(\eta), \quad \eta = \sqrt{\frac{b}{\nu(1-\alpha t)}} y$$

$$C(x, y, t) = C_\infty + C_{ref} \frac{bx^2}{2\nu(1-\alpha t)^{3/2}} \phi(\eta), \quad \phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty} \tag{7}$$

Where,  $\psi$  is the stream function with  $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$  and  $\eta$  is the stream variable.

Using equation (7), the equations (2), and (3) reduce to

$$f''' + ff'' - f'^2 + \beta(2ff'f'' - f^2f''') - A \left( f' + \frac{\eta}{2} f'' \right) - k_p f' - M^2(f' - \beta ff'') = 0 \tag{8}$$

$$\phi'' + Sc \left( f\phi' - 2f'\phi - \frac{A}{2}(3\phi + \eta\phi') - \gamma\phi \right) = 0 \tag{9}$$

The boundary conditions (4) and (5) in dimensionless form

$$f(0) = 0, f'(0) = 1 + hf''(0), \phi'(0) = -Bi(1 - \phi(0)) \tag{10}$$

$$f'(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \tag{11}$$

Where  $\beta = \lambda_0 b$  is the Maxwell Parameter,  $A = \alpha/b$  is the unsteadiness parameter,  $M = \sigma B_0^2 / \rho b$  is the magnetic field parameter,  $k_p = \nu / b k_p^*$  is the porosity parameter,  $h = N_0 \sqrt{b\nu}$  is velocity slip parameter,  $Sc = \nu / D$  is the Schmidt number,  $\gamma = k_0 / b$  is the chemical reaction parameter and  $Bi = \frac{h_s}{D} \sqrt{\nu b / U_w}$  is the mass transfer Biot number.

The non-dimensional expressions of the skin friction coefficient and Sherwood numbers can be written as follows;

$$C_f \sqrt{Re_x} = (1 + \beta) f''(0), Sh_x / \sqrt{Re_x} = -\phi'(0) \tag{22}$$

Where  $Re_x = u_w x / \nu$  is a local Reynolds number.

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

The ordinary differential equations (8) and (9) are coupled and highly non – linear. These equations with the boundary conditions (10) and (11) are solved using Runge-Kutta Fehlberg method and obtained numerical solutions. The results of the present paper has been compared with those SwatiMukhopadhyay [9] and Haritha et al. [10] and found to be good agreement (Table 1).

**Table 1:** Values of  $f''(0)$  for various values of Maxwell parameter  $\beta$  when  $M = 0.2$  with  $A = k_p = h = Bi = 0$

$\beta$	SwatiMukhopadhyay [9]	Haritha et al. [10]	Present results
0.0	1.095444	1.095378	1.095378
0.2	1.188270	1.188243	1.188243
0.4	1.275878	1.275810	1.275810
0.6	1.358732	1.358698	1.358698

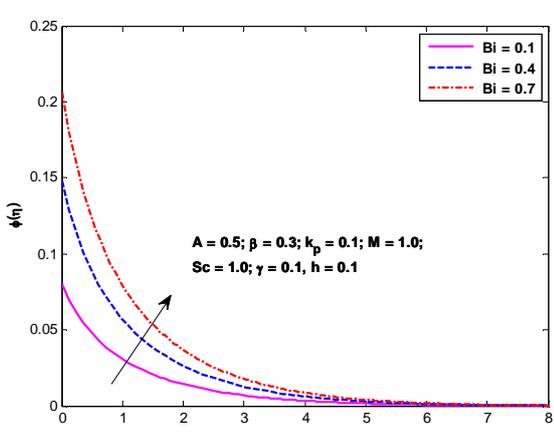
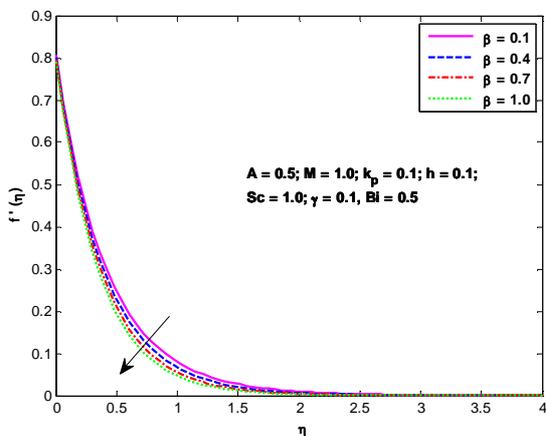
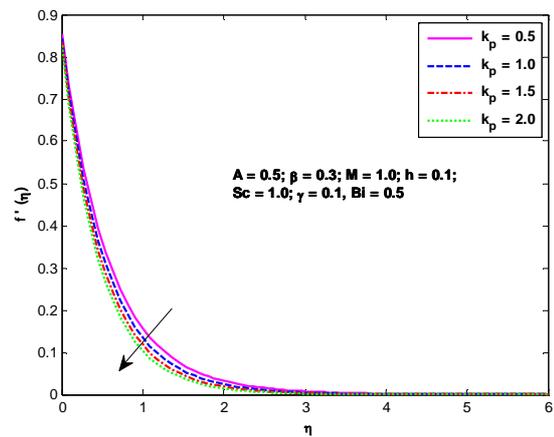
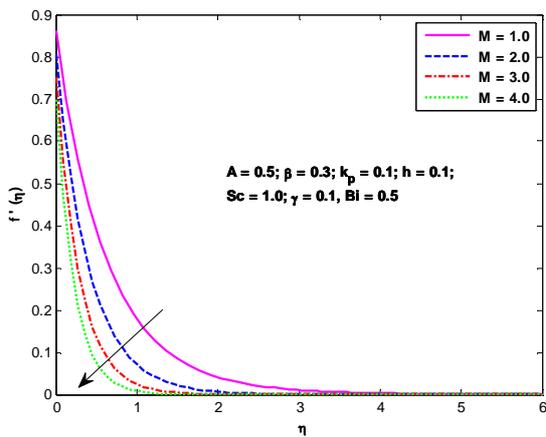
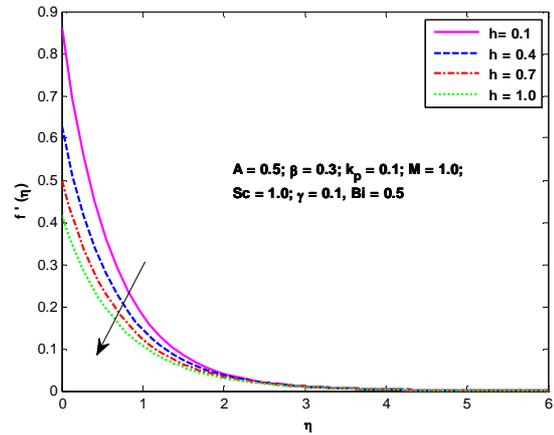
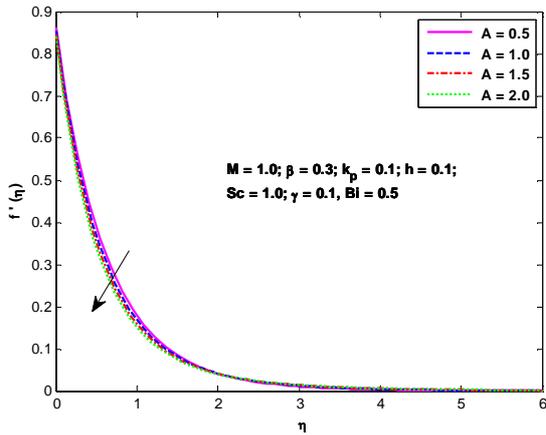
### III. RESULTS AND DISCUSSIONS

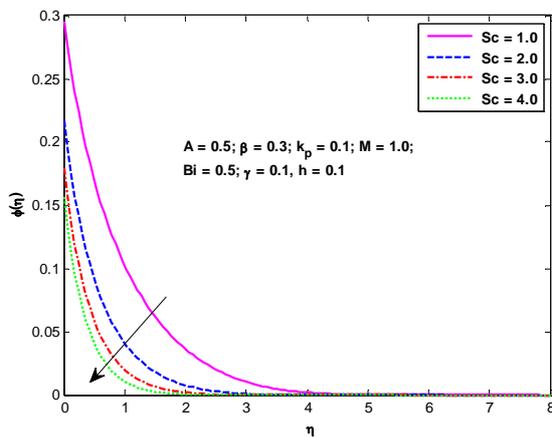
The physical interpretation and behavior of the embedded parameters are shown graphically in Figs 1-11.

**Fig. 1** represents the effect of unsteadiness parameter (A) on the velocity component. It is seen that the velocity decreases with the increase of A and implies reduction in the thickness of boundary layer near the wall. As the unsteadiness parameter (A) increases less mass is transferred from the fluid hence concentration decreases. **Fig. 2** exhibits the velocity distribution for different values of magnetic parameter M. It shows that the velocity decreases with increasing values of magnetic field parameter (M). This is due to the resistance offered by Lorentz force acting in the normal direction.

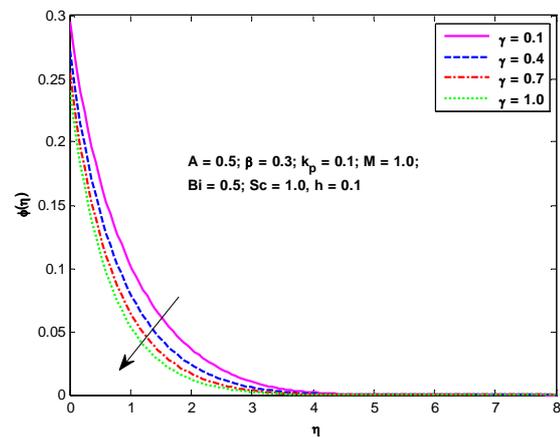
Effect of Maxwell parameter ( $\beta$ ) on velocity is shown in **Fig. 3**. Increasing values of  $\beta$  reduces the velocity and hence decreases the boundary layer thickness. **Fig. 4** depicts the variation of velocity with increasing slip parameter (h). It shows that the velocity decreases with increasing values of h. The effect of permeability parameter ( $k_p$ ) on velocity profile is exhibited in **Fig 5**. As the porosity of the medium increases the value of  $\alpha$  decreases and for smaller values of  $\alpha$  velocity increases. Thus an increase in the permeability parameter leads to a decrease of the horizontal velocity.

**Fig. 6** shows that the concentration profile increases with the increase of mass transfer Biot number Bi. The variation of concentration with chemical reaction parameter  $\gamma$  and Schmidt number Sc are illustrated in **Figs. 7 and 8**. It shows that with the increasing values of  $\gamma$  and Sc the concentration decreases.





**Fig. 7:** Variation of  $Sc$  on  $\theta(\eta)$



**Fig. 8:** Variation of  $\gamma$  on  $\theta(\eta)$

#### IV. CONCLUSION

At the end we conclude that

- It is shown that the fluid velocity decreases at the sheet due to an increase in the unsteadiness parameter.
- An increase in the Maxwell parameter  $\beta$  leads to decrease in velocity of the fluid.
- The fluid velocity decreases with the increase of slip parameter
- The concentration is enhanced by increasing the mass transfer Biot number.
- The fluid velocity decreases with the increase of porous parameter.
- The concentration decreases for increasing values of Schmidt's number and chemical reaction parameter.

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