

# Nanofluid Flow past a Wedge in the Presence of Thermal Radiation with Convective Boundary Condition

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**ABSTRACT:** The purpose of the present paper is to investigate the steady boundary layer flow heat and mass transfer characteristics of incompressible nanofluid over a wedge in the presence of thermal radiation, thermophoresis, Brownian motion. The governing nonlinear partial differential equations of mass, momentum, energy and concentration are transformed into a set of ordinary differential equations by using suitable similarity transformations and are solved numerically by using Keller-Box method. The effect of pertinent parameters on velocity, temperature and concentration profiles as well as the skin friction, Nusselt number and Sherwood number are analyzed.

**KEYWORDS:** Nanofluid, Wedge, Thermal Radiation, Convective Boundary Condition.

## I. INTRODUCTION

Nanofluids are prepared by engineered colloidal suspensions of metallic and non-metallic nano particles base in fluids such as water, ethylene glycol, oil and polymer solutions. In recent years, great attention has been developed in the study of nanofluids, which was coined by Choi [1]. Terkrov et al [2] concluded that the nanoparticles added in base fluid causes an increase in thermal conductivity and therefore heat transfer when compared to non – metallic fluid. This increase is due to various types of nanoparticle materials, which refers to alumina, titanium oxide, copper, silver, their volume fraction and other factors. Buongiorno [3] proved that intensified turbulence and thermal dispersion are caused by the motion of nanoparticles by assuming seven slip mechanisms. That is Brownian diffusion, diffusiophoresis, fluid drainage, inertia, thermophoresis, Magnus effects and gravity. Thermophoresis and Brownian diffusion are the most important slip mechanisms among the seven in nanofluids. Nield and Kuznetsov [4] studied onset of convection in a horizontal layer through a porous medium saturated by nanofluid and later, considered three cases of boundaries such as free-free, rigid-rigid and rigid-free boundaries using Brinkman model. The problem of Cheng-Minkowycz of natural convection over a vertical plate embedded in a porous medium saturated by a nanofluid was examined by Nield and Kuznetsov [5]. Thermal radiation plays a vital role in controlling heat transfer in industry. At high temperature, thermal radiation can significantly effect the temperature distribution and the heat transfer in the boundary layer flow. Recently Mukungu James et al [6] studied the effects of variable viscosity of nanofluid past a permeable wedge through porous medium in the presence of thermal radiation and chemical reaction. The effects of thermal radiation and velocity slip condition on MHD stagnation point flow of nanofluid due to stretching sheet was studied by Rizwan Ul Haq [7]. The fluid flows over a wedge have several thermal engineering applications such as ground water pollution, crude oil extraction and heat exchangers etc., Falkner and Skan [8] pioneer two-dimensional wedge flows. Postelnicu and Pop[9] analyzed the effects of Falkner-Skan boundary layer flow of power-law fluid over a stretching wedge. Srinivasacharya et al [10] studied the steady laminar MHD boundary layer flow, heat and mass transfer in a nanofluid past a wedge with variable magnetic field. Ishak et al [11] made an analysis on steady MHD boundary-layer flow along a wedge saturated in micro polar fluid with variable magnetic field. The convective heat transfer has great importance in procedures where high temperatures are involved. For example, nuclear plants, storage of thermal energy etc., Due to its wide range of engineering and industrial processes the convective boundary conditions are more practical including material drying, transpiration cooling process etc., and the boundary layer flow

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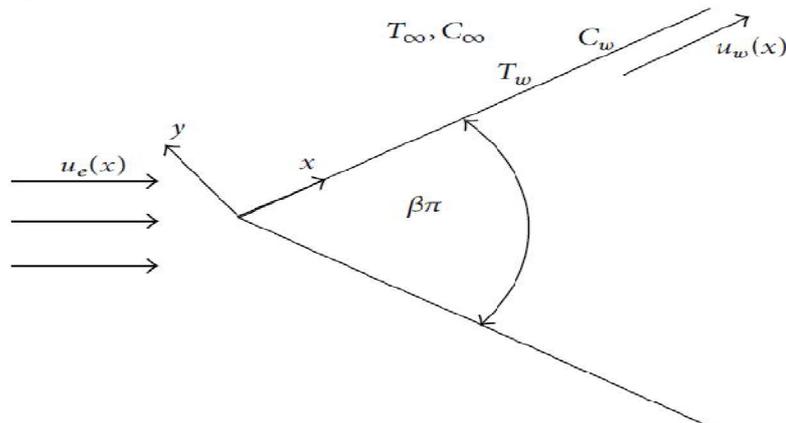
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of a nanofluid over a stretching sheet with convective boundary condition was analysed by Makinde et al [12]. Nagendramma et al [13] analyzed the effects of MHD heat and mass transfer flow over a stretching wedge in the presence of viscous dissipation and wall slip with convective boundary condition.

## II. MATHEMATICAL FORMULATION

Let us consider a steady two-dimensional boundary layer flow along a stretching wedge heated by flowing fluid at a temperature  $T_f$  with variable heat transfer coefficient  $h(x)$  saturated in a nanofluid with velocity  $u_w(x)$  in the presence of thermal radiation as shown in fig 1. The x-axis is along the stretching surface of the wedge and y-axis normal to it. The angle of the wedge is given by  $\Omega = \beta\pi$  where  $\beta = \frac{2m}{m+1}$  is the wedge angle parameter and m is the pressure gradient parameter. It should be observed that the case of  $u_w(x) > 0$  relates to a stretching wedge and  $u_w(x) < 0$  relates to a shrinking wedge. We assume that the free stream velocity is  $u_e(x)$ ,  $T_w$  and  $C_w$  are the temperature and concentration at the wall of the wedge respectively,  $T_\infty$  and  $C_\infty$  are the ambient temperature and ambient concentration respectively.



**Fig. 1 Physical model of the problem**

The thermophoresis and Brownian motion are taken into account. The basic governing equations describing the conservation of mass, momentum, energy and concentration respectively are as follows.

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{K}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \tau \left[ D_B \left( \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where u and v are the velocity components in x and y directions respectively,  $\nu$  is the kinematic viscosity,  $\rho$  is the density of the fluid, T is the fluid temperature, C is the fluid concentration,  $c_p$  is the specific heat at constant pressure, K is the thermal conductivity of the medium,  $q_r$  is the radiative heat flux, D is the mass diffusivity,  $\tau$  is the ratio of the effective heat capacity of the ordinary fluid and heat capacity of the nanofluid,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is thermophoresis diffusion coefficient.

Using Rosseland approximation, the radiative heat flux is given by

$$q_r = -\frac{4\sigma^* \partial T^4}{3K^* \partial y} \quad (5)$$

Where  $\sigma^*$  is the Stefan-Boltzmann constant and  $K^*$  is the absorption coefficient.

We consider the following boundary conditions

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$$u = u_w(x) = -\lambda u_e(x), -k \frac{\partial T}{\partial y} = h(T_f - T), C = C_w \text{ at } y = 0, \\ u = u_e(x), T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty \tag{6}$$

where  $\lambda = c/a$  is the constant moving parameter ( $\lambda < 0$  corresponds to stretching wedge and  $\lambda > 0$  corresponds to shrinking wedge),  $k$  is the thermal conductivity and  $h$  is the heat transfer parameter

We assume that  $u_w(x) = ax^m$ ,  $u_e(x) = cx^m$  where  $a$ ,  $c$  and  $m$  are constants.

We introduce the following non-dimensional variables

$$\eta = \left( \frac{(1+m)u_e}{2xv} \right)^{1/2} y \quad \psi(x, y) = \left( \frac{2u_e x v}{1+m} \right)^{1/2} f(\eta) \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Substituting above similarity variable in equations (2) – (4) we obtain the following non-linear differential equations

$$f''' + ff'' + \beta(1 - f'^2) = 0 \tag{7}$$

$$\frac{1}{Pr} (1 + Nr) \theta'' + f\theta' + N_b \phi' \theta' + N_t \theta'^2 = 0 \tag{8}$$

$$\phi'' + Le f \phi' + \frac{N_t}{N_b} \theta'' = 0 \tag{9}$$

and the transformed boundary conditions

$$f(0) = 0, f'(0) = -\lambda, \theta'(0) = -\gamma(1 - \theta(0)), \phi(0) = 1 \\ f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) \rightarrow 0 \tag{10}$$

Where  $\beta$  is the wedge angle parameter,  $Pr = \frac{\rho c_p}{k}$  Prandtl number,  $Nr = \frac{16\sigma_s T_\infty^3}{3Kk^*}$  is the thermal radiation parameter,  $Nb = \frac{\tau D_B C_\infty}{\nu}$  is the Brownian motion parameter,  $Nt = \frac{\tau D_T \Delta T}{T_\infty \nu}$  is the thermophoresis parameter,  $Le = \frac{\alpha}{D_B}$  is the Lewis number and  $\gamma = \frac{h}{k} \sqrt{\frac{2vL}{U_0}}$  is the Generalized Biot number

The local skin friction coefficient, heat transfer rate and mass transfer rate are given by

$$\frac{1}{2} C_{f_x} \sqrt{Re_x} = f''(0), Nu_x / \sqrt{Re_x} = -\theta'(0), Sh_x / \sqrt{Re_x} = -\phi'(0) \tag{11}$$

Where  $Re_x = x u_w / \nu$  is the Reynolds number.

### III. RESULTS AND DISCUSSIONS

In the numerical solutions, the effect of thermal radiation, Thermophoresis, Brownian motion and convective boundary condition on heat and mass transfer characteristics of a stretching/shrinking wedge in a nanofluid were considered. The transformed ordinary differential equations subject to the boundary conditions were solved numerically by using Runge - Kutta method. Velocity, temperature and concentration profiles were obtained and the local skin friction, Nusselt number and Sherwood number are calculated. The numerical results were discussed for different values of the pertinent parameters. To validate the accuracy of the results, comparisons were made with those of [17 – 20] in the absence of constant moving parameter ( $\lambda$ ). As shown in table. 1 the results are found to be nice agreement. Figs. 2(a) – 2(d) exhibit the effect of stretching/shrinking parameter on velocity and temperature profiles. With the enhancing stretching parameter ( $\lambda < 0$ ) fluid velocity is found to enhance [Fig. 2(a)] while for shrinking parameter ( $\lambda > 0$ ), velocity diminishes. Fig. 2(c) elucidates that the temperature decreases with increase of stretching parameter where as it increases with shrinking parameter. The effect of thermophoresis parameter  $N_t$  on temperature for two different values of Brownian motion parameter  $N_b$  is depicted in Fig. 3. It is observed that the temperature increases with increasing values of both thermophoresis and Brownian motion parameters. Fig. 4 illustrates the influence of thermal radiation parameter on temperature for both stretching/shrinking cases. It is observed the temperature is enhanced for increasing values of  $Nr$  and thus resulting in the increasing of the thickness of thermal boundary layer.

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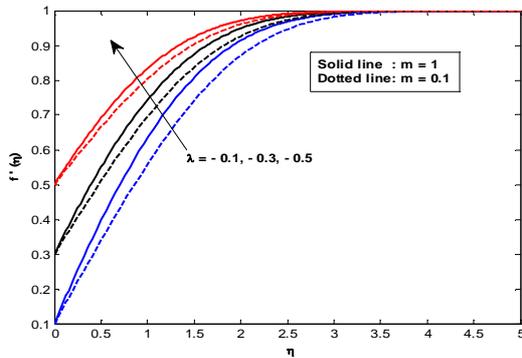


Fig. 2(a) Velocity profiles for different values of stretching parameter  $\lambda$  with  $Pr = 1; Nr = 1; Nt = 0.1; Nb = 0.1; Le = 1; \gamma = 0.1;$

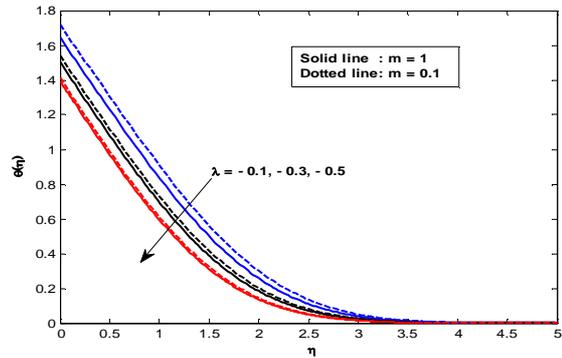


Fig. 2(b) Temperature profiles for different values of stretching parameter  $\lambda$  with  $Pr = 1; Nr = 1; Nt = 0.1; Nb = 0.1; Le = 1; \gamma = 0.1;$

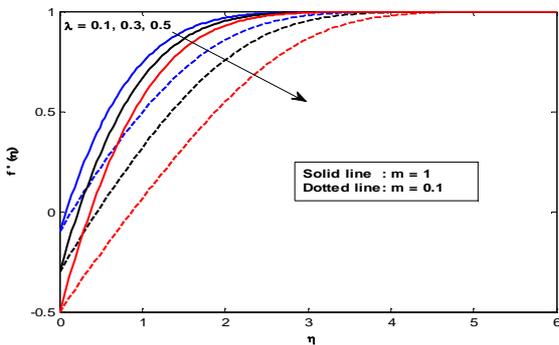


Fig. 2(c) Velocity profiles for different values of shrinking parameter  $\lambda$  with  $Pr = 1; Nr = 1; Nt = 0.1; Nb = 0.1; Le = 1; \gamma = 0.1;$

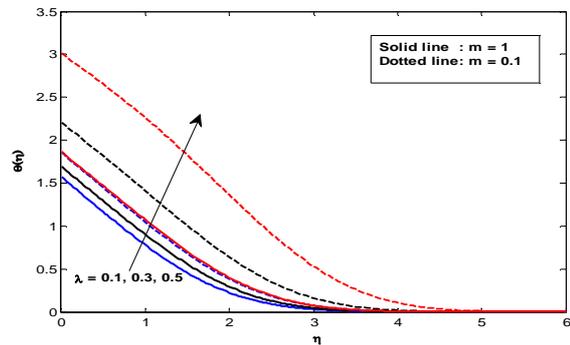


Fig. 2(d) Temperature profiles for different values of shrinking parameter  $\lambda$  with  $Pr = 1; Nr = 1; Nt = 0.1; Nb = 0.1; Le = 1; \gamma = 0.1;$

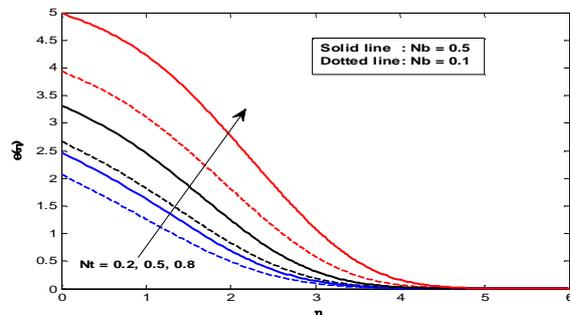


Fig. 3 Temperature profiles for different values of  $Nt$  with  $m = 1; \lambda = 0.5; Pr = 1; Nr = 1; Le = 1; \gamma = 0.1;$

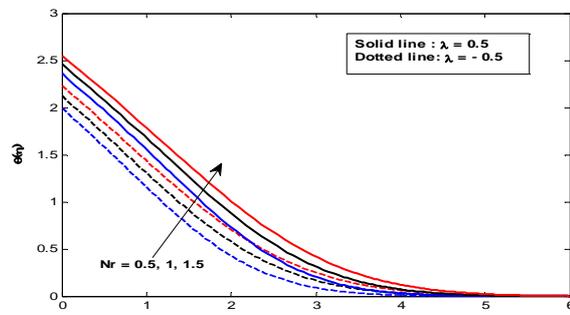


Fig. 4 Temperature profiles for different values of  $Nr$  with  $Pr = 1; Nr = 1; Nt = 0.1; Nb = 0.1; Le = 1; \gamma = 0.1;$

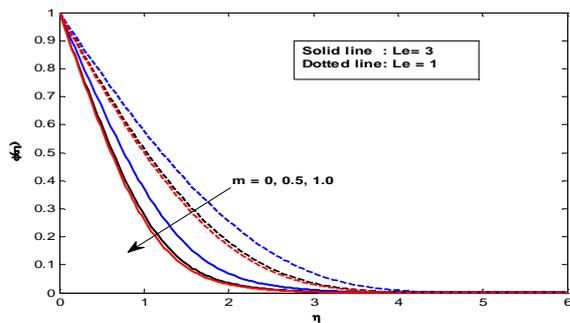


Fig. 5 Nanoparticle volume fraction profiles for different values of  $m$  with  $\lambda = 0.5; Pr = 1; Nt = 0.1; Nb = 0.1; Nr = 1; \gamma = 0.1;$

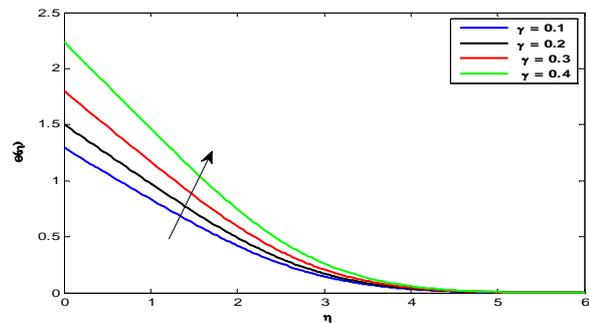


Fig. 6 Temperature profiles for different values of  $Nr$  with  $Pr = 1; Nr = 1; Nt = 0.1; Nb = 0.1; Le = 1; \gamma = 0.1;$

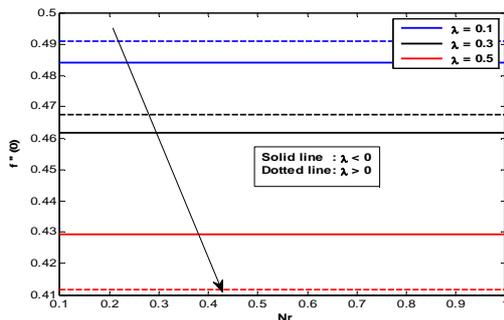


Fig. 7 Variation of skin friction coefficient with  $Nr$  for different values of  $\lambda$

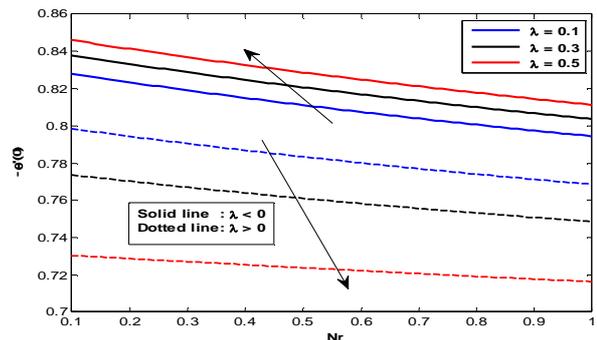


Fig. 8 Variation of Nusselt number with  $Nr$  for different values of  $\lambda$

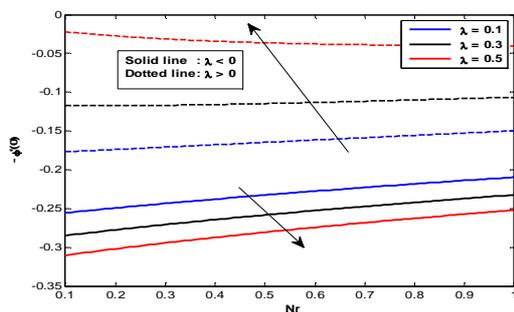


Fig. 9 Variation of Sherwood number with  $Nr$  for different values of  $\lambda$

**Table 1:** A comparison table of  $f''(0)$  for different values of  $m$

$m$	Yih [14]	White [15]	Khan and Pop [16]	Present results
0	.4696	0.4696	0.4696	0.469645
1/11	0.6550	0.6550	0.6550	0.654513
1/5	0.8021	0.8021	0.8021	0.802127
1/3	0.9276	0.9277	0.9277	0.927625
1/2	—	1.0389	1.0389	1.038904
1	—	1.2326	1.2326	1.232588

The thermal radiation facilitates additional means to diffuse energy as an enhancement in the radiation parameter which corresponds to a reduction in the Rosseland mean absorption coefficient  $K^*$  for fixed values of  $T_\infty$  and  $k$ . The variation of nanoparticle volume fraction for various values of wedge parameter with Lewis number is depicted in Fig. 5. We notice that the solutal boundary layer thickness decreases with increasing values of wedge for two different values of Lewis number. Fig. 6 exhibits the effect of Biot number on nanofluid temperature. It is noted that enhancing values of Biot number amounts to stronger convection and produces higher temperature on the wedge surface. The local skin friction coefficient versus thermal radiation ( $Nr$ ) for different values of both stretching/shrinking parameter ( $\lambda$ ) is show in Fig. 7. It is observed that the local skin friction diminishes with

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increasing values of  $\lambda$  and  $N_r$ . The local Nusselt number and Sherwood number are found to increase with increasing values of stretching parameter where as an opposite trend is noticed with shrinking wedge parameter [Figs. 8 – 9].

## IV. CONCLUSIONS

In this article, we study a boundary layer flow, heat and mass transfer along a moving wedge saturated in a water base nanofluid in the presence of thermal radiation and convective boundary condition. The effects of various parameters on velocity, temperature, nanoparticle volume fraction, skin friction, Nusselt number and Sherwood number are analyzed and presented graphically. Concludes that velocity enhances/diminishes with stretching/shrinking parameter and an opposite trend is noticed for dimensionless temperature. Temperature increases with thermophoresis parameter, Brownian motion parameter, Biot number and thermal radiation parameter. The effect of wedge parameter on nanofluid is to diminish the nanoparticle volume fraction where as the surface convection parameter enhances. The skin friction coefficient decrease for increasing values of both stretching and shrinking parameter. It is clear that with increasing values of stretching (shrinking) parameter the rate of heat transfer increases (decreases) where as the opposite trend noticed for the mass transfer rate.

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