

Non-Newtonian Nanofluid Flow over an Exponentially Stretching Sheet with Ohmic Effects

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ABSTRACT: In this article we studied the effects of thermal radiation and viscous dissipation on boundary layer flow, heat and mass transfer characteristics of Maxwell fluid due to an exponentially stretching sheet in the presence of suction and convective boundary conditions with nano particles. The basic equations governing the flow, heat and mass transfer are converted into a set of ordinary differential equations by using suitable similarity variables. The transformed equations are solved numerically using Runge-Kutta Fourth order method with shooting technique. The effects of pertinent flow parameters on velocity, temperature and nanoparticle volume fraction characteristics are elucidated graphically. The obtained numerical results are compared with previous literature and are found to be in good agreement.

KEYWORDS: MHD, Maxwell fluid, Nano fluid, Exponentially stretching sheet.

I. INTRODUCTION

The study of non-Newtonian fluids is extended due to its practical applications in technology and industrial process. Several fluids in our daily life such as ketchup, paper pulp, cyme, shampoos and blood at lower shear rate etc. shows the characteristics of non-Newtonian fluids. There are various fluids in nature which shows the non linear relationship between the deformation rate and the stress. These fluids are called as non-Newtonian fluids. Generally, non-Newtonian fluids are classified into three types. A simplest subclass of rate type fluid is Maxwell model. The fluid flow of Viscoelastic non-Newtonian fluids has vast applications in power engineering, petroleum production, cooling of a metallic plate in a cooling bath etc. The first viscoelastic rate type fluid is Maxwell fluid model introduced by Maxwell [1]. Mukhopadhyay et al [2] reported an unsteady flow of a Maxwell fluid past a stretching sheet with first order chemical reaction. Wang and Tan [3] explored the double diffusive convective stability analysis of a Maxwell fluid through a porous medium. Wang and Hayat [4] investigated the effects of fluctuating flow of a Maxwell fluid with a porous plate in the presence of variable suction. Hayat et al [5] studied the mixed convection flow of the Maxwell fluid with thermal radiation and thermal stratification. Noor et al [6] studied the MHD boundary layer flow of a Maxwell fluid over a vertical stretching sheet with thermophoresis and chemical reaction.

A mixture of nanoparticles and a base fluid is known as a nanofluid. Nanofluid has numerous applications in industries such as thermal storage, thermal management technologies and thermal therapy for cancer treatment as well as heating, cooling, ventilation, and air-conditioning. Choi [7] was the first author to introduce the concept of nanofluid in his paper entitled enhancing the thermal conductivity of fluids with nano-sized particles. The transport properties of heat transfer can be enhanced with the help of Nanofluids. For example, Xuan and Li [8] witnessed a 39% rise in the heat transfer using nanofluid having 2% copper nanoparticles. The impact of MHD boundary layer flow and heat transfer due to a stretching/shrinking sheet in a nanofluid with thermal radiation and viscous dissipation is analyzed by Sulochana et al [9]. Krishnamurthy et al [10] investigated the MHD steady, two-dimensional boundary layer flow of nanofluid due to an exponentially stretching sheet with viscous dissipation and fluid particle suspension.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

Ferdows et al [11] reported the effects of mixed convective boundary layer flow of nanofluid from an exponentially stretching surface in the presence of magnetic field and viscous dissipation through a porous medium.

The convective heat transfer is significant where high temperatures are involved. Referring to numerous industrial and engineering processes the convective boundary conditions are more practical including material drying, transpiration cooling process etc. Battler [12] investigated the Blasius and Sakiadis flows in a viscous fluid with convective boundary conditions. The heat transfer of a stretching sheet with convective boundary conditions has been studied by Yao et al [13]. Makinde et al [14] made an analysis on boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition.

The purpose of present analysis is to study the flow, heat and mass transfer over an exponentially stretching sheet in the presence of thermal radiation, viscous dissipation, joule heating, thermophoresis, Brownian motion and convective boundary condition. Using suitable similarity transformations the partial differential equations are converted into ordinary differential equations. The numerical calculations were carried out by using Runge-kutta fourth order method. Graphical results for different values of governing parameters are discussed to gain through insight towards the physics of the problem. Under some special conditions the present results have a good agreement with previous studies.

II. MATHEMATICAL FORMULATION

The steady two dimensional flow of an incompressible Maxwell nanofluid past an exponentially stretching sheet in a porous medium is considered. The x-axis is oriented along the stretching sheet in the direction of motion and y axis is normal to it. A transverse magnetic field $B(x) = B_0 e^{x/2L}$ is applied in y-direction and induced magnetic field is ignored. Thermophoresis and Brownian motion are taken into account. We assume convective heating process which is characterized by heat transfer coefficient h and temperature on the surface T_f .

The basic governing equations of the problem under the usual boundary layer approximations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda_1 \left[u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u, \tag{2}$$

$$\rho c_p u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \left(\frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_B}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho c_p} u^2, \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_B}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \tag{4}$$

Where u and v are the velocities in the x and y directions respectively, ν is the coefficient of viscosity, λ_1 is the relaxation time, σ is the electrical conductivity, B is the uniform Magnetic field, ρ is the density of the fluid, k^* is the permeability parameter of the porous medium, α is the thermal diffusivity, μ is dynamic viscosity, D is the mass diffusivity, c_p is the specific heat at constant pressure, T is the temperature of the fluid, C is the local nano particle volume fraction, D_B and D_T symbolizes the coefficients of Brownian motion and thermophoretic diffusion respectively. The associated boundary conditions of the problem are

$$u = U_w(x); v = 0; -k \frac{\partial T}{\partial y} = h_w(T_f - T); D_B \frac{\partial C}{\partial y} + \frac{D_B}{T_\infty} \frac{\partial T}{\partial y} =; \text{ at } y = 0 \tag{5}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } y \rightarrow \infty \tag{6}$$

where $U_w(x) = U_0 e^{x/L}$ is the velocity of the sheet, k is the thermal conductivity and $h_w = h e^{x/2L}$ is the heat transfer coefficient.

Since we are interested in similarity solutions of Eqs (1)-(4) subject to the boundary conditions (5)-(6) we introduce the following similarity variables

$$\eta = \sqrt{\frac{U_0}{2\nu L}} e^{x/2L} y, \psi(x, y) = \sqrt{2U_0 \nu L} e^{x/2L} f(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty} \text{ and } \phi = \frac{C}{C_\infty} \tag{7}$$

Using above transformations, equation (1) is identically satisfied and Equations (2)-(6) reduce to

$$f''' + ff'' - 2f'^2 - D \left(3ff'f'' + \frac{\eta}{2} f'^2 f'' - \frac{1}{2} f''' f^2 - 2f'^3 \right) - Mf' = 0 \tag{8}$$

$$\theta'' + \text{Pr}(f\theta' + Nb\theta'\phi' + Nt\theta'^2 + \text{Ec}f'^2 + \text{MEc}f'^2) = 0 \tag{9}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$\phi'' + Scf\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{10}$$

$$f(0) = 0, f'(0) = 1, \theta'(0) = -\gamma(1 - \theta), Nb\phi'(0) + Nt\theta'(0) = 0 \tag{11}$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \tag{12}$$

where $D = \frac{Re_x \lambda_1 \nu}{2L^2}$ is the local Deborah number (At lower Deborah numbers, the material behaves in a more fluid-like manner, almost Newtonian viscous flow. At higher Deborah numbers, the material behaves as non-Newtonian regime, increasingly dominated by elasticity and demonstrating solidlike behavior), $M = \frac{2\sigma B_0^2 L}{\rho U_0}$ is the Hartmann number,

$Ec = \frac{U_0^2}{c_p \Delta T}$ is the Eckert number, $Nb = \frac{\tau D_B C_\infty}{\nu}$ is the Brownian motion parameter, $Nt = \frac{\tau D_T \Delta T}{T_\infty \nu}$ is the thermophoresis

parameter, $Sc = \frac{D_B}{\nu}$ is the Schmidt number and $\gamma = \frac{h_w}{K} \sqrt{\frac{2\nu L}{U_0}}$ is the Generalized Biot number

The physical quantities of the engineering interest of the problem are skin friction coefficient and heat transfer coefficient and are given by

$$C_{fx} = \frac{\tau_w}{\rho U_0^2} \text{ and } Nu_x = -\frac{xq_w}{k(T_w - T_\infty)} \text{ where } \tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using the similarity transformation, we obtain

$$C_f Re_x^{\frac{1}{2}} = f''(0) \text{ and } Nu Re_x^{-\frac{1}{2}} \frac{2L}{x} = -\theta'(0) \text{ where } Re_x = \frac{2U_0 e^{x/L}}{\nu}$$

It is noted that Sherwood number which is dimensionless mass flux Sh_x is now identically zero

Results and Discussion:

The system of non linear ordinary differential equations (8) – (10) with boundary conditions (11) – (12) has been solved numerically by fourth order Runge Kutta method with shooting technique. The velocity, temperature and nano particle volume fraction profiles for Newtonian and non – Newtonian fluid cases are elucidated graphically. The parameters of interest for the present analysis are the Deborah number(D), Hartman number (M), Prandtl number (Pr), Eckert number (Ec), Schmidt number (Sc) and Biot number (γ). Comparison of $-\theta'(0)$ values with Mustafa et al [15] are presented in table 1.

Fig. 1-3 show the effect of Deborah number (D) on velocity, temperature and nano particle volume fraction. It is interesting to note that the velocity diminishes for increasing values of D. Figs. 2 & 3 exhibit the temperature and nano particle volume fraction profiles for various values of Deborah number. We observe that the temperature and nanoparticle fraction enhance for increasing values of D. This is due to the fact that physically the Deborah number depends on the relaxation time. It is clear that the fluid flow resists with larger relaxation time. Here, larger Deborah number tends to larger relaxation time. Therefore, the velocity and momentum boundary layer thickness diminish when Deborah number enhances. From fig. 1 we notice that the results when $D = 0$ corresponds to viscous fluid results.

The effect of Hartmann number (M) on velocity profile for both $D = 0$ and $D = 0.5$ are depicted in fig. 4. We observed that the strength of magnetic field increases velocity for both cases, because introduction of transverse magnetic field has a tendency to create drag known as Lorentz force which resists the flow. The temperature and nano particle volume fraction of magnetic parameter (M) is depicted in figs. 5 & 6. Physically the magnetic field is normal to the fluid, so for larger value of M, it enhances both temperature and nano particle volume fraction.

Fig.7 describes the effect of viscous dissipation in terms of Eckert number on temperature profile. Physically, viscous dissipation changes the temperature distribution by acting a role like an energy source. which tends to affect heat transfer rates. Here the temperature enhances with hike in the values of Ec.

From fig. 8 we noticed that the enhancing values of thermophoresis parameter leads to an increment in temperature and thermal boundary layer thickness. This is due to fact that an increase in N_t leads to an enhancement in the thermophoresis force which tends to move nano particles from hot to cold surfaces. Fig.9 depicts that an hike in N_t causes an increase in the nano particle concentration and its associated boundary layer thickness.

Fig. 10 portrays the effect of Biot number on temperature profile. An increment in Biot number causes a stronger convection and produces higher temperature on the surface and hence the thermal effects penetrate deep into the fluid.

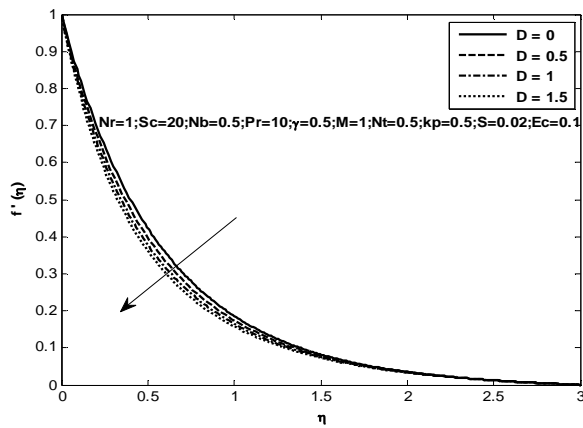


Fig. 1. Velocity profiles for different values of D

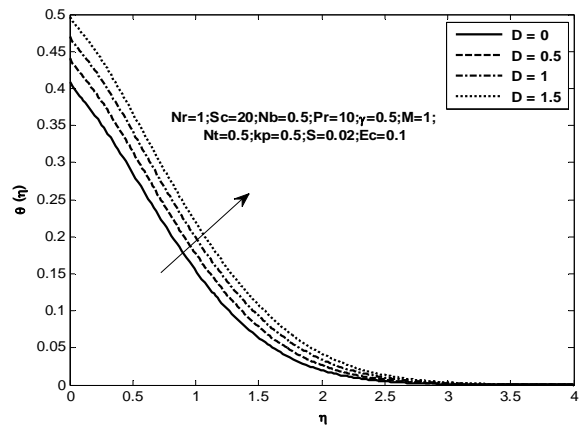


Fig. 2. Temperature profiles for different values of D

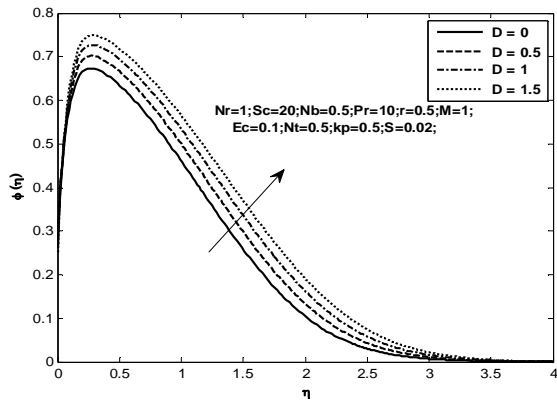


Fig. 3. Concentration profiles for different values of D

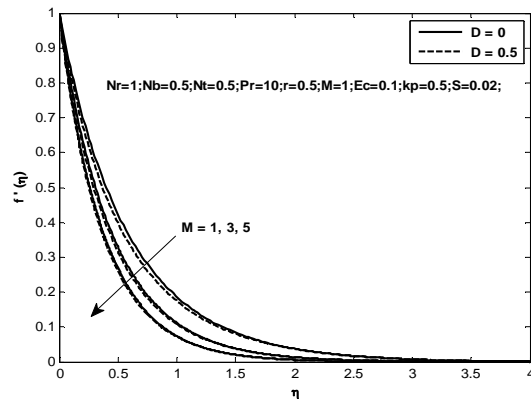


Fig. 4. Velocity profiles for different values of M

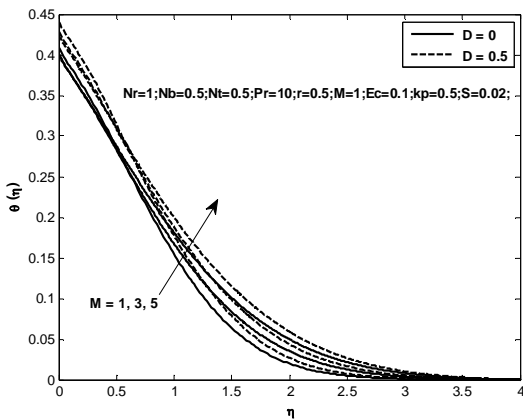


Fig. 5. Temperature profiles for different values of M

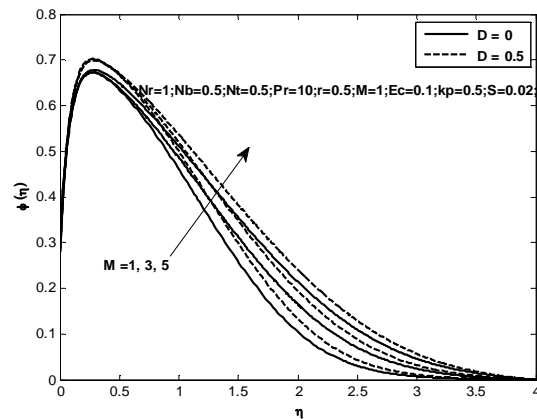


Fig. 6. Concentration profiles for different values of M

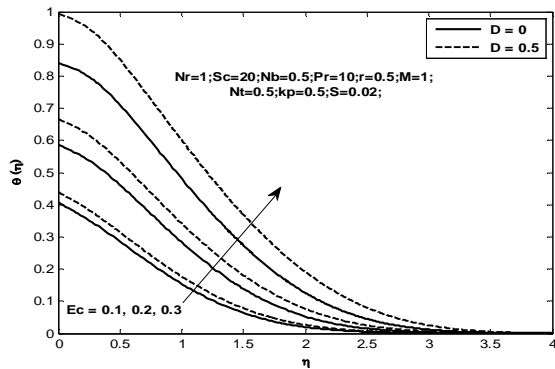


Fig. 7. Temperature profiles for different values of Ec

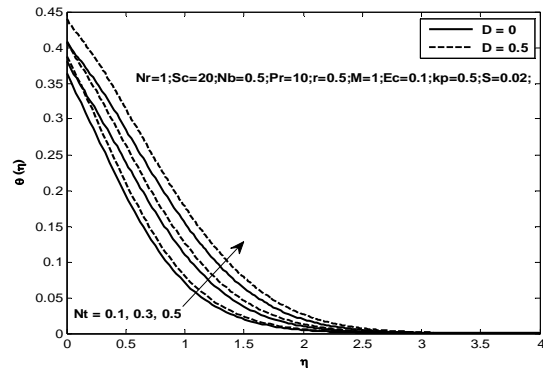


Fig. 8. Temperature profiles for different values of Nt

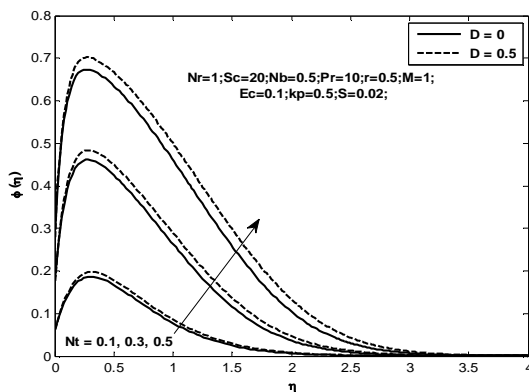


Fig. 9. Concentration profiles for different values of Nt

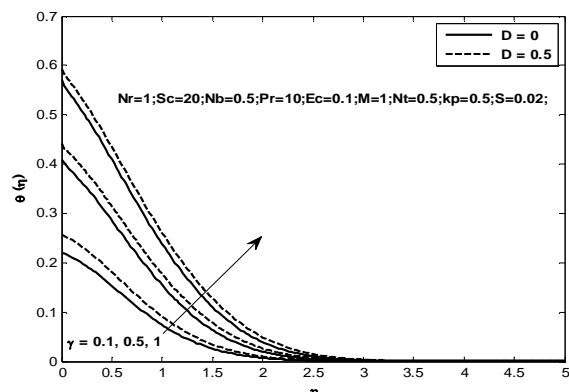


Fig. 10. Temperature profiles for different values of γ

III. CONCLUSIONS

The steady boundary layer flow of an incompressible, electrically conducting Maxwell nanofluid past an exponentially stretching sheet in the presence of magnetic field, thermal radiation, viscous dissipation subject to the suction/injection and convective boundary condition is analysed. The partial differential equations describing the problem are transformed into ordinary differential equations employing similarity transformations and obtained numerical solutions. From the analysis, the salient features are presented below:

- ❖ The velocity decelerates with suction, magnetic field parameter, permeable parameter, Deborah number, thermophoresis parameter while an opposite trend is observed with injection.
- ❖ The temperature and concentration increases with increase of magnetic field parameter, Deborah number, permeable parameter, Biot number and thermophoresis parameter.
- ❖ The thickness of the thermal boundary enhances with increasing values of blowing, thermal radiation parameter and viscous dissipation parameter and decreases with Prandtl number and suction.
- ❖ The solutal boundary layer thickness decreases with suction, Prandtl number and Brownian motion parameter while it enhances with blowing.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

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