

Influence of Soret and Dufour Effects on MHD Casson Fluid over a Stretching Surface in the Presence of Thermal Radiation

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ABSTRACT: An analysis has been carried out to study the influence of thermal radiation, heat source and velocity slip on MHD Casson fluid flow over a stretching sheet in the presence of combined effects of Soret and Dufour. The governing differential equations are transformed in to a set of non linear coupled ordinary differential equations which are solved numerically using Runge-Kutta-Fehlberg method with shooting technique using appropriate boundary conditions for various physical parameters. Comparisons of the present numerical results are found to be in excellent with the earlier published results under limiting cases.

KEYWORDS: Casson fluid, MHD flow, Velocity Slip, Soret and Dufour effects, chemical reaction.

I. INTRODUCTION

In recent years, the study of non-Newtonian fluids has gained great importance due to a variety of applications in engineering and industry. The relationship between shear stress and rate of shear strain is non-linear. The success of theoretical and experimental investigations made Casson fluid model more popular than the other non-Newtonian fluid models. Casson fluids exhibit a yield stress. When the shear stress applied is less than the yield stress it behaves as a solid and if greater than the yield stress it starts to move [1, 2]. Casson fluids have applications in the fields of food processing, metallurgy, bio-engineering operations and drilling operators.

The effect of external magnetic field on magnetohydrodynamic (MHD) flow over a stretching sheet have applications in glass manufacturing purification of crude oil, geo physics, paper production etc. Crane [3] provided a solution for flow of viscous, incompressible boundary layer flow of fluids generated by a stretching surface. Hassani et al. [4] investigated the analytical solutions for the boundary flow of a nano fluid over a stretching surface. The slip effects on a second grade fluid past a stretched surface in a porous space was discussed by Hayat et al. [5]. The heat and mass transfer characteristics in the presence of Soret and Duffour effects was studied by Rahman [6]. Hayat et al. [7] analysed the flow of a Maxwell fluid with heat and mass transfer in the presence of a chemical reaction. Muhaimina et al. [8] discussed the boundary layer flow of a viscous fluid over a porous shrink sheet in the presence of suction.

In this paper diverse physical aspects are applied on the flow. An analysis has been carried out to study the influence of thermal radiation, heat source and velocity slip on MHD Casson fluid flow over a stretching sheet in the presence of combined effects of Soret and Dufour. The governing differential equations are transformed in to a set of non linear coupled ordinary differential equations which are solved numerically using Runge-Kutta-Fehlberg method with shooting technique using appropriate boundary conditions for various physical parameters. Comparisons of the present numerical results are found to be in excellent with the earlier published results under limiting cases.

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II. MATHEMATICAL FORMULATION

Let us consider the steady state MHD flow of an incompressible Casson fluid over a heated stretching surface at $y = 0$. The surface is elastic. The x -axis is chosen to be parallel to the surface and y -axis is normal to the surface. The motion in an incompressible fluid is induced because of the stretching property. This occurs in view of the elastic properties of the surface parallel to the x -axis through equal and opposite forces when the origin is fixed. A constant magnetic field B is exerted in the transverse direction to the surface. The induced magnetic field is negligible due to small magnetic Reynolds numbers. We also considered the heat and mass transfer processes in the presence of Soret and Dufour effects. The rheological equation of state for an isotropic and incompressible flow of the Casson fluid is

$$\tau_{ij} = \begin{cases} 2(\mu_B + P_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c \\ 2(\mu_B + P_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c \end{cases}$$

Here, $\pi = e_{ij}e_{ij}$ and e_{ij} are the $(i, j)^{th}$ component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluids and P_y is the yield stress of the fluid.

The resulting boundary layer equations in the MHD flow under consideration are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \left[\left(K + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{D_m D k_T}{C_s c_p} \frac{\partial^2 C}{\partial y^2} + Q_0 (T - T_\infty) \right] \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_\infty) \tag{4}$$

The boundary conditions are

$$u = U_w + N_1 v \frac{\partial u}{\partial y}, v = -V_w, T = T_w, C = C_w \text{ at } y = 0, \tag{5}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty$$

where u, v are the fluid velocity components along x and y axes respectively, ν is the kinematic viscosity, ρ is the density of the fluid, $\beta = \mu_B \sqrt{2\pi_c}/P_y$ is parameter of the Casson fluid, T is the fluid temperature, C is the fluid concentration, σ is the electrical conductivity, σ^* is the Stefan-Boltzman constant and K^* is the absorption coefficient, c_p is the specific heat at constant pressure, C_s is the concentration susceptibility, K is the thermal conductivity of the medium, k_T is the thermal-diffusion density, q_r is the radiation heat flux, Q_0 is the uniform volumetric heat generation and absorption, D_m is the mass diffusivity, T_m is the fluid mean temperature, k_1 is the chemical reaction.

III. METHOD OF SOLUTION

The governing partial differential equations (2) - (4) can be reduced to a set of ordinary differential equations on introducing the following similarity variables:

$$\eta = \sqrt{\frac{c}{\nu}} y, u = c x f'(\eta), v = -\sqrt{c\nu} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{6}$$

Using equation (6) in equations (2)-(4) we obtain the following set of ordinary differential equations:

$$\left(1 + \frac{1}{\beta}\right) f''' + ff'' - f'^2 - M^2 f' = 0 \tag{7}$$

$$\theta'' \left(1 + \frac{4}{3} Nr\right) + Pr(f\theta' - f'\theta + D_f \phi'' + Q\theta) = 0 \tag{8}$$

$$\phi'' + Sc(f\phi' - f'\phi + S_r \theta'' - \gamma\phi) = 0 \tag{9}$$

The corresponding boundary conditions are

$$\eta = 0 : f = f_w, f' = 1 + \lambda f'', \theta = 1, \phi = 1, \tag{10}$$

$$\eta \rightarrow \infty : f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0, \tag{11}$$

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Where the primes denote the differentiation with respect to η , $M = \frac{\sigma B^2}{\rho c}$ is magnetic parameter, $Pr = \frac{\rho c_p \nu}{k}$ is Prandtl number, $Nr = \frac{4\sigma^* T_\infty^3}{Kk^*}$ is thermal radiation parameter, $D_f = \frac{D_m k_T (C_w - C_\infty)}{C_s c_p \nu (T_w - T_\infty)}$ is Dufour number, $S_r = \frac{D_m k_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}$ is Soret number, $Sc = \frac{\nu}{D_m}$ is Schmidt number, $\gamma = \frac{k_1}{c}$ is Chemical reaction parameter, $Q = \frac{Q_0}{c_p c_p}$ is heat source/sink parameter, $f_w = \frac{v_w}{\sqrt{c\nu}}$ is mass transfer parameter, which is positive for suction and negative for injection and $\lambda = N_1 \sqrt{\nu c}$ is momentum slip parameter, The ordinary differential equations (7) – (9) are coupled and highly non – linear. These equations with the boundary conditions (10) and (11) are solved using Runge-Kutta fourth order method with shooting technique and obtained numerical solutions. The accuracy of the numerical scheme is ensured by comparing the present results with those of Magyari and Keller [9], Bidin and Nazar [10] and Pramanik [11] in the absence of magnetic field, Dufour effect, heat generation/absorption and suction for a steady flow i.e. $M = Nr = D_f = Q = f_w = \phi = 0$ for different values of Pr . The compared values of $-\theta'(0)$ are presented in Table 1 which is found to be in good agreement with them.

Table 1 values of $[-\theta'(0)]$ for several values of Prandtl number for Newtonian fluid.

Pr	Magyari and Keller [9]	Bidin and Nazar [10]	Pramanik [11]	Present results
1	0.9548		0.9547	0.9549
2		0.9547	1.4714	1.4714
3	1.8691	1.4714	1.8691	1.8691
5	2.5001	1.8691	2.5001	2.5001
10	3.6604		3.6603	3.6603

IV. RESULTS AND DISCUSSION

Fig. 1 represent the effect of Casson parameter (β) on velocity in the presence of suction/injection. It is observed that the velocity decreases with increasing Casson parameter. The presence of yield stress reduces the velocity. The velocity of the fluid is much reduced in the case of suction than that of injection. It is also noted that the boundary layer thickness decreases with increasing β . Fig. 2 reveals the variation of velocity with magnetic field parameter (M). The magnetic field is due to the Lorentz force which opposes the flow. Hence for higher values of magnetic field the velocity decreases. The flow velocity and the boundary layer thickness reduce with an increase in M . The influence of velocity slip parameter (λ) on velocity is presented in Fig. 3. It is observed that the velocity decreases with increase in velocity slip. Fig. 4 shows that the velocity significantly decreases as suction ($f_w > 0$) increases where as an opposite behaviour is observed in blowing ($f_w < 0$). Suction causes reduction in the thickness of the boundary layer. Increasing values of suction further reduces the thickness of the boundary layer.

From Fig. 5 for increasing values of Prandtl number (Pr) the temperature profile decreases rapidly. A reduction in the thickness of the thermal boundary layer is observed for increasing values of Prandtl number. Fig. 6 exhibits the effects of thermal radiation (Nr) on temperature profile at the boundary. It is found that as the radiation parameter increases the temperature enhances. As thermal radiation increases the thickness of the thermal boundary layer also increases. The temperature profiles for heat source parameter (Q) are represented in Fig. 7. It is noticed that the temperature enhances for increasing values of Q . The effect of Dufour number (D_f) on temperature profile is depicted in Fig. 8. It is observed that the temperature increases as Dufour number increases.

Fig. 9 shows that an increase in Soret number (Sr) increases the concentration and the boundary layer thickness is also increased. Fig. 10 depicts that the impact of chemical reaction (γ) on concentration profiles. It is observed that for increasing value of chemical reaction, the concentration of the species in the boundary layer decreases.

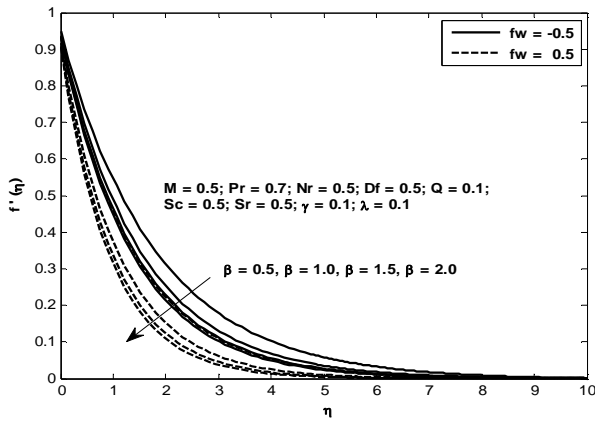


Fig. 1: Velocity profiles for different values of β

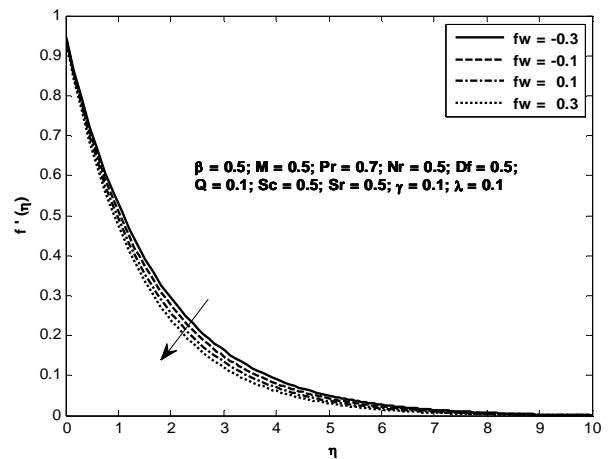


Fig. 4: Velocity profiles for different values of fw

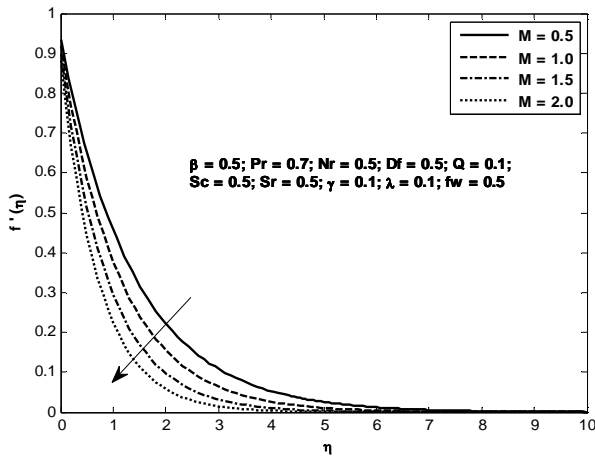


Fig. 2: Velocity profiles for different values of M

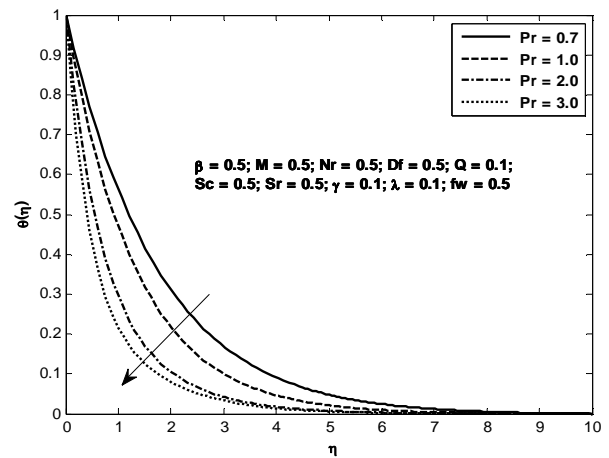


Fig. 5: Temperature profiles for different values of Pr

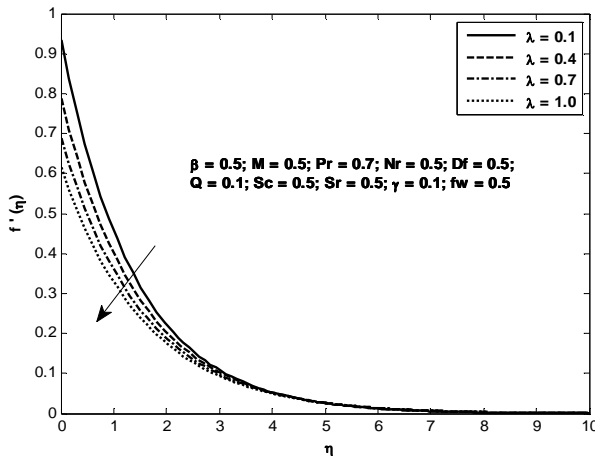


Fig. 3: Velocity profiles for different values of λ

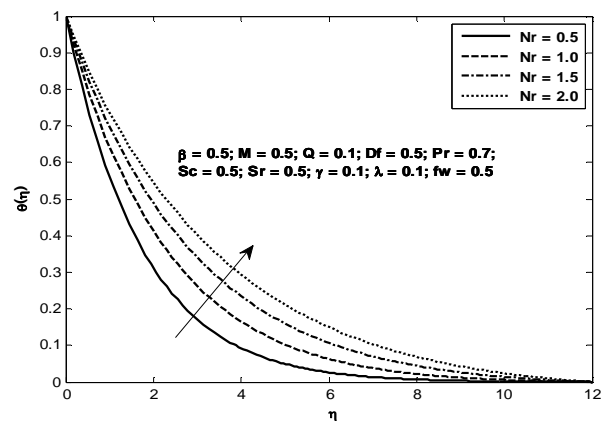


Fig. 6: Temperature profiles for different values of Nr

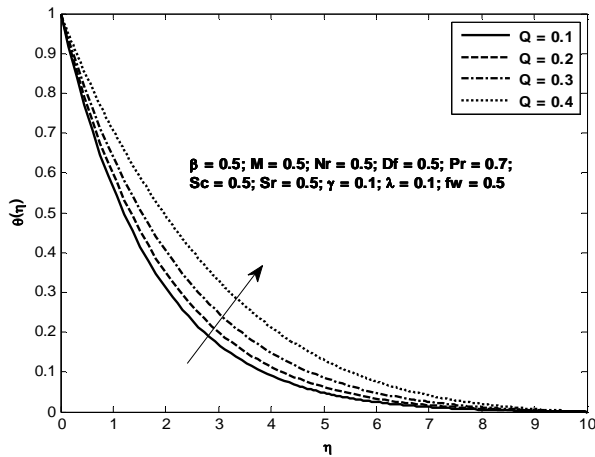


Fig. 7: Temperature profiles for different values of Q

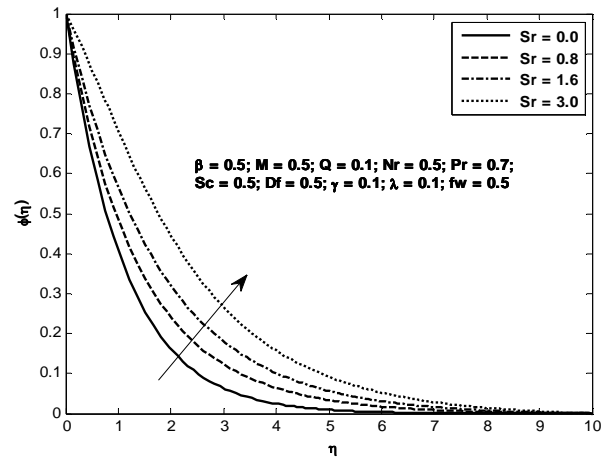


Fig. 9: Concentration profiles for different values of Sr

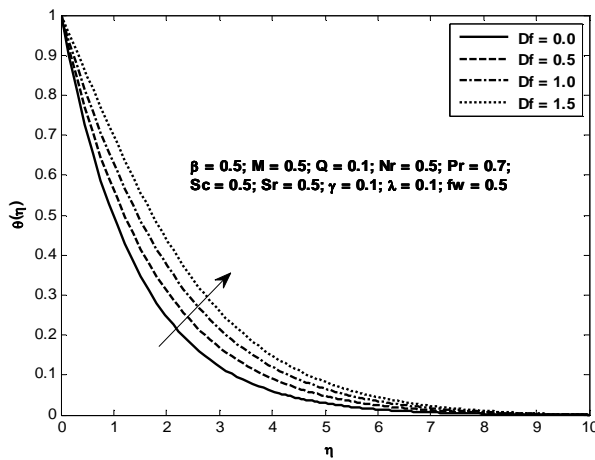


Fig. 8: Temperature profiles for different values of Df

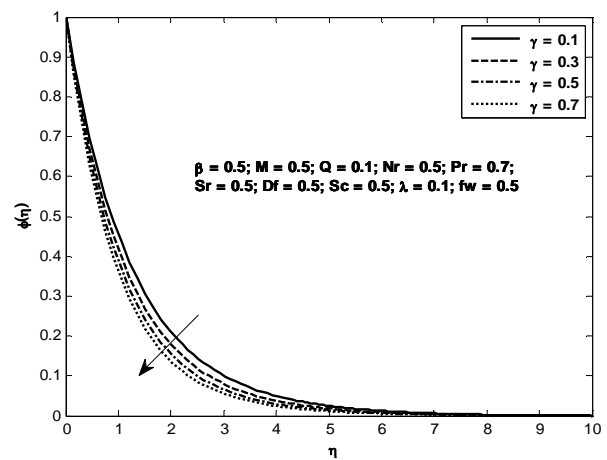


Fig. 10: Concentration profiles for different values of γ

V. CONCLUSION

The following conclusions are drawn:

- ❖ It is observed that the velocity decreases with Casson parameter, suction and magnetic field parameter.
- ❖ The thickness of the thermal boundary layer increases with increasing values of radiation parameter and heat generation parameter while a reduction is noticed with Prandtl number.
- ❖ The chemical reaction parameter has a reducing influence on concentration resulting in thinner solutal boundary layers.

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