

Melting Heat Transfer in a Steady Stagnation Point Flow of a Jeffrey Fluid over a Stretching Sheet with Radiation and Velocity Slip

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ABSTRACT: In this analysis the effect of melting heat transfer on the stagnation point flow of a chemically reactive Jeffrey fluid with thermal radiation, Newtonian heating and wall slip studied. Similarity variables have been employed to transform the governing non-linear boundary layer equations into a set of non-linear ordinary differential equations. Numerical solutions are obtained for the velocity, temperature and concentration using shooting method and Runge-Kutta-Fehlberg method. Comparison of our results under limiting cases is made with the earlier published results. Velocity is found to be an increasing function of melting heat transfer parameter and Deborah number and is a decreasing function of magnetic field parameter, velocity slip parameter and ratio of relaxation and retardation times.

KEYWORDS: Jeffrey fluid, Melting heat transfer, Stagnation point flow, Velocity slip.

I. INTRODUCTION

Interest in boundary layer flows of non-Newtonian fluids has increased due to their abundant applications in science and engineering including thermal oil recovery, food and slurry transportation, polymer and food processing etc. A variety of non-Newtonian fluid models have been proposed in the literature keeping in view of their several rheological features. There is one subclass of non-Newtonian fluids known as Jeffrey fluid which has been attracted by many of the researchers in view of its simplicity. This fluid model is capable of describing the characteristics of relaxation and retardation times. Hayat et al. [1] examined the combined effect of Newtonian heating and magnetic field on the stagnation point flow of a Jeffrey fluid over a radially stretching surface. Shehzad et al. [2] examined the heat and mass transfer characteristics of MHD radiative flow of an incompressible Jeffrey fluid on a linearly stretched surface in the presence of Joule heating and thermophoresis. Ahmed et al. [3] obtained an exact solution of the convective heat transfer of a MHD Jeffrey fluid flow past a stretching sheet considering the effects of Joule and Newtonian heating, internal heat source/sink and thermal radiation.

The phenomenon of melting heat transfer has been a subject of intensive experimental and theoretical investigations for several years as it is closely related to a wide variety of technologically important processes. Some of the important applications are melting of soil, artificial freezing of ground for mining and construction purposes, magma solidification, melting of permafrost, thawing of frozen grounds, freezing of soil around the heat exchanger coils of a ground based pump, preparation of semiconductor-material, thermal energy storage, the freeze treatment of sewage and the casting and welding of a manufacturing process. Roberts [4] is the pioneer to examine the melting of an ice body with its flat surface facing a hot air stream. Bourhan [5] examined the effect of buoyancy and magnetic field on the melting process about a vertical plate embedded in a non-Darcy saturated porous medium. Yacob et al. [6] studied the melting heat transfer in a steady boundary layer stagnation point flow towards a stretching/shrinking sheet in a micropolar fluid. Das et al. [7] examined numerically the effect of melting heat transfer and thermal radiation on the MHD stagnant point flow of a Jeffrey fluid over a stretching surface with wall slip.

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Literature survey reveals that not much work has been done on the melting effect in Jeffrey fluid flows. To the best of knowledge of the author the effect of melting heat transfer on the stagnation point flow of a chemically reactive Jeffrey fluid with thermal radiation, Newtonian heating and wall slip is yet to be studied.

II. MATHEMATICAL FORMULATION

We Consider the two-dimensional stagnation-point flow of a steady incompressible Jeffrey fluid towards a horizontal linearly stretching sheet at $y = 0$, which is melting at a steady rate. The x -coordinate is measured along the plate and the y -coordinate normal to it. The flow being confined to $y \geq 0$. The flow is generated, due to stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x -axis. The sheet is stretched, keeping the origin fixed, with a velocity $u_w(x) = ax$ where 'a' is a positive constant with dimensions of (time)⁻¹. It is assumed that the velocity of the external flow is $u_e(x) = bx$ where 'b' is a positive constant with dimensions of (time)⁻¹. It is also assumed that the temperature of the melting surface is T_m , while the temperature in the free stream condition is T_∞ where $T_m > T_\infty$. In addition, the temperature of the solid medium far from the interface is constant and is denoted by T_s where $T_s < T_m$. The species concentration C_w is assumed to be constant at the stretching surface. When y tends to infinity, the ambient value of the concentration is denoted by C_∞ .

The constitutive relationships of Jeffrey fluid are:

$$\mathbf{T} = -P\mathbf{I} + \mathbf{S},$$

$$\mathbf{S} = \frac{\mu}{(1+\lambda_1)} \left(\dot{\boldsymbol{\gamma}} + \lambda \frac{d}{dt} \dot{\boldsymbol{\gamma}} \right),$$

where P denotes the pressure, \mathbf{I} is the identity tensor, μ is the dynamic viscosity, λ_1 the ratio of relaxation and retardation times and λ is the retardation time. The quantities $\dot{\boldsymbol{\gamma}}$ and $\ddot{\boldsymbol{\gamma}}$ are defined by

$$\dot{\boldsymbol{\gamma}} = \nabla \mathbf{v} + (\nabla \mathbf{v})^T,$$

$$\ddot{\boldsymbol{\gamma}} = \frac{d(\dot{\boldsymbol{\gamma}})}{dt} = \frac{\partial(\dot{\boldsymbol{\gamma}})}{\partial t} + (\mathbf{v} \cdot \nabla) \dot{\boldsymbol{\gamma}}.$$

The governing equations of momentum, thermal energy and species concentration equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \frac{v}{1+\lambda_1} \left[\frac{\partial^2 u}{\partial y^2} + \lambda \left(u \frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} \right) \right] - \frac{\sigma B_0^2}{\rho} (u - u_e) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^* \rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty) \tag{4}$$

where, u and v are fluid velocity components along x and y -axes, respectively, ν is the kinematic viscosity, ρ is the density of the fluid, σ is the electrical conductivity, B_0 is the strength of the magnetic field, T is the fluid temperature, C is the fluid concentration, k is thermal conductivity of the fluid, c_p is the specific heat at constant pressure, σ^* is the Stefan-Boltzman constant, k^* is the absorption coefficient, D is the mass diffusivity and k_0 is the chemical reaction.

The boundary conditions for the present flow are:

$$u = u_w + N_1 \nu \frac{\partial u}{\partial y}, T = T_m, C = C_w \quad \text{at } y = 0 \tag{5}$$

$$u \rightarrow u_e = bx, \frac{\partial u}{\partial y} \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \tag{6}$$

$$k \left(\frac{\partial T}{\partial y} \right)_{y=0} = \rho [\lambda_2 + c_s (T_m - T_0)] \nu(x, 0), \tag{7}$$

where, N_1 is the velocity slip factor, λ_2 is the latent heat of the fluid and c_s is the specific heat of the solid surface and T_0 is temperature of the solid. Equation (7) indicates that the heat conducted to the melting surface is equal to the sum of heat of melting and the heat required to raise the temperature T_0 of the solid surface to its melting temperature.

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III. METHOD OF SOLUTION

We introduce the following transformations:

$$\psi = (av)^{\frac{1}{2}}xf(\eta), \quad \eta = (a/v)^{1/2}y$$

$$\theta(\eta) = \frac{T-T_m}{T_\infty-T_m}, \quad \phi(\eta) = \frac{c-c_\infty}{c_w-c_\infty} \tag{8}$$

The stream function is defined such that $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

With the help of the above transformations, equation (1) is identically satisfied, and equations (2)-(7) along with boundary conditions take the following forms:

$$f''' + \beta_2(f''^2 - ff''''') + (1 + \lambda_1)(ff'' - f'^2) - (1 + \lambda_1)M(f' - A) + (1 + \lambda_1)A^2 = 0 \tag{9}$$

$$(1 + \frac{4}{3}Nr)\theta'' + Pr(f\theta' + Ec f''^2) = 0 \tag{10}$$

$$\phi'' + Scf\phi' - Sc\gamma\phi = 0 \tag{11}$$

The boundary conditions will take the following form:

$$f'(0) = 1 + hf''(0), Prf(0) + Me\theta'(0) = 0, \theta(0) = 0, \phi(0) = 1 \tag{12}$$

$$f'(\infty) \rightarrow A, f''(\infty) \rightarrow 0, \theta(\infty) \rightarrow 1, \phi(\infty) \rightarrow 0 \tag{13}$$

where, $\beta_2 = \lambda a$ is the Deborah number, $M = \sigma B_0^2/\rho a$ is the magnetic field parameter, $A = b/a$ is the stretching ratio parameter, $Pr = \nu/\alpha$ is the Prandtl number, $Nr = 4\sigma^*T_\infty^3/kk^*$ is the thermal radiation parameter, $Ec = U_w^2/c_p(T_\infty - T_m)$ is the Eckert number, $h = N_1\sqrt{av}$ is the velocity slip parameter, $Sc = \nu/D$ is the Schmidt's number, $\gamma = k_0/a$ is the chemical reaction parameter and $Me = \frac{c_p(T_\infty - T_m)}{\lambda_2 + c_s(T_m - T_0)}$ is the melting heat transfer parameter.

The non-dimensional expressions of the skin friction coefficient, local Nusselt and Sherwood numbers can be written as follows;

$$C_f(Re_x)^{\frac{1}{2}} = \left(\frac{1+\beta_2}{1+\lambda_1}\right)f''(0), Nu_x(Re_x)^{-\frac{1}{2}} = -\theta'(0), (Re_x)^{-\frac{1}{2}} = -\phi'(0) \tag{14}$$

Where $Re_x = U_w x/\nu$ is the local Reynolds number.

The non-dimensional coefficient of frictional drag calculated in our analysis for various values of stretching ratio parameter (A) in the absence of magnetic field and melting effects is compared with those evaluated by Mahapatra and Gupta [8], Hayat et al. [9] and Ibrahim et al. [10] in **Table 1** and it is clear that for each value of 'A' a good agreement between the two is seen. Hence, we are confident of accuracy of the present results.

Table 1 Comparison values of $f''(0)$ when $Me = M = \beta_2 = \lambda_1 = h = Pr = Nr = Ec = Sc = \gamma = 0$

A	Mahapatra and Gupta [8]	Hayat et al. [9]	Ibrahim et al. [10]	Present results
0.01	-	-0.9982	-0.9980	-0.99813
0.1	-0.9694	-0.9695	-0.9694	-0.96942
0.2	-0.9181	-0.9181	-0.9181	-0.91811
0.5	-0.6673	-0.6673	-0.6673	-0.66729
2.0	2.0175	2.0176	2.0175	2.01754
3.0	4.7293	4.7296	4.7292	4.72956

IV. RESULTS AND DISCUSSION

Our focus is to explore the influence of melting heat transfer and magnetic field on stagnation point flow of a Jeffrey fluid over a stretching sheet. Computational results of the numerical solutions showing the influence of different physical parameters are presented here.

Fig. 1 is the plots of velocity distribution for a variation of the stretching ratio parameter (A). For small values of A in the range $0 \leq A < 1$ the velocity tends to reduce along the sheet, and it is uniform when $A = 1.0$ and the flow gets accelerated with a rise in the velocity near the boundary and remains uniform away from the boundary. **Fig. 2** illustrates the variation of melting heat transfer parameter (Me) on velocity. It is observed that the velocity and species concentration increase with intense melting. **Fig. 3** presents the effects of Deborah number (β_2) on velocity. It is clear that velocity is enhanced as Deborah number increases for higher values of Deborah number, since higher values of higher β_2 imply lesser viscosity of the fluid. The corresponding boundary layers are thicker. **Fig. 4** shows the influence of magnetic field parameter (M) on velocity. Velocity is suppressed as expected due to the Lorentz force. **Fig. 5** is the plots of velocity, temperature and concentration depicting the influence of ratio of relaxation and retardation times (λ_1). For increasing values of ratio of relaxation to retardation times, velocity experiences a decrease in magnitude. The influence of slip effects corresponding to velocity is displayed in **Fig. 6**. It is observed that increasing values of velocity slip parameter (h) reduces the velocity.

Fig. 7 depicts the influence of melting heat transfer parameter (Me) on temperature profile. It is seen that increasing values of melting heat transfer parameter diminish the temperature giving rise to thinner thermal boundary layers. However, the variation is small. From **Fig. 8** it is observed that presence of the thermal radiation (Nr) decreases the temperature significantly throughout the region. The temperature is further decreased for increasing thermal radiation parameter.

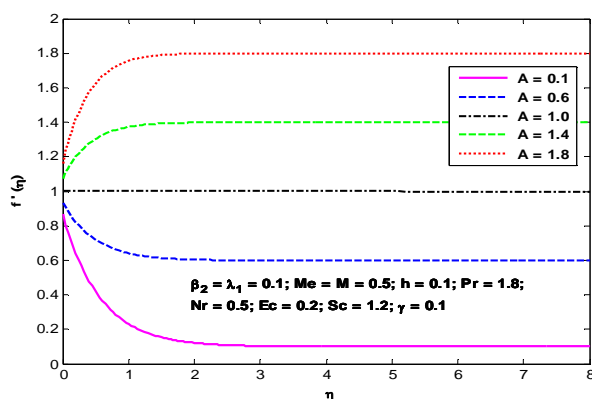


Fig. 1: Variation of A on f'

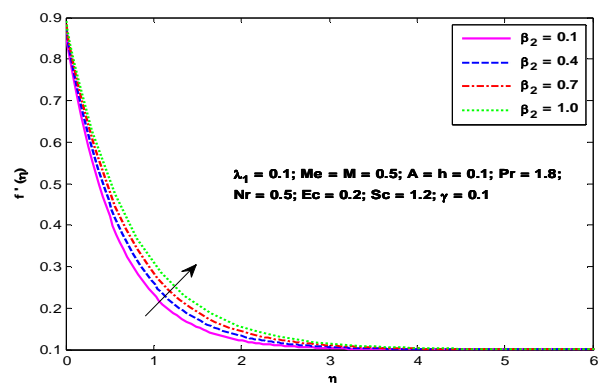


Fig. 3: Variation of β_2 on f'

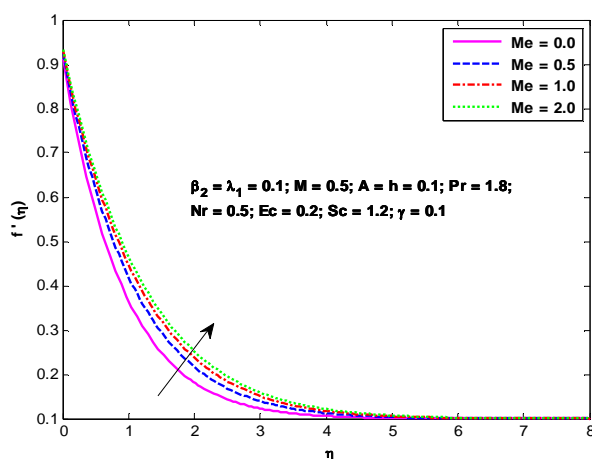


Fig. 2: Variation of Me on f'

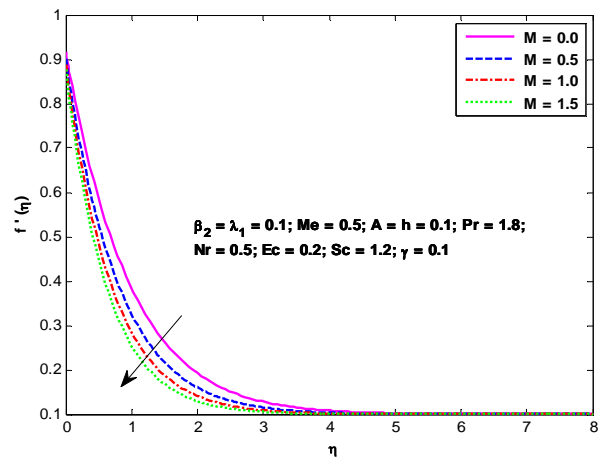


Fig. 4: Variation of M on f'

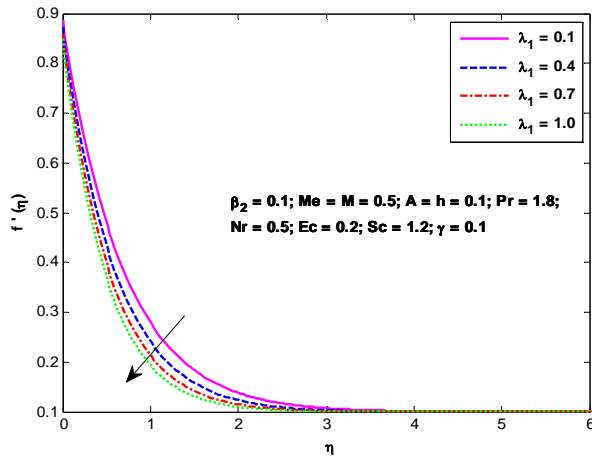


Fig. 5: Variation of λ_1 on f'

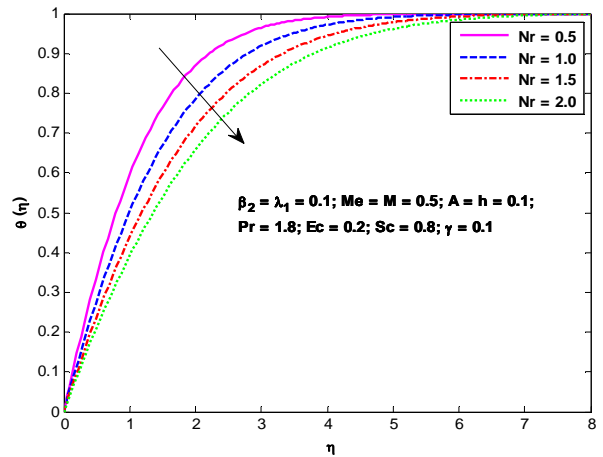


Fig. 8: Variation of Nr on θ

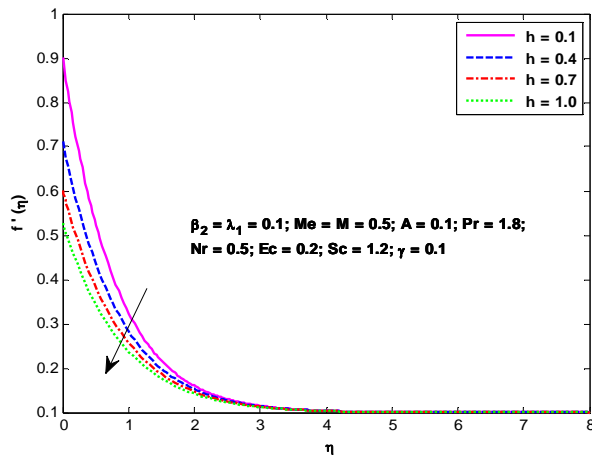


Fig. 6: Variation of h on f'

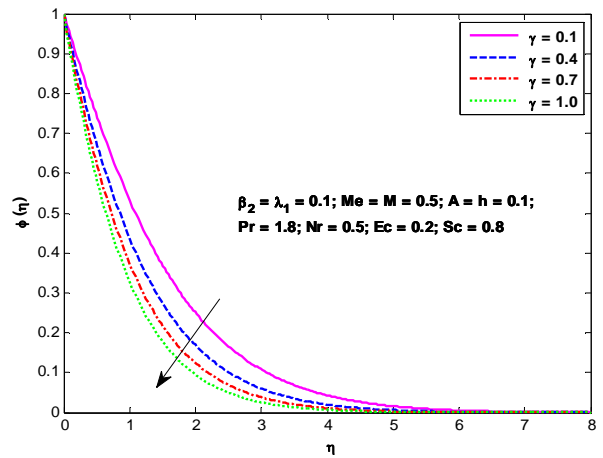


Fig. 9: Variation of γ on ϕ

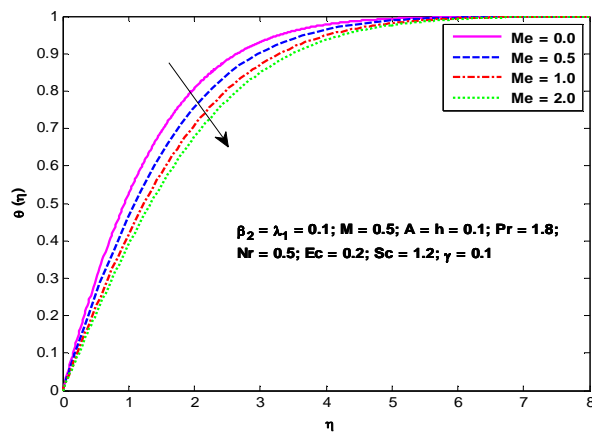


Fig. 7: Variation of Me on θ

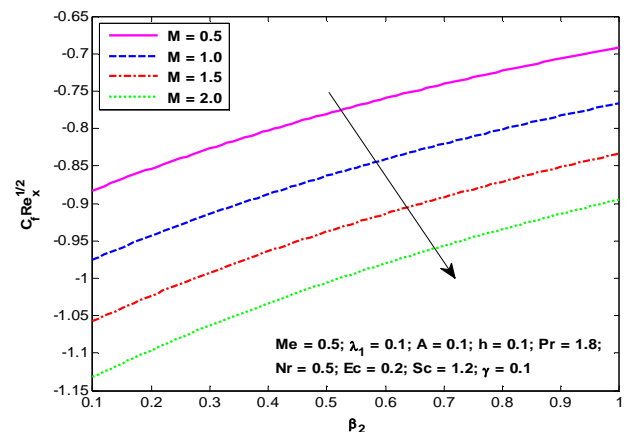


Fig. 10: Variation of β_2 and M on $C_f Re_x^{1/2}$

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The influence of chemical reaction parameter (γ) on the species concentration for generative chemical reaction is depicted in **Fig. 9**. It is found that species concentration with its highest value at the plate decreases slowly till it reaches the minimum value i.e., zero at the far downstream. Further, increasing of the chemical reaction decreases concentration of the species in the boundary layer due to the fact that destructive chemical reaction reduces the thickness of the solutal boundary layer and increases the mass transfer.

Fig. 10 represents the skin friction coefficient versus Deborah number for a variation in the magnetic field. It is observed that the Skin friction coefficient (c_f) increases monotonically with Deborah number and the values of c_f significantly decrease with increase in the magnetic field strength.

V. CONCLUSION

The main findings from our analysis are:

- ❖ Velocity is an increasing function of melting heat transfer parameter and Deborah number and is a decreasing function of magnetic field parameter, velocity slip parameter and ratio of relaxation and retardation times.
- ❖ Temperature experiences reduction with higher values of thermal radiation and melting heat transfer parameters.
- ❖ Species concentration is a decreasing function of chemical reaction parameter.
- ❖ Deborah number has an increasing influence on skin friction coefficient.

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