

Oscillatory Hydro magnetic Convective Heat Transfer through a porous medium in a converging channel

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ABSTRACT: A series of numerical simulations were performed in order to study with unsteady convective heat transfer of a viscous, electrically conducting fluid through a porous medium in horizontal channel bounded by corrugated walls, by taking into account converging channel. The unsteadiness in the flow is due to the imposed oscillatory flow on the convective flow through the channel. The velocity and the temperature profiles were plotted and their behaviour is discussed in detail. The shear stress and the average Nusselt number are also calculated and tabulated for these sets of parameters. The effect of oscillatory flux on the interplay of the viscous, Lorentz and Buoyancy forces has been clearly brought out. The results of this study are expected to lead to guide lines that will allow the improving the design of the heat transfer equipment with converging channel.

KEYWORDS: Heat transfer, porous medium, Hydromagnetic convection, converging channel,

I. INTRODUCTION

Unsteady convection flows play an important role in aerospace technology, turbo machinery and chemical Engineering. Such flows arise due to either in steady motion of a boundary or boundary temperature. In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchange equipment through augmentation of the heat transfer. The heat transfer through wavy channel has been a topic of interest in recent times owing to its applications in technological areas viz., transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching of an ablative surface and film vapourisation in combustion chambers [1-3], and also the early research which used suspension and dispersion of millimeter- and micrometer-sized particles, faced the major problem of poor suspension stability. Thus, a new class of fluid for improving both thermal conductivity and suspension stability is required in the various industrial fields. This motivation leads to development of nanofluids[4]. Channels with diverging-converging geometries augments the transportation of heat transfer and momentum. It has been established that a variable geothermal reservoir for power generation must have a hot heat source for the continuous supply of energy and a highly permeable formulation to ensure sustained delivery of fluids to production wells at adequate rates[5-9]. Natural convection in porous medium with internal heat generation is of interest in such situations as post accident heat removal in nuclear power reactors and the geophysical problem arising during the storage of nuclear waste in the earth [10,11]. The MHD effects on the oscillatory convective flows have been discussed by the authors[12,13]. This problem has been extended to the case of non isothermal surface and also vertical cylinder cavity.

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II. MATHEMATICAL FORMULATION

We consider the unsteady motion of viscous, incompressible electrically conducting fluid through a porous medium in a horizontal channel bounded by wavy walls. In the presence of a constant heat source/sink. We choose a rectangular Cartesian system $0(x, y)$ with x-axis in the direction of motion and y-axis in the vertical direction and the walls are taken at $y = \pm Lf\left(\frac{\delta x}{L}\right)$, where $2L$ is the distance between the walls, f is a twice differentiable function and δ is a small parameter proportional to the boundary slope. The flow is maintained by an oscillatory volume flux rate for which a characteristic velocity is defined as

$$q(1 + e^{i\omega t}) = \left(\frac{1}{L}\right) \int_{-Lf}^{Lf} u dy \quad (1)$$

The flow being two-dimensional in order to maintain the compatibility of the momentum equations the temperature in the flow field must be a general function of x and y . However temperature over the boundary walls may be kept constant assuming the temperature dependence on the walls to be a function of

$$\eta = \frac{y}{f(x)} \quad (2)$$

The boundary conditions for the velocity and temperature fields are

$$u = 0, v = 0, T = T_1(\eta) \text{ on } y = -L f(\delta x/L) \quad (3)$$

$$u = 0, v = 0, T = T_2(\eta) \text{ on } y = L f(\delta x/L) \quad (4)$$

Eliminating pressure p the equations governing the flow in terms of ψ are

$$\left[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x \right] = \nu \nabla^4 \psi + \beta g (T - T_0)_x - \left(\frac{\nu}{k}\right) \nabla^2 \psi - \left(\frac{\sigma \mu_e H_o^2}{\rho_e}\right) \frac{\partial^2 \psi}{\partial y^2} \quad (5)$$

$$\begin{aligned} \rho_e C_p \left(\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) &= k_1 \nabla^2 \theta + Q \\ + \mu \left(\left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left(\frac{\partial^2 \psi}{\partial x^2} \right)^2 \right) &+ \left(\frac{\mu}{k} \right) \left(\left(\frac{\partial \psi}{\partial x} \right)^2 + \left(\frac{\partial \psi}{\partial y} \right)^2 \right) + M^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right) \end{aligned} \quad (6)$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\gamma \left((\nabla^2 \psi)_t + R \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} \right) = \nabla^4 \psi + \left(\frac{G}{R}\right) \theta_x - D^{-1} \nabla^2 \psi - M^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right) \quad (7)$$

and the energy equation in the non-dimensional form is

$$P\gamma^2 \left(\frac{\partial \psi}{\partial t} \right) + PR \left(\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta + \alpha \quad (8)$$

with the corresponding boundary conditions

$$\psi(+1) - \psi(-1) = (1 + ke^{i\omega t}) \quad (9)$$

$$\frac{\partial \psi}{\partial x} = 0, \frac{\partial \psi}{\partial y} = 0, \theta = \frac{T_1 - T_e}{T_2 - T_e} = h, \text{ say on } y = -f(\delta x) \quad \frac{\partial \psi}{\partial x} = 0, \frac{\partial \psi}{\partial y} = 0, \theta = 1 \text{ on } y = f(\delta x) \quad (10)$$

$$\frac{\partial \theta}{\partial y} = 0, \text{ at } y = 0 \quad (11)$$

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Where

$$G = \frac{\beta g (T_2 - T_e) L^3}{\nu^2} \quad (\text{Grashof number}), R = \frac{qL}{\nu} \quad (\text{Reynolds number}), P = \frac{\mu c_p}{k_1} \quad (\text{Prandtl number}),$$

$$D^{-1} = \frac{L^2}{k} \quad (\text{Darcy parameter}), M^2 = \left(\frac{\sigma \mu_e^2 H_o^2 L^2}{\nu^2} \right) \quad (\text{Hartman Number}), \gamma^2 = \frac{\omega L^2}{\nu} \quad (\text{The Wormsley number})$$

$$\alpha = \frac{QL^2}{k} \quad (\text{The Heat source parameter})$$

III. SHEAR STRESS AND JUSETL NUMBER

The shear stress on the channel walls is given by

$$\tau = \frac{\sigma_{xy}(1 - f'^2) + (\sigma_{yy} - \sigma_{xx})f'^2}{(1 + f'^2)} \quad (12)$$

where

$$(\tau)_{\eta=\pm 1} = \left[\frac{\left((1 - f'^2) f^2 (e_{48} + ke^{it} e_{50}) + \delta (1 - f'^2) f^2 (g_{21} + ke^{it} g_{10}) - \left(\frac{2f'}{f} \right) (g_{25} + ke^{it} g_{26}) + O(\delta^2) \right)}{(1 + f'^2)} \right] \quad (13)$$

$$\sigma_{ij} = -p\delta_{ij} + 2\mu e_{ij} \quad (14)$$

$$\sigma_{xx} = \frac{\partial u}{\partial x}, \sigma_{yy} = \frac{\partial v}{\partial y}, \quad (15)$$

$$\sigma_{zz} = \frac{\partial w}{\partial z}, \sigma_{xy} = 0.5 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (16)$$

and the corresponding expressions are

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{\eta=\pm 1} \quad (17)$$

$$\text{where } \theta_m = 0.5 \int_{-1}^1 \theta dy$$

$$(\tau)_{\eta=-1} = \left[\frac{\left((1 - f'^2) f^2 (e_{48} + ke^{it} e_{50}) + \delta (1 - f'^2) f^2 (g_{22} + ke^{it} g_{20}) - \left(\frac{2f'}{f} \right) (g_{29} + ke^{it} g_{31}) + O(\delta^2) \right)}{(1 + f'^2)} \right] \quad (18)$$

and the corresponding expressions are

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$$(Nu)_{\eta=+1} = \frac{2(e_{40} + \delta(e_{41} + ke^{it} e_{42}))}{f(e_3 + \delta(e_{30} + ke^{it} e_{39}) - 1)} \quad (19)$$

$$(Nu)_{\eta=-1} = \frac{2(e_{43} + \delta(e_{44} + ke^{it} e_{45}))}{f(e_{37} + \delta(e_{38} + ke^{it} e_{39}) - h)} \quad (20)$$

IV. RESULTLS AND DISCUSSIONS

The primary aim of our analysis is to investigate the effect of the magnetic effect on the temperature induced buoyancy flows through a porous medium taking into account the effects of surface geometry and the wall temperature ratio. The flow phenomena is analysed for different sets of the parameters, M, β . It should be noted that the flow is basically asymmetric due to distinct surface temperatures. For computational purpose we assume that the boundaries to be $y = \pm f(\bar{x}) = \pm(1 + \beta \exp(-\bar{x}^2))$. $\beta > 0$ corresponds to a dilated channel and $\beta < 0$ corresponds to a constricted channel. In this analysis we confined our attention to the constricted case. The transformation $\eta = \frac{y}{f(x)}$ reduces the

boundaries to be $\eta = \pm 1$. The non-uniformity in the boundary gives rise to a secondary transverse flow and hence the general pattern of the flow can be judged by the behaviour of the resultant of primary and secondary velocities. The computation of the individual velocity components would enable us to investigate the effect of each force acting on the flow and its relative influence on the primary and secondary flows.

Fig.1 shows that The reversed flow appears in the region for all values of M . The magnitude of u enhances with increase in $M \leq 4$ and for further increase in M , $|u|$ experiences a depreciation with M

Also greater the constriction of the boundaries larger the magnitude of u for $|\beta| \leq 0.1$ and for further increase in the constriction $|u|$ experiences a depreciation (Fig.2).

An increase in M enhances $|\theta|$ in the upper half and in a narrow region abutting the lower boundary while reduces it in the region $-0.6 \leq \eta \leq 0$ (Fig.3)

Higher the constriction smaller the magnitude of θ in the lower half and smaller $|\theta|$ in the upper half of the flow region (Fig.4)

The shear stress (τ) on the boundary walls $y = \pm 1$ is exhibited in (Tables 1 and 2) the stress on the lower wall in general is positive for the heating of the channel walls and negative in the cooling case and is positive on the upper wall The magnitude of the stress on the lower wall decreases for an increase in $G > 0$ and increase with $|G| (< 0)$ for $D^{-1} > 3 \times 10^3$ while for $D^{-1} \leq 2 \times 10^3$, $|\tau|$ decreases with $G > 3 \times 10^3$ and increases with higher $G > 5 \times 10^3$. Also lesser the permeability of the porous medium larger the magnitude of τ .

The rate of heat transfer on $y = \pm 1$ are show in (Tables 3 and 4). The magnitude of Nu on the lower wall enhances with increase in G and that on the upper wall it enhances in the heating case and reduces in the cooling case. For an increase in $D^{-1} \leq 2 \times 10^3$ and for further increase in

$D^{-1} \geq 5 \times 10^3$, $|Nu|$ decreases in the cooling case and enhances in the heating case. The Nusselt number on the upper wall decreases with increase in D^{-1} in the heating case and enhances with D^{-1} in the cooling case. An increase in M enhances Nu on the upper wall while on the lower wall $|Nu|$ decreases with $M \leq 4$ in both heating and cooling cases and enhances with $M \geq 5$. Nusselt number on the lower wall enhances with $\alpha > 0$ and decreases with increase in $|\alpha| (< 0)$ while on the upper wall $|Nu|$ depreciates with increase in $|\alpha| (< 0)$. Nu with γ shows that an increase in γ enhances $|Nu|$ on the lower and upper walls in the heating case and depreciates in the cooling case and for further increase in $\gamma \geq 10$ we notice a depreciation in $|Nu|$ in both heating and cooling cases (Tables 3 and 4).

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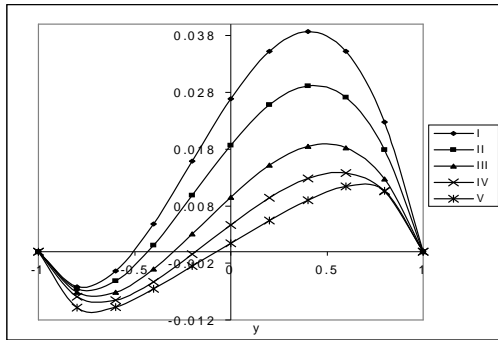


Figure.1 u with $M\beta = -0.5$

I	II	III	IV	V	
M	2	4	6	8	10

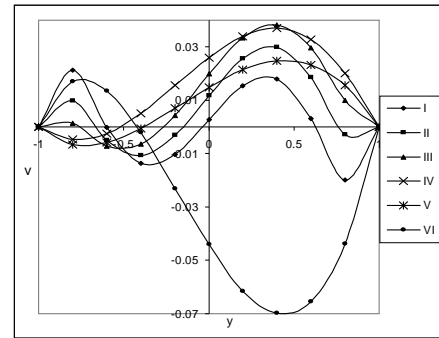


Figure.2 u with β

I	II	III	IV	V	VI	
β	0.1	0.3	0.5	0.7	1.0	1.5

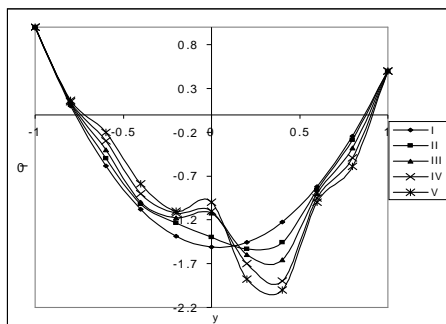


Figure.3 θ with $M\beta = -0.5$

I	II	III	IV	V	
M	2	4	6	8	10

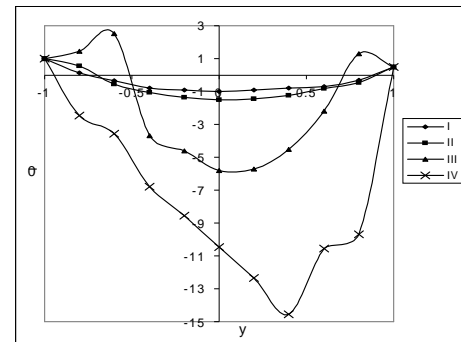


Figure.4 θ with β

I	II	III	IV	
β	0.1	0.3	0.5	1.5

Table.1: Shear Stress (τ) at lower wall $y = 1$ $P=0.71, M=2, \alpha=2, \beta=0.5$

G	I	II	III	IV	V
10^3	0.3444164	2.574206	4.236074	2.923205	-1.309867
3×10^3	-1.980491	0.989382	3.346382	1.831807	-0.277811
5×10^3	-1.728171	-0.227059	2.544121	-0.227059	11.12218
-10^3	5.225984	4.526155	5.213046	4.526155	7.891261
-3×10^3	12.64516	6.843977	6.277137	6.843977	27.19082
-5×10^3	22.58188	9.526412	7.428193	9.526411	56.45404
	I	II	III	IV	V
D-1	10^3	3×10^3	5×10^3	10^3	10^3
M	2	2	2	4	6

Table.2: Shear Stress(τ) at lower wall $y = -1$ $P=0.71, R=35, M=2, D^{-1} = 10^3, \beta = 0.5$

G	I	II	III	IV	V
10^3	0.3444164	-1.83389	4.000422	45.75865	0.3444164
3×10^3	-1.980491	1.517621	8.926012	234.4567	-1.980491
5×10^3	-1.728171	21.02951	16.31624	74.76517	-1.728171
-10^3	5.225984	10.71075	1.549492	-0.401713	5.225984

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-3×10^3	12.64516	38.88734	1.583229	5.6651284	12.64516
-5×10^3	22.58188	82.43164	4.111656	27.631464	22.58188

	I	II	III	IV	V
α	2	5	-2	-5	2
γ	2	2	2	2	5

Table.3: Average Nusselt Number(Nu) at lower wall $y = 1$ $P=0.71, M=2, \alpha=2, \beta= 0.5$

G	I	II	III	IV	V
10^3	0.4073	0.4028	0.4013	0.4073	0.4152
3×10^3	0.4256	0.4092	0.4042	0.4256	0.4591
5×10^3	0.4484	0.4163	0.4073	0.4484	0.5289
-10^3	0.3926	0.3969	0.3985	0.3926	0.3862
-3×10^3	0.3809	0.3916	0.3958	0.3809	0.3668
-5×10^3	0.3717	0.3869	0.3932	0.3717	0.3552

	I	II	III	IV	V
D-1	10^3	3×10^3	5×10^3	10^3	10^3
M	2	2	2	4	6

Table.4: Average Nusselt Number(Nu) at lower wall $y = -1$ $P=0.71, R=35, M=2, D^{-1} = 10^3, \beta = 0.5$

G	I	II	III	IV	V
10^3	0.4073	0.7451	3.9742	1.1071	0.4073
3×10^3	0.4256	0.9372	4.0547	1.4567	0.4256
5×10^3	0.4484	1.2928	4.3262	2.5287	0.4484
-10^3	0.3926	0.6353	4.0694	1.5335	0.3926
-3×10^3	0.3809	0.5651	4.3722	2.2495	0.3809
-5×10^3	0.3717	0.5227	4.9817		0.3717

	I	II	III	IV	V
α	2	5	-2	-5	2
γ	2	2	2	2	5

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