

Heat Transfer of non-Newtonian Williamson Fluid Flow past a Circular Cylinder with Suction and Injection

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ABSTRACT: We have explored heat transfer nature of Williamson fluids towards a circular cylinder. The governing equations of the flow are transformed using non-similarity variables and solved by employing finite difference Keller-Box technique. The behaviour of flow, velocity, temperature fields for different physical aspects is discussed through graphical illustrations. Also computed the coefficient of skin friction, local Nusselt and presented through tables.

KEY WORDS: Heat transfer, Williamson fluid, Suction/Injection, Skin friction, Nusselt number.

I. INTRODUCTION

Viscosity is an internal possession of a fluid that offers resistance to flow. Viscosity is an essential property of fluids which is important in many ways such as it is used in coating and printing, petroleum, industrial chemistry, food and beverages, organic chemistry etc. In rheological science viscosity is taken as a function of either temperature or pressure. The pressure dependence of the viscosity plays an important role in fluid film lubrication, micro-fluidics and geophysics. In geophysical flows the fluid viscosity changes with the depth of the fluids. After the early study discussed by Stoke's the pressure dependent viscosity has been considered by many researchers. Bridgman [1, 2] examined experimentally the pressure dependent viscosity for various organic liquids. Williamson [3] discussed the flow of pseudo plastic materials and proposed a model equation to describe the flow of pseudo plastic fluids and experimentally verified the results. Malic et al [4] discussed the flow of Homogeneous heterogeneous reactions in Williamson fluid model over a stretching cylinder by using Keller box method. Malic et al [5] investigate the Numerical solution of MHD stagnation point flow of Williamson fluid model over a stretching cylinder. Lyubimov and Perminov [6] discussed the flow of a thin layer of a Williamson fluid over an inclined surface in the presence of a gravitational field. Nagaraja et al [12] studied Convection boundary layer flow and heat transfer in an Eyring-Powell fluid past a horizontal circular cylinder in Porous medium.

The purpose of the present paper is to present the modeling of a two-dimensional Williamson fluid over a circular cylinder. The non-linear partial differential equations are simplified by using suitable non-similarity transformations and the resulting reduced equations are then solved with the help of finite difference Keller-Box method. The results are discussed from end to end graphs for various corporal parameters. The coefficient of skin friction, nusselt number is discussed tabular form.

II. MATHEMATICAL FLOW MODELLING

The steady, laminar, two-dimensional, viscous, incompressible, convection heat transfer flow from a horizontal circular cylinder embedded in a Williamson non-Newtonian fluid. The cylinder stretch with x-coordinate is measured along the circumference of the horizontal cylinder from the lowest point and the y-coordinate is measured normal to the surface, with 'a' denoting the radius of the horizontal cylinder. x/a is the angle of the y-axis with respect to the

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vertical ($0 \leq \Phi \leq \pi$). The gravitational acceleration, 'g' acts downwards. Both the horizontal cylinder and the fluid are maintained initially at the same temperature. Instantaneously they are raised to a temperature $T_w > T_\infty$ the ambient temperature of the fluid which remains unchanged. The governing conservation equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2\nu} \Gamma \left(\frac{\partial u}{\partial y} \right) \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty) \sin\left(\frac{x}{a}\right) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where u and v are the velocity components in the x - and y -directions respectively, ν -the kinematic viscosity of the conducting fluid, α -the thermal diffusivity, T -the temperature, T_∞ -the free stream temperature. The boundary conditions are prescribed at the circulation and the edge of the boundary layer regime, respectively as follows:

$$\text{As } y = 0 \quad : \quad u = u_w, \quad v = -v_w, \quad T = T_w \tag{4}$$

$$\text{At } y \rightarrow \infty \quad : \quad u \rightarrow 0, \quad T \rightarrow T_\infty$$

Where the stream function is given by

$$\psi = \nu \xi Gr^{1/4} f \tag{5}$$

Where the stream function ψ is defined by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, and therefore, the continuity equation is automatically satisfied. In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\xi = \frac{x}{a}, \eta = \frac{y}{a} Gr^{1/4}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, Pr = \frac{\nu}{\alpha}, Gr = \frac{g \beta (T_w - T_\infty) a^3}{\nu^3} \tag{6}$$

In view of non-dimensional quantities (6), Equations (1)-(3) reduce to the following coupled, nonlinear, dimensionless partial differential equations for momentum and energy for the region.

$$f''' - f'^2 + ff'' - \lambda f''^2 f''' + \frac{\sin \xi}{\xi} \theta = \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \tag{7}$$

$$\frac{1}{Pr} \theta'' + f \theta' = \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \tag{8}$$

The transformed dimensionless boundary conditions are:

$$\text{At } \eta = 0, \quad f = 0, \quad f' = 0, \quad \theta = 1 \tag{9}$$

$$\text{As } \eta \rightarrow \infty \quad f' \rightarrow 0 \quad \theta \rightarrow 0$$

The automated design quantities of physical attention include the skin-friction factor and Nusselt number, are given by,

$$C_f Gr^{-3/4} = f''(0) - \frac{\delta}{3} \varepsilon \xi^3 (f''(\xi, 0))^3 \tag{10}$$

$$\frac{Nu}{\sqrt[4]{Gr}} = -\theta'(0) \tag{11}$$

III. NUMERICAL SOLUTION

Fixed boundary layer equations in (η, ξ) , coordinate system is strongly nonlinear. A numerical method, implicit finite difference Keller-Box method, is deployed to solve the boundary value problem defined by Eqs. (7)-(8) with boundary conditions (9). This technique has been described succinctly in Cebeci and Bradshaw [7] and Keller [8]. It has been used recently in polymeric flow dynamics by Subba Rao *et al.* [9-11] for visco-elastic models. The key stages involved are as follows:

- Reduction of the Nth order partial differential equation system to N first order equations.
- Finite difference discretization of reduced equations.
- Quasilinearization of non-linear Keller algebraic equations.
- Block -tridiagonal elimination of linearized Keller algebraic equations.

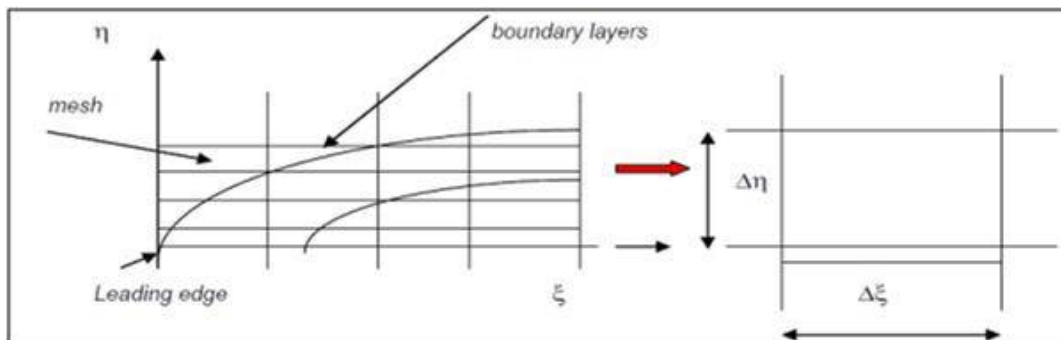


Fig. 1. Keller box computational domain

1. IV. RESULTS & DISCUSSIONS

The velocity, temperature are discussed here. Impact of different parameters Williamson fluid parameter λ , Prandtl number Pr and suction and injection parameter f_w have been examined.

Table 1 it can be observed from the table that increases Prandtl number Pr , Williamson fluid parameter λ behavior of skin friction coefficient and nusselt number. Fig. 2(a)-2(b) are plotted to see the Prandtl number (Pr) effect on the axial velocity, temperature in the curves. It can be seen that when increase the prandtl number, the axial velocity, temperature is decreases. Whereas the skin friction, heat transfer rates are decrease, increase respectively. Fig. 3(a)-3(b) elucidate the influence of Williamson fluid parameter (λ) on the axial velocity, temperature. It is noted that the axial velocity and temperature are decrease, increase respectively. Fig. 4(a)-4(b) represents the influence of suction and injection parameters (f_w) on the axial velocity, temperature. It is noted that the axial velocity and temperature are decreases.

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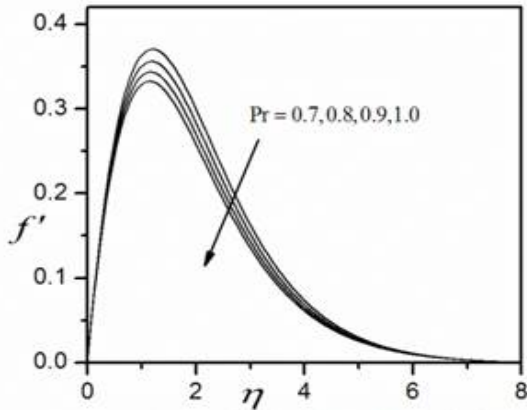


Fig. 2(a). Velocity profiles for different values of Pr for fixed $\lambda = 0.3$

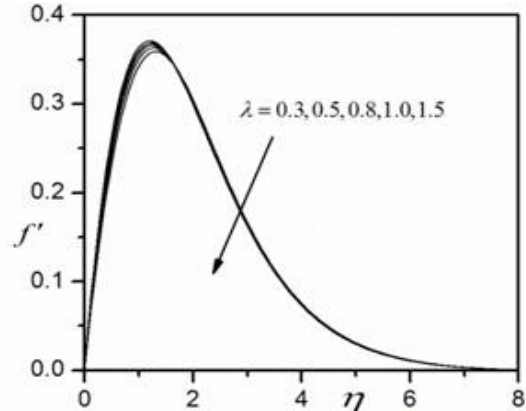


Fig. 3(a). Velocity profiles for different values of λ for fixed $Pr = 0.7$

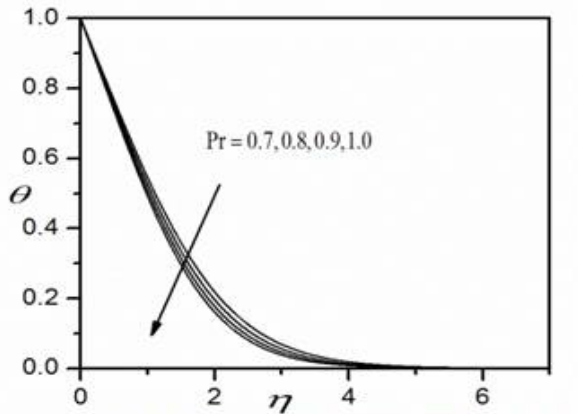


Fig. 2(b). Temperature profiles for different values of Pr for fixed $\lambda = 0.3$

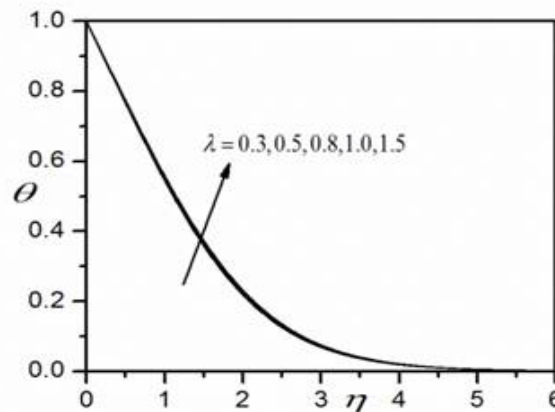


Fig. 3(b). Temperature profiles for different values of λ for fixed $Pr = 0.7$

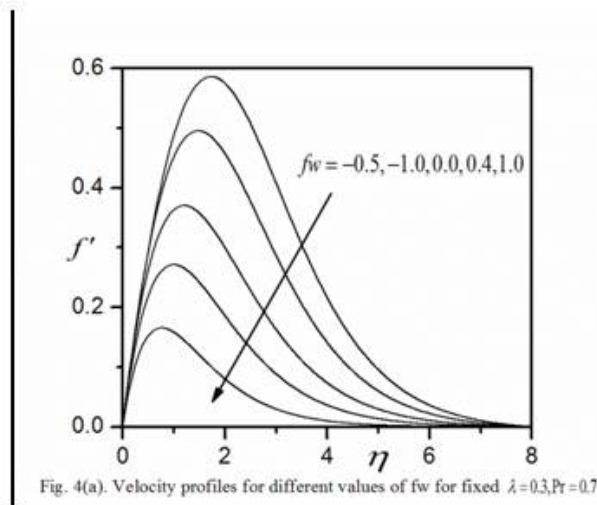


Fig. 4(a). Velocity profiles for different values of f_w for fixed $\lambda = 0.3, Pr = 0.7$

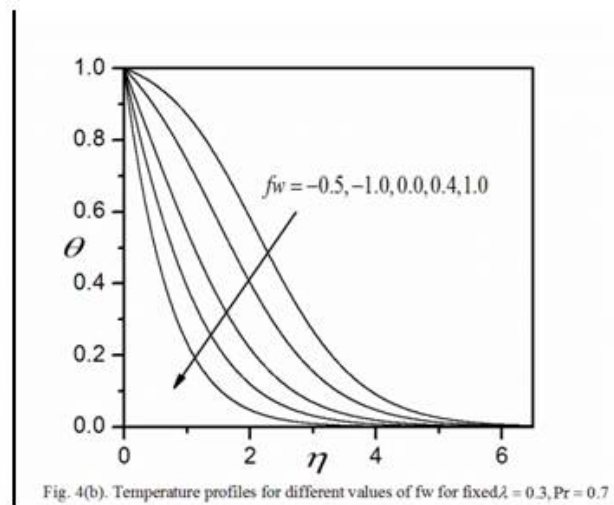


Fig. 4(b). Temperature profiles for different values of f_w for fixed $\lambda = 0.3, Pr = 0.7$

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Table 1: Skinfriction coefficient(C_f) and nusselt number(Nu) for different values of Pr & λ

λ	Pr	C_f	Nu
0.3	0.7	0.668784	0.39965
	0.8	0.65602	0.418813
	0.9	0.644664	0.436241
	1.0	0.634478	0.452257
0.5		0.643205	0.397409
0.8		0.621437	0.395362
1.0		0.593926	0.392574
1.5		0.557581	0.388537

V. CONCLUSIONS

Effects of suction and injection parameters in the boundary layer flow of non-Newtonian Williamson fluid over a steady circular cylinder are discussed.

1. The prandtl number (Pr) is increase, the axial velocity, temperature and skin friction coefficients are decrease, nusselt number is enhance.
2. The Williamson fluid parameter (λ) increases, the axial velocity, nusselt number and skin friction coefficients are decrease, temperature is enhance.
3. The suction and injection parameter (f_w) is increases, the axial velocity, temperature are decreases.

REFERENCES

- [1] Kannan K, Rajagopal KR. Flow through porous media due to high pressure gradients. *Appl Math Comput* 2008;199:748–59.
- [2] Bridgman PW. *The physics of high pressure*. New York: MacMillan; 1931.
- [3] Williamson, R. V., *The flow of pseudoplastic materials*. *Industrial & Engineering Chemistry Research*, 21, No. 11, 1108 (1929).
- [4] Malik MY, Salahuddin T, Hussain A, Bilal S, Awais M. Homogeneous heterogeneous reactions in Williamson fluid model over a stretching cylinder by using Keller box method. *AIP Adv* 2015;5(10):107227.
- [5] Malik MY, Salahuddin T. Numerical solution of MHD stagnation point flow of Williamson fluid model over a stretching cylinder. *Int J Nonlinear Sci Numer Simul* 2015;16(3–4):161–4.
- [6] Lyubimov, D. V. and Perminov, A. V., Motion of a thin oblique layer of a pseudoplastic fluid. *Journal of Engineering Physics and Thermophysics*, 75, No. 4, 920 (2002).
- [7] Cebeci T., Bradshaw P. 1984 *Physical and Computational Aspects of Convective Heat Transfer*, Springer, New York.
- [8] Keller H.B. 1970 A new difference method for parabolic problems, J. Bramble (Editor), *Numerical Methods for Partial Differential Equations* Academic Press, New York, USA
- [9] Subba Rao A, Prasad V.R, Nagendra N, Murthy K.V.N, Reddy N.B and Beg O.A 2015 Numerical Modeling of Non-Similar Mixed Convection Heat Transfer over a Stretching Surface with Slip Conditions *World Journal of Mechanics* 5 117- 128.
- [10] Ramachandra Prasad V, Subba Rao A and Anwar Bég O, Flow and Heat Transfer of Casson Fluid from a horizontal Circular Cylinder with Partial Slip in non-Darcy porous Medium, *Journal of Applied & Computational Mathematics* 2013, 2:3
- [11] V Ramachandra Prasad, A Subba Rao, N Bhaskar Reddy, B Vasu and O Anwar Be'g, Modelling laminar transport phenomena in a Casson rheological fluid from a horizontal circular cylinder with partial slip, *J Process Mechanical Engineering* 227(4) 309–326
- [12] L. Nagaraja, A.Subba rao M.Sudhakar reddy,M.Surya narayana reddy, Convection boundary layer flow and heat transfer in an Eyring-Powell fluid past a horizontal circular cylinder in Porous medium, *i-manager's Journal on Mathematics*, Vol. 6 , No. 1, January - March 2017