

Some Star Related Cubic Harmonious Graphs

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ABSTRACT: In this paper we have investigated some new cubic graceful graph the connection with star. A (n,m) graph $G=(V,E)$ is said to be **Cubic Harmonious Graph(CHG)** if there exists an injective function $f:V(G)\rightarrow\{1,2,3,\dots\dots m^3+1\}$ such that the induced mapping $f^*_{chg}:E(G)\rightarrow\{1^3,2^3,3^3,\dots\dots m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. In this paper, focus will be given on the result “cubic harmonious labeling of Hoffman tree, Coconut tree, Bananna tree, Complete binary tree.

KEYWORDS: , Bananna tree, Coconut tree, Cubic harmonious graph, Cubic harmonious labeling, Graceful graph, Star graph .

I. INTRODUCTION

For standard and terminology and notation we follow Graham and Sloane [4]. Graham and Sloane[4] defined a (n,m) -graph G of order n and size m to be harmonious, if there is an injective function $f:V(G)\rightarrow Z_m$, where Z_m is the group of integers modulo m , such that the induced function $f^*:E(G)\rightarrow Z_q$, defined by $f^*(uv) = f(u) + f(v)$ for each edge $uv \in E(G)$ is a bijection. Square harmonious graphs were introduced in [9]. Cubic graceful graphs were introduced in [5]. Cubic harmonious graphs were defined in [6]. We consider in this paper, simple, finite, connected and undirected graph.

Definition 1.1

The path on n vertices is denoted by P_n .

Definition1.2

A complete bipartite graph $K_{1,n}$ is called a star and it has $(n+1)$ vertices and n edges

Definition1.3

The bistar graph $B_{m,n}$ is the graph obtained from a copy of a star $K_{1,m}$ and a copy of star $K_{1,n}$ by joining the vertices of maximum degree by an edge.

Definition1.4

A complete binary tree is an 2-ary tree in which every internal vertex has exactly 2 children and all leaves are at the same level.

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Definition1.5

The coconut tree graph is obtained by identifying the vertex of $K_{1,m}$ with a pendant vertex of the path P_n

Definition1.5

An (n,k) – banana tree is a graph obtained by connecting one leaf of each of n copies of n k -star graph with a single root vertex that is distinct from all the stars.

II. MAIN RESULTS

Theorem 2.1

If $G_r = K_{1,m}$ for $1 \leq r \leq n$ be the given graph. Let $(K_{1,m}; G)$ be the graph obtained from ' r ' copies of $K_{1,m}$ by joining u_{rm} with $u_{(r+1)1}$ for $1 \leq r \leq n-1$. Then $(K_{1,m}; G)$ is cubic harmonious graph for all m and n .

Proof: Let $(K_{1,m}; G)$ be the given graph,

Let $V(G) = \{u_r, u_{rs}\}; \quad 1 \leq r \leq n, 1 \leq s \leq m$

and

$$E(G) = \begin{cases} \{u_r u_{rs}\}; & 1 \leq r \leq n, 1 \leq s \leq m \\ \{u_{rm} u_{(r+1)1}\} & 1 \leq r \leq n-1 \end{cases}$$

So

$$n(G) = n(m+1) \quad \text{and} \quad m(G) = [n(m+1)-1]$$

Define $f: V(G) \rightarrow \{1, 2, \dots, (n(m+1)-1)^3 + 1\}$ by

$$f(u_1) = (n(m+1)-1)^3 + 1$$

$$f(u_{1s}) = [n(m+1)-s]^3 + (n(m+1)-1)^3 + 1 - f(u_1); \quad 1 \leq s \leq m$$

$$f(u_r) = [(m+1)(n-r+1) - 1]^3 + (n(m+1)-1)^3 + 1 - f(u_{r1}); \quad 2 \leq r \leq n$$

$$f(u_{r1}) = [(m+1)(n-r+1)]^3 + (n(m+1)-1)^3 + 1 - f(u_{(r-1)3}); \quad 2 \leq r \leq n$$

$$f(u_{rs}) = [(m+1)(n-r+1) - s]^3 + (n(m+1)-1)^3 + 1 - f(u_r); \quad 2 \leq r \leq n,$$

$$2 \leq s \leq m$$

The induced edge mapping are

$$f^*(u_r u_{rs}) = (n(m+1) - ((m+1)(r-1) - s))^3; \quad 1 \leq s \leq m,$$

$$f^*(u_{r3} u_{(r+1)1}) = [(m+1)(n-r)]^3; \quad 1 \leq r \leq n-1$$

The induced edge labels are distinct and they are labeled as $\{1^3, 2^3, 3^3, \dots, (n(m+1)-1)^3\}$. Hence the theorem.

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Theorem 2.2

The graph $K_{1,p} \odot K_{1,q}$ is cubic harmonious for $p, q \geq 1$.

Proof :

Let $K_{1,p} \odot K_{1,q}$ be the given graph G with $(p + 1)(q + 1)$ vertices.

Let the vertex set be

$$V(K_{1,p} \odot K_{1,q}) = u, u_r, u_{rs}, w_s; \quad 1 \leq r \leq p, \quad 1 \leq s \leq q$$

and

$$V(K_{1,p} \odot K_{1,q}) = uu_r, u_r u_{rs}, u_{rs} w_s; \quad 1 \leq r \leq p, \quad 1 \leq s \leq q$$

Then $n(G) = (p + 1)(q + 1)$ and $m(G) = q(p + 1) + p$

Define an injection $f: V(K_{1,p} \odot K_{1,q}) \rightarrow \{1, 2, \dots, (q(p + 1) + p)^3 + 1\}$ by

$$f(u) = (q(p + 1) + p)^3 + 1$$

$$f(u_r) = [p + pq - r + 1]^3; \quad 1 \leq r \leq p$$

$$f(w_s) = [q(p + 1) + p - s + 1]^3; \quad 1 \leq s \leq q$$

$$f(u_{rs}) = [(r - 1)q + s]^3 + (q(p + 1) + p)^3 + 1 - [p(q + 1) - r + 1]^3; \quad 1 \leq r \leq p, \quad 1 \leq s \leq q$$

Then f induces a bijection $f^*: E(K_{1,p} \odot K_{1,q})$

$$f^*(uu_r) = [p + pq - r + 1]^3; \quad 1 \leq r \leq p$$

$$f^*(u_r u_{rs}) = [qr - q + s]^3; \quad 1 \leq r \leq p, \quad 1 \leq s \leq q$$

$$f^*(u_{rs} w_s) = [p(q + 1) + q + 1 - s]^3; \quad 1 \leq s \leq q$$

The induced edge labels are distinct and cubic. The edge mappings are $\{1^3, 2^3, 3^3, \dots, (q(p + 1) + p)^3\}$ Hence the theorem.

Theorem 2.3

If $G_r = K_{1,p}$ be the given graph for $1 \leq r \leq q$ with vertex set $V(G_r) = \{u_r, u_{rs}; 1 \leq r \leq q, 1 \leq s \leq p\}$. Let G be the given graph formulated by associate u_{rp} with $u_{(r+1)1}$ for $1 \leq r \leq q - 1$, Then G is a cubic harmonious for all p and q .

Proof :

The graph G consisting of the vertex set

$$V(G) = \{u_r, u_{11}, u_{rs}; 1 \leq r \leq q, \quad 2 \leq s \leq p\}$$

and the edge set

$$E(G) = \begin{cases} u_1 u_{11}; \\ u_r u_{rs}; \end{cases} \quad 1 \leq r \leq q, \quad 2 \leq s \leq p$$

Then $n(G) = pq + 1$ and $m(G) = pq$

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Let $f: V(G) \rightarrow \{1, 2, 3, \dots, (pq)^3 + 1\}$ be defined as follows.

$$f(u_1) = (pq)^3 + 1$$

$$f(u_{11}) = 2(pq)^3 + 1 - f(u_1)$$

$$f(u_r) = (pq - p(r-1))^3 + (pq)^3 + 1 - f(u_{(r-1)p}); \quad 2 \leq r \leq p$$

$$f(u_{rs}) = [pq - ((s-1) + (r-1)p)]^3 + (pq)^3 + 1 - f(u_r); \quad 1 \leq r \leq q, \quad 2 \leq s \leq p$$

Then the induced edge mapping are

$$f^*(u_1 u_{11}) = (pq)^3;$$

$$f^*(u_r u_{rs}) = \{pq - ((s-1) + (r-1)p)\}^3, \quad 1 \leq r \leq q, \quad 2 \leq s \leq p$$

The vertex labels are in the set $\{1, 2, 3, \dots, (pq)^3 + 1\}$. Then the edge label arranged in the set

$\{1^3, 2^3, \dots, (pq)^3\}$. So the vertex labels are distinct and edge labels are also distinct and cubic. So the graph G is cubic harmonious for all p and q .

Theorem 2.4

The coconut tree graph is cubic harmonious for all $m \geq 1$ and $n \geq 1$

Proof:

Let G be a coconut tree graph.

$$\text{Let } V(G) = \{u_r, v_s\}; \quad 1 \leq r \leq m; \quad 0 \leq s \leq n-1$$

and

$$E(G) = \{v_0 u_r, v_s v_{s+1}\}; \quad 1 \leq r \leq m; \quad 0 \leq s \leq n-2;$$

$$\text{So, } |n(G)| = m + n \quad \text{and} \quad |m(G)| = m + n - 1$$

Define $f: V(G) \rightarrow \{1, 2, \dots, (m+n-1)^3 + 1\}$ by

$$f(v_0) = (m+n-1)^3 + 1$$

$$f(u_r) = (m+n-r)^3; \quad 1 \leq r \leq m$$

$$f(v_s) = (n-s)^3 + (m+n-1)^3 + 1 - f(v_{s-1}); \quad 1 \leq s \leq n-1$$

The induced edge mapping are

$$f^*(v_0 u_r) = [m+n-r]^3; \quad 1 \leq r \leq m$$

$$f^*(v_s v_{s+1}) = (n-s-1)^3; \quad 0 \leq s \leq n-2$$

The vertex labels are in the set $\{0, 1, 2, \dots, (m+n-1)^3 + 1\}$. Then the edge label arranged in the set $\{1^3, 2^3, \dots, (m+n-1)^3\}$. So the vertex labels are distinct and the edge labels are also distinct and cubic. So the coconut tree graph is cubic harmonious for all $m \geq 1$ and $n \geq 1$.

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Theorem 2.5

Banana Tree $B_{n,k}$ is cubic harmonious for all $n, k, \geq 1$

Proof:

Let G be the graph $B_{n,k}$

Let $V(G) = u_i, u_r, w_s ; \quad 1 \leq r \leq nk, 1 \leq s \leq n$

and

$$E(G) = \begin{cases} uu_s; & 1 \leq s \leq n \\ w_s u_r; & 1 \leq r \leq nk \end{cases}$$

Then $n(G) = n(k+1) + 1$ and $m(G) = n(k+1)$

Let $f: V(G) \rightarrow \{1, 2, \dots, (n(k+1))^3 + 1\}$ be defined as follows.

$$f(u) = (n(k+1))^3 + 1$$

$$f(u_s) = [n(k+1) - s + 1]^3; \quad 1 \leq s \leq n$$

$$f(w_s) = (nk - s + 1)^3 + (n(k+1))^3 + 1 - f(u_s); \quad 1 \leq s \leq n,$$

$$f(u_r) = [n(k+1) - (r+1)]^3 + (n(k+1))^3 + 1 - f(u_{r-(n+i)});$$

$$(n+1) \leq r \leq k+1, \quad 0 \leq i \leq k-2$$

$$f(u_r) = [n(k+1) - (r+1)]^3 + (n(k+1))^3 + 1 - f(w_{r-(n-i(2-k)+j)});$$

$$1 \leq i \leq n-1, \quad (n+k) \leq r \leq nk, \quad 0 \leq j \leq k-2$$

The induced mapping are

$$f^*(uu_s) = [n(k+1) - (s-1)]^3; \quad 1 \leq s \leq n$$

$$f^*(w_s u_r) = [(nk - r + 1)]^3; \quad 1 \leq s \leq n, 1 \leq r \leq nk$$

The vertex labels are in the set $\{1, 2, \dots, (n(k+1))^3 + 1\}$. Then the edge label arranged in the set $\{1^3, 2^3, \dots, [n(k+1)]^3\}$. So the vertex labels are distinct and edge labels are also distinct and cubic. Hence the theorem

Theorem 2.6

A complete binary tree is cubic harmonious for all $n \geq 3$.

Proof:

Let G be the complete binary tree

Let $V(G) = u_r; \quad 1 \leq r \leq n$

and

$$E(G) = u_r u_{2r}; u_r u_{2r+1}; \quad 1 \leq r \leq \frac{n-1}{2}$$

Then $n(G) = n$ and $m(G) = (n-1)$

Let $f: V(G) \rightarrow \{1, 2, \dots, (n-1)^3 + 1\}$ be defined as follows:

$$f(u_1) = (n-1)^3 + 1$$

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$$f(u_{2r}) = (n-2r+1)^3 + (n-1)^3 + 1 - f(u_r); \quad 1 \leq r \leq \frac{n-1}{2};$$

$$f(u_{2r+1}) = (n-2r)^3 + (n-1)^3 + 1 - f(u_r); \quad 1 \leq r \leq \frac{n-1}{2};$$

The induced edge mapping are

$$f * (u_r u_{2r}) = [n - 2r + 1]^3; \quad 1 \leq r \leq \frac{n-1}{2};$$

$$f * (u_r u_{2r+1}) = [n - 2r]^3; \quad 1 \leq r \leq \frac{n-1}{2};$$

The vertex labels are in the set $\{1, 2, \dots, (n-1)^3 + 1\}$. Then the edge label arranged in the set $\{1^3, 2^3, 3^3, \dots, (n-1)^3\}$. So the vertex labels are distinct and the edge labels are also distinct and cubic. So the complete binary tree is cubic harmonious for all $n \geq 3$.

Corollary 1

A full binary tree is cubic harmonious for all $n \geq 3$.

Proof:

It can be proved by using evidence similar to those which led to theorem 2.9. So the given full binary tree is cubic harmonious for all $n \geq 3$.

Theorem 2.7

The graph G is obtained from $K_{1,a}$ by attaching $K_{1,b}$ at each pendant vertices of $K_{1,a}$ (child vertices) is cubic harmonious for all $a, b \geq 1$.

Proof: Let G be the graph obtained from joining the pendant vertices of $K_{1,a}$ by $K_{1,b}$.

$$\text{Let } V(G) = \{u, u_r, u_{rs}\}; \quad 1 \leq r \leq a, \quad 1 \leq s \leq b$$

$$\text{and } E(G) = \{uu_r, u_r u_{rs}\}; \quad 1 \leq r \leq a, \quad 1 \leq s \leq b$$

$$\text{So } |n(G)| = a(l+b) + 1$$

$$|m(G)| = a(l+b)$$

Define $f: V(G) \rightarrow \{0, 1, 2, \dots, [a(1+b)]^3 + 1\}$ by

$$f(u_0) = [a(1+b)]^3 + 1$$

$$f(u_r) = [a(1+b) - (r-1)]^3; \quad 1 \leq r \leq a$$

$$f(u_{rs}) = [b(r-1) + s]^3 - [a(1+b)]^3 + 1 - f(u_r); \quad 1 \leq r \leq a; \quad 1 \leq s \leq b;$$

The induced edge mapping are

$$f^*(u_0 u_r) = [a + ab - r + 1]^3; \quad 1 \leq r \leq a$$

$$f^*(u_r u_{rs}) = [b(r-1) + s]^3; \quad 1 \leq r \leq a, \quad 1 \leq s \leq b$$

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The vertex labels are in the set $\{1, 2, \dots, [a(1+b)]^3 + 1\}$. Then the edge label arranged in the set $\{1^3, 2^3, \dots, [a(1+b)]^3\}$. So the vertex labels are distinct and edge labels are also distinct and cubic. So the graph G is cubic harmonious for all $a, b \geq 1$.

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