

A Study on Locating Chromatic Number of Some Special Graphs

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ABSTRACT : The minimum number of colours needed in a locating colouring of G is the locating chromatic number of G denoted $\chi_L(G)$. In this paper, We determine the locating chromatic number of join of graphs, when G_1 and G_2 are two connected graphs with diameter at most two, then $\chi_L(G_1 + G_2) = \chi_L(G_1) + \chi_L(G_2)$, where $G_1 + G_2$ is the join of G_1 and G_2 . Also derive the conditions for the locating chromatic number for corona product of two connected graph $G \odot H$ and the exact value of locating-chromatic number for corona product of certain graphs.

KEYWORDS: Colouring, Chromatic Number, Locating Chromatic Number, Heighbour locating Colouring, Corona Product.

I. INTRODUCTION

Let f be a proper k -coloring of a connected graphs. If distinct vertices have distinct color codes then f is called locating coloring. The minimum number of colors needed in a locating coloring of G is the locating chromatic number of G denoted $\chi_L(G)$. In this chapter, We show that when G_1 and G_2 are two connected graphs with diameter at most two, then $\chi_L(G_1 + G_2) = \chi_L(G_1) + \chi_L(G_2)$, where $G_1 + G_2$ is the join of G_1 and G_2 . Also, we determine the locating chromatic numbers of the join of paths, cycles and complete multipartite graph. For any given graphs G and H , define the corona product $G \odot H$ between G and H as the graph obtained from G and H by taking one copy of G and $|V(G)|$ copies of H and joining all the vertices of the i^{th} -copy of H with the i^{th} -vertex of G . In this chapter, we derive the upper bound of the locating-chromatic number for corona product of two connected graph $G \odot H$ and the exact value of locating-chromatic number for corona product of certain graphs.

1.1 HISTORY OF GRAPH THEORY

Bridges in Konigsberg is about the Swiss mathematician Leonhard Euler (1707-1783). While he lived in the Prussian town of Konigsberg at the Baltic sea, he was thinking about the following problem. Konigsberg is divided into four parts by the river Pregel, and connected by seven bridges. Is it possible to tour Konigsberg along a path that crosses every bridge once, and at most once? You can start and finish wherever you want, not necessarily in the same place. The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Konigsberg bridge problem. The Konigsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one seven bridges that span a forked river flowing past an island-but without crossing any bridge twice. Euler argued that no such path exists. Graph theory refers to a set of vertices and of edges that connect the vertices. When any two vertices are joined by more than one edge, the graph is called a multigraph. In 1857, the Irish mathematician William Rowan Hamilton invented a puzzle ("The Lcosian Game"). In 1750 Euler discovered the formula $V-E+F=2$ relating the number of vertices (V), edges (E) and faces (F) of a polyhedron. The Chinese postman problem which was solved in the 1960's and the travelling salesman problem, data products and people work on such problems is related to the field of linear programming which was founded in the mid-20th century by the American mathematician Dantzi.

1.2 APPLICATION OF GRAPH THEORY

- In recent years, there has been another important use of Graph Theory: the internet. Every page in the internet could be a vertex in a graph, and whenever there is a link between two pages, there is an edge between the corresponding vertices. The resulting graph is of course very, very large.

- Early web search engines had a very big problem: they could search the web for a particular keyword, but they could not determine whether a page is "good" or just spam. If you searched for 'London', you might get hundreds of websites of small shops in London, or people who live there, before the official London.gov website.
- Google found a solution to this: any page that is very good will have many other pages linking to it. Pages linking to it. Pages that are rarely visited, or not very interesting, will be very "lonely" in the internet graph with only few other pages linking to it. This gives a way to *rank* websites and allows Google to display the best results at the beginning.
- There is a digital graph, of which you yourself are a part: Facebook. All the users are friends they are linked by an edge. Graph theory can help web developers improve the performance of social networking sites, and it can help us understand Facebook better.
- If I were to choose two users of Facebook completely at random, how many of these friendship edges do I need to get from one to the other? 10? 100? Surprisingly the answer, on average, is only six. On average you are less than six friends away from any other Facebook user: that's globalization!
- Experiments have been conducted to test this and the longest distance ever found on Facebook was only 12. The number of **Degrees of separation** has inspired countless writers and film makers. How many degrees are you away from the Queen? Or the American president? Nobody knows the exact answer, but it is probably a lot less than you think.

1.3 OPERATIONS OF THE GRAPH G_1 AND G_2

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs with $V_1 \cap V_2 = \emptyset$. Then, **The union** $G_1 \cup G_2$ to be (V, E) where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. **The sum** $G_1 + G_2$ as $G_1 \cup G_2$ together with all the edges joining vertices of V_1 to vertices of V_2 . **The product** $G_1 \times G_2$ as having $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent if $u_1 = v_1$ and u_2 adjacent v_2 in G_2 or $u_2 = v_2$ and u_1 adjacent v_1 in G_1 . The **corona** of two graphs G_1 and G_2 , defined as $G = G_1 \odot G_2$, formed from one copy G_1 and $|V(G_1)|$ copies of G_2 . Where i^{th} vertex of G_1 is adjacent to every vertex in i^{th} copy of G_2 .

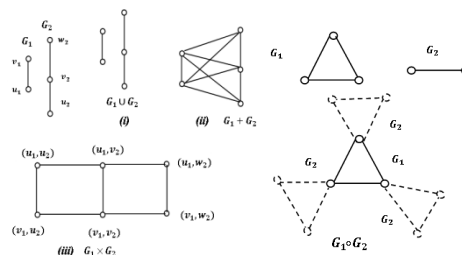


Figure.1.3. Operations of the graphs G_1 and G_2

1.4 COLOURING

An assignment of colors to the vertices of a graph G so that no two adjacent vertices get the same color is called a **coloring** of a graph. For each color, the set of all points which get that color is independent and is called **color class**. A coloring of a graph G using at most n colors is called **n coloring**.

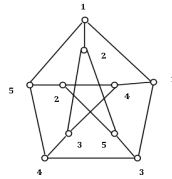


Figure.1.4. vertex coloring of the Petersen graph

1.5 CHROMATIC NUMBER

The **chromatic number** $\chi(G)$ of graph G is the minimum number of colors needed to color G . A graph G is called, **n -colorable** if $\chi(G) \leq n$.

Graph (G)	K_p	K_{p-x}	\bar{K}_p	$K_{m,n}$	C_{2n}	C_{2n+1}
Chromatic Number $\chi(G)$	p	$p-1$	1	2	2	3

Chromatic number of graphs

The **edgechromatic number** (also called **line chromatic number** or **chromatic index**) $\chi'(G)$ of graph G is the minimum number of colors needed to edge color G . A graph G is called, **n -edge colorable** if $\chi'(G) \leq n$.

1.6 LOCATING CHROMATIC NUMBER

The **locating chromatic number** $\chi_L(G)$ is the minimum number of colors in a locating coloring of G .

II. RESULT AND DISCUSSION

We begin with the following straightforward observations.

2.1 Theorem

For two arbitrary graphs G_1 and G_2 , we have $\chi_L(G_1 + G_2) = \chi_{L2}(G_1) + \chi_{L2}(G_2)$.

Proof

The diameter of $G_1 + G_2$ is at most two and using "If G is a connected graph with diameter at most two, then $\chi_L(G) = \chi_{L2}(G)$ ".

$$\chi_L(G_1 + G_2) = \chi_{L2}(G_1 + G_2).$$

Let $k_1 = \chi_{L2}(G_1)$, $k_2 = \chi_{L2}(G_2)$, and $k = \chi_{L2}(G_1 + G_2)$.

Let f be a neighbor locating k -coloring of $G_1 + G_2$ vertices of G_1 are adjacent to the vertices of G_2 and hence,

$$\{f(u) : u \in V(G_1)\} \cap \{f(v) : v \in V(G_2)\} = \emptyset$$

Let $k'_1 = |\{f(u) : u \in V(G_1)\}|$ and $k'_2 = |\{f(v) : v \in V(G_2)\}|$ thus, $k = k'_1 + k'_2$. Assume that u and u' are two vertices of G_1 with the same color. Since f is a neighbor locating coloring and $V(G_2) \subseteq N_{G_1+G_2}(u) \cap N_{G_1+G_2}(u')$, Since $f(N_{G_1}(u)) \neq f(N_{G_1}(u'))$. Therefore, f on $V(G_1)$ is a neighbor locating k'_1 -coloring of G_1 . Hence $k_1 \leq k'_1$. A similar argument holds for G_2 . Thus,

$$k_1 + k_2 \leq k'_1 + k'_2 = k.$$

Now let f_1 be a neighbor locating k_1 -coloring of G_1 with the color set $\{1, 2, \dots, k_1\}$, and f_2 be a neighbor locating k_2 -coloring of G_2 with the color set $\{k_1 + 1, k_2 + 2, \dots, k_1 + k_2\}$. Define the $(k_1 + k_2)$ -coloring f' of $G_1 + G_2$ as $f'(u) = f_1(u)$ when $u \in V(G_1)$ and $f'(v) = f_2(v)$ when $v \in V(G_2)$.

If z_1 and z_2 are two vertices in $G_1 + G_2$ with $f'(z_1) = f'(z_2)$, then $\{z_1, z_2\} \subseteq V(G_1)$ or $\{z_1, z_2\} \subseteq V(G_2)$.

Assume that $\{z_1, z_2\} \subseteq V(G_1)$. since f_1 is a neighbor locating k_1 -coloring and $V(G_2) \subseteq N_{G_1+G_2}(z_1) \cap N_{G_1+G_2}(z_2)$,

We have $f'(N_{G_1+G_2}(z_1)) \neq f'(N_{G_1+G_2}(z_2))$. This means that f' is a neighbor locating $k_1 + k_2$ -coloring of $G_1 + G_2$ and hence $k \leq k_1 + k_2$. Thus $k = k_1 + k_2$, which completes the proof.

2.2 Theorem

Let G and H be two connected graphs with diameter of $H \leq 2$ then $\chi_L(G \odot H) \leq \chi_L(G) + \chi_L(H)$.

Proof

Let Π_G and Π_H be minimum locating coloring of G and H , respectively. Let $|V(G)| = n$,

For $i = 1, 2, \dots, n$ color/partition all the vertices of each H_i according to Π_H ,

say $\{V(H_i)^1, V(H_i)^2, \dots, V(H_i)^s\}$, where $s = pd(H)$.

Consider the coloring/partition $\Pi = \Pi_1 \cup \Pi_2$ on $G \odot H$

where $\Pi_1 = \{ \cup_{i=1}^n V(H_i)^1, \cup_{i=1}^n V(H_i)^2, \dots, \cup_{i=1}^n V(H_i)^s \}$ and $\Pi_2 = \Pi_G$.

To prove that Π is locating coloring of $G \odot H$.

Since the diameter of H is at most 2, then the distance of any vertices $u, v \in V(H_i)$, for any i , under the corona graph $G \odot H$ is the same as its distance under the original graph H .

Therefore, if the vertices $u, v \in V(H_i)$, for any i , are distinguishable by Π_H then they are distinguishable too by Π_1 .

Let u and v be any two vertices of $G \odot H$. If $u, v \in V(H_i)$ then, if it is distinguished by $\cup_{i=1}^n V(H_i)^t$ for some t .

If $u, v \in V(G)$ then, if it is distinguished by some set in Π_G .

Assume that $u \in V(H_i)$ and $v \in V(G)$. If $u \in \cup_{i=1}^n V(H_i)^t$ for some t , Then the distances between u and v to $\cup_{i=1}^n V(H_i)^t$ is 0 and 1, respectively. Therefore, u and v are distinguished.

Now, the only case we have not considered is $u \in V(H_i)$ and $v \in V(H_j)$, for $i \neq j$.

If $u, v \in \cup_{i=1}^n V(H_i)^t$ for some t then u, v are distinguished by some set in Π_G .

Since Π_G is locating coloring for G .

2.3 Theorem

For $n, m \geq 1$, the locating-chromatic number of $K_n \odot \overline{K_m}$ is as follows:

$$\chi_L(K_n \odot \overline{K_m}) = \begin{cases} m+1, & \text{if } n \leq m+1 \\ n, & \text{if } n > m+1 \end{cases}$$

Proof

Let $V(K_n \odot \overline{K_m}) = \{x_1, x_2, \dots, x_n\} \cup V(H_1) \cup V(H_2) \cup \dots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the i^{th} -copy of $\overline{K_m}$. Using "Let G be a connected non trivial graph. Let c be a locating coloring for G and $u, v \in V(G)$. If $d(u, w) = d(v, w)$ for all $w \in V(G) - \{u, v\}$, then u and v must be in different color". Every two vertices in $V(H_i)$ must be in different color classes.

Since x_i is adjacent to all vertices of H_i , then x_i must be in a different color class other than all the color classes $V(H_i)$.

Therefore, $\chi_L(K_n \odot \overline{K_m}) \geq m+1$. If $n \leq m+1$ then define a mapping c on $V(K_n \odot \overline{K_m})$ such that $c(x_i) = i$ and $c(V(H_i)) = [1, m+1] - \{i\}$ for each i .

This mapping c is a locating coloring. Therefore, $\chi_L(K_n \odot \overline{K_m}) = m+1$ for $n \leq m+1$.

Now, consider $n > m+1$. Since every two x_i s must be in different classes then $\chi_L(K_n \odot \overline{K_m}) \geq n$. By constructing a mapping c such that

$$c(x_i) = i, \\ c(V(H_i)) = \begin{cases} [1, m+1] - \{i\} & \text{if } i \leq m+1 \\ [1, m] & \text{otherwise} \end{cases}$$

To prove that c is a locating coloring. If $d(u, x_1) = d(v, x_1)$ then we have:

- (i) $u = x_k$ and $v = x_l$ for some k, l ;
- (ii) $u = x_i$ and $v = a_{1k}$ for some $i \neq 1$ and k , or
- (iii) $u = a_{il}$ and $v = a_{js}$ for some i, j, l, s .

If (i) holds then u and v must be in different classes.

If (ii) holds then $d(u, S_{i-1}) \neq d(v, S_{i-1})$.

If (iii) holds then $d(u, S_i) \neq d(v, S_i)$. Therefore, c is a locating coloring and so $\chi_L(K_n \odot \overline{K_m}) = n$.

2.4 Theorem

For $n, m \geq 1$, the locating-chromatic number of $K_n \odot K_m$ is as follows:

$$\chi_L(K_n \odot K_m) = \begin{cases} m+1, & \text{if } n=1 \\ m+2, & \text{if } 2 \leq n \leq m+2 \\ n, & \text{if } n > m+2. \end{cases}$$

Proof

Let $V(K_n \odot K_m) = \{x_1, x_2, \dots, x_n\} \cup V(H_1) \cup V(H_2) \cup \dots \cup V(H_m)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the i^{th} -copy of K_m . Using "Let G be a connected non trivial graph. Let c be a locating coloring for G and $u, v \in V(G)$. If $d(u, w) = d(v, w)$ for all $w \in V(G) - \{u, v\}$, then u and v must be in different color", every two vertices in $V(H_i)$ must be in different color classes.

Furthermore, x_i is adjacent to all vertices of H_i , then x_i is in a different color class other than all the color classes $V(H_i)$.

Therefore, $\chi_L(K_n \odot K_m) \geq m+1$. If $n=1$ then it is clear that $\chi_L(K_n \odot K_m) = m+1$. If $2 \leq n \leq m+2$ then $\chi_L(K_n \odot K_m) \geq m+2$.

Let us define a mapping c satisfying:

$$c(x_i) = i, \\ c(V(H_i)) = \begin{cases} [1, m+2] - \{i, i+1\} & \text{if } i \neq m+1, \\ [1, m+2] & \text{otherwise.} \end{cases}$$

To prove that c is locating coloring on $(K_n \odot K_m)$. If $d(u, x_1) = d(v, x_1)$ then we have:

- (i) $u = x_k$ and $v = x_l$ for some k, l ;
- (ii) $u = x_i$ and $v = a_{ik}$ for some $i \neq 1$ and k , or
- (iii) $u = a_{il}$ and $v = a_{js}$ for some i, j, l, s .

If (i) holds then u and v must be in different classes.

If (ii) holds then $d(u, S_2) \neq d(v, S_2)$.

If (iii) holds then $d(u, S_{i+1}) \neq d(v, S_{i+1})$ or $d(u, S_1) \neq d(v, S_1)$.

Therefore, c is a locating coloring and so $\chi_L(K_n \odot K_m) = m+2$ for $2 \leq n \leq m+2$. Next, if $n > m+2$ then $\chi_L(K_n \odot K_m) \geq n$.

Now, let us define a mapping c such that $c(x_i)$, and $c(V(H_i)) = X_i$, where X_i is any m -subset of $\{1, 2, \dots, n\}$ not containing i and $i+1 \pmod n$, for each i . It can be verified that c is a locating coloring.

Therefore $\chi_L(K_n \odot K_m) = n$.

III. CONCLUSION

In this paper, we studied coloring of graph and analyze locating chromatic number of a graphs. We discussed the locating chromatic number of join of graphs under the conditions for the graphs are connected. We studied necessary and sufficient conditions for a disjoint union of graphs having a finite locating chromatic number. Also studied the upper bound of the locating-chromatic number for corona product of two connected graphs and the exact value of locating-chromatic number for corona product of certain graphs.

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