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On The Ternary Quadratic Diophantine Equation

$$a(x^2 + y^2) - (2a - 1)xy - k(x + y) + k^2 = (b^2 + 4a - 1)z^2$$

Shreemathi Adiga^{1*}, N. Anusheela², M.A. Gopalan³

Assistant Professor, Department of Mathematics, Government First Grade College, Koteswara, Kundapura Taluk,
Udupi, Karnataka, India^{1*}

Assistant Professor, Department of Mathematics, Government Arts College, Udthagamandalam, The Nilgiris, India.²

Professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy, Tamil Nadu, India³

ABSTRACT: The quadratic equation with three unknowns given by $a(x^2 + y^2) - (2a - 1)xy - k(x + y) + k^2 = (b^2 + 4a - 1)z^2$ is considered. Different sets of non-zero distinct integer solutions are exhibited.

KEYWORDS: quadratic equation, non-homogeneous quadratic equation, ternary quadratic, integer solutions.

I. INTRODUCTION

The theory of diophantine equations is one of the fascinating subjects occupying a vital place in the history of number theory. The beauty of diophantine equations is that the number of unknowns is bigger than the number of equations. It is worth mentioning that the diophantine problems offer an unlimited field for research by reason of their variety [1, 2]. Special choices of ternary quadratic diophantine equations for their solution in integers are considered by [3-13].

This communication concerns with the problem of determining solutions in integers to the ternary quadratic equation represented by $a(x^2 + y^2) - (2a - 1)xy - k(x + y) + k^2 = (b^2 + 4a - 1)z^2$. Various sets of integer solutions to the above equation are obtained.

II. METHOD OF ANALYSIS

The quadratic equation in three unknowns to be solved is given by

$$a(x^2 + y^2) - (2a - 1)xy - k(x + y) + k^2 = (b^2 + 4a - 1)z^2 \quad (1)$$

where a, b are non-zero integers.

Introduction of the linear transformations

$$x = u + v, \quad y = u - v, \quad u \neq v \neq 0 \quad (2)$$

in (1) gives

$$(u - k)^2 + (4a - 1)v^2 = (b^2 + 4a - 1)z^2 \quad (3)$$

Now, assume

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Vol. 7, Issue 7, July 2018

$$z = \alpha^2 + (4a-1)\beta^2 \tag{4}$$

Using (4) in (3) and applying the method of factorization, define

$$u - k + i\sqrt{4a-1}v = (b + i\sqrt{4a-1})(\alpha + i\sqrt{4a-1}\beta)^2$$

Equating the real and imaginary parts, one obtains

$$u = k + b(\alpha^2 - (4a-1)\beta^2) - 2(4a-1)\alpha\beta$$

$$v = \alpha^2 - (4a-1)\beta^2 + 2b\alpha\beta$$

In view of (2), the values of x and y are given by

$$\left. \begin{aligned} x &= k + (b+1)(\alpha^2 - (4a-1)\beta^2) + 2\alpha\beta(b-4a+1) \\ y &= k + (b-1)(\alpha^2 - (4a-1)\beta^2) - 2\alpha\beta(b+4a-1) \end{aligned} \right\} \tag{5}$$

Thus, (4) and (5) represent the integer solutions to (1).

However, there are other choices of solutions to (1) which are illustrated below:

Choice: 1

The substitution of the linear transformations

$$\left. \begin{aligned} z &= X \pm (4a-1)T \\ v &= X \pm (b^2 + 4a-1)T \end{aligned} \right\} \tag{6}$$

in (3) leads to

$$b^2 X^2 = b^2 (b^2 + 4a-1)(4a-1)T^2 + (u-k)^2$$

which is satisfied by

$$T = 2brs$$

$$u = k + b^2 (b^2 + 4a-1)(4a-1)r^2 - b^2 s^2$$

$$X = b(b^2 + 4a-1)(4a-1)r^2 + bs^2$$

Substituting the above values of X, T in (6), it is seen that

$$v = b(b^2 + 4a-1)(4a-1)r^2 + bs^2 \pm (b^2 + 4a-1)2brs$$

$$z = b(b^2 + 4a-1)(4a-1)r^2 + bs^2 \pm (4a-1)2brs \tag{7}$$

In view of (2), one obtains

$$\left. \begin{aligned} x &= k + b(b^2 + 4a-1)(4a-1)r^2 (1+b) + bs^2(1-b) \pm (b^2 + 4a-1)2brs \\ y &= k + b(b^2 + 4a-1)(4a-1)r^2 (b-1) - bs^2(1+b) \mp (b^2 + 4a-1)2brs \end{aligned} \right\} \tag{8}$$

Thus, (7) and (8) represent the integer solutions to (1).

Choice: 2

(3) is written in the form of ratio as

$$\frac{u-k+bz}{z+v} = \frac{(4a-1)(z-v)}{u-k-bz} = \frac{\alpha}{\beta}, \beta \neq 0$$

which is equivalent to the system of equations

$$\beta(u-k) + (b\beta - \alpha)z - \alpha v = 0$$

$$\alpha(u-k) - (b\alpha + (4a-1)\beta)z + (4a-1)\beta v = 0$$

Applying the method of cross-multiplication, the values of u, v, z are given by

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Vol. 7, Issue 7, July 2018

$$\begin{aligned} u &= k + b\alpha^2 + 2(4a-1)\alpha\beta - b(4a-1)\beta^2 \\ v &= 2b\alpha\beta - \alpha^2 + (4a-1)\beta^2 \\ z &= \alpha^2 + (4a-1)\beta^2 \end{aligned} \tag{9}$$

In view of (2), the values of x, y are found to be

$$\left. \begin{aligned} x &= k + (b-1)\alpha^2 + 2\alpha\beta(b+4a-1) + (4a-1)\beta^2(1-b) \\ y &= k + (b+1)\alpha^2 + 2\alpha\beta(4a-1-b) - (4a-1)\beta^2(b+1) \end{aligned} \right\} \tag{10}$$

Thus, (9) and (10) represents the integer solutions to (1).

Choice: 3

One may write (3) as

$$(u-k)^2 + (4a-1)v^2 = (b^2 + 4a-1)z^2 * 1 \tag{11}$$

Consider 1 as

$$1 = \frac{(2a-1+i\sqrt{4a-1})(2a-1-i\sqrt{4a-1})}{4a^2} \tag{12}$$

Using (4), (12) in (11) and employing the method of factorization, define

$$u-k+i\sqrt{4a-1}v = (b+i\sqrt{4a-1}) \frac{(2a-1+i\sqrt{4a-1})}{2a} (\alpha+i\sqrt{4a-1}\beta)^2$$

On equating real and imaginary parts in the above equation and performing an algebra, it is seen that

$$\begin{aligned} u &= k + [b(2a-1) - 4a + 1][2ap^2 - (4a-1)2aq^2] - 4apq(4a-1)(b+2a-1) \\ v &= (b+2a-1)(2ap^2 - 2a(4a-1)q^2) + 4apq(b(2a-1) - 4a + 1) \\ z &= 4a^2(p^2 + (4a-1)q^2) \end{aligned} \tag{13}$$

Substituting the above values of u and v in (2), it is seen that

$$\left. \begin{aligned} x &= k + 4a^2(p^2 - (4a-1)q^2)(b-1) - 8a^2pq(b+4a-1) \\ y &= k + 2a(p^2 - (4a-1)q^2)(b(2a-2) - 6a + 2) - 4apq(b(6a-2) + (4a-1)(2a-2)) \end{aligned} \right\} \tag{14}$$

Thus, (13) and (14) represent the integer solutions to (1).

Remark: 1

It is worth to mention that (3) may be written as

$$(b^2 + 4a-1)z^2 - (4a-1)v^2 = (u-k)^2 = (u-k)^2 * 1$$

Assume

$$1 = \frac{(\sqrt{b^2 + 4a-1} + \sqrt{4a-1})(\sqrt{b^2 + 4a-1} - \sqrt{4a-1})}{b^2}$$

Following the procedure as presented above, one obtains another set of solutions to (1).

Remark: 2

For particular values of a, b and k , the above solutions are found to be in agreement with [3-11].

As quadratic diophantine equations are rich in variety, one may attempt to determine integer solutions to multi-variable quadratic equations as well as higher degree diophantine equations with suitable properties.

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Vol. 7, Issue 7, July 2018

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