

Modeling the Capacitated-Time Windows Vehicle Routing Problem with the Existence of Closed Route

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ABSTRACT: Nowadays, due to the increase of population number, distribution systems of products have become more complex regarding to the time-consuming demands of customers and the variable product characteristics. For this reason, the companies that are working in this area have to make their delivery plans more carefully. It is aimed to supply the demands with minimum spending. Car locating problems are a problem of a distribution system. Problems dealing with the distribution of goods between the warehouse and the customers are referred to as vehicle routing problem (VRP). This optimization problem is to specify a homogeneous set of vehicles and routes, in which each vehicle starts from a depot and traverses along a route in order to serve a set of customers. The objective is to find the minimum total travel cost (related to the travel times or distances) and operational cost (related to the number of vehicles used). In this paper we present a variant of the VRP with time windows given a set of avoiding paths and to find the minimum total travel cost. We use integer programming model to describe the problem. A feasible neighbourhood approach is proposed to solve the model.

KEYWORDS: Vehicle routing problem with time windows, avoiding path, integer programming, feasible neighbourhood search

I. INTRODUCTION

The Vehicle Routing Problem (VRP) which is popular for solving distribution problem, is designed to seek a set of minimum-cost routes for the fleet of vehicles of a logistics company. The model is conditioned that the vehicles must start at the depot (distribution center), and then to deliver demands of a set of geographically scattered customers, and finally return to the depot. It is required that each customer can only be visited once by one single vehicle. The total demand of each route cannot exceed the capacity of the vehicle and the duration of each route cannot exceed a given limit.

Collection of household waste, gasoline delivery trucks, goods distribution, snowplough and mail delivery are the most used applications of the VRP. Huge research efforts have been devoted to studying the VRP since 1959 where Dantzig and Ramser have described the problem as a generalised problem of Travelling Salesman Problem (TPS). Thousands of papers have been written on several VRP variants such as Vehicle Routing Problem with Time Windows (VRPTW), Vehicle Routing Problem with Pick-Up and Delivery (VRPPD) and Capacitated Vehicle Routing Problem (CVRP) by Christofides et al., [1]. However, many gaps may exist between the basic VRP and real applications; for example, the number of depots, customer requirements with multiple pickups, delivery, type of vehicles with different travel times, travel costs and capacity, time windows restrictions, and route restrictions for vehicles. In [2] discuss comprehensive details on VRPs consisting of its variants, formulation, and solution methods. In [3] presents a compact review of vehicle routing literature.

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II. RELATED WORKS

As the problem is challenging, there are a lot variant of VRP have been intensively explored, see for example in [4]; [5]; [6]. Vehicle routing problems with time windows (VRPTW) form a large proportion of the matters studied. One example is the request for a vehicle to start service within a given time interval (i.e. time window). There are two types of model constraint in VRPTW, i.e., hard (strict) or soft (loose). In the hard time case, customers refuse if the arrival of vehicles to deliver is late, as can be seen in ([7]; [13]; [14]). In the soft time window case, customers accept the vehicle service regardless of arrival time, but nonetheless penalties for earliness or tardiness are incurred by Taillard et al. [8]; Chiang and Russell [9]). In both the hard and soft cases, early arrival vehicles must wait until the customer's requested service time arrives.

In graph the VRPTW can be described visually as follows: Let $G = (V, A)$ be a connected digraph consisting of a set of $n + 1$ nodes, each of which can be travelled only within a given time interval or time window, and a set A of arcs with non-negative weights representing travel distances (or cost) and associated travel times. Let one of the nodes be designated as the depot (center of distribution). Each node i , except the depot, requests a service of demand with size $q_i > 0$.

This paper discusses a variant of VRPTW in which there may be closed route(s). The occurrence of closed route (path) can happen frequently and can turn up dynamically due to rush hour, lane closures, construction, etc. Longer closed or forbidden subpaths are less common, but can arise, for example if heavy traffic makes it impossible to turn left soon after entering a multi-lane roadway from the right. If we are routing a single vehicle it is more natural to find a detour from the point of failure when a closed path is discovered.

Logically, a model whereby an algorithm identifies a potential path, and then this path is tried out on the actual network. In case of failure, further tests can be done to pinpoint a minimal closed subpath. Because such tests are expensive, a routing algorithm should try out as few paths as possible. In particular it is practically impossible to identify all closed paths ahead of time—we have an exponential number of possible paths to examine in the network. Therefore we impose an assumption of having no a priori knowledge of the forbidden paths, and of identifying forbidden paths only by testing feasibility of a path.

III. MATHEMATICAL FORMULATION OF THE VRPTW

Consider a vehicle of capacity Q is to deliver goods from a depot $N_0 = \{0\}$ to a set of customer nodes $N_1 = \{1, \dots, n\}$ in a complete directed graph with arc set A . A cost c_{ij} is incurred and a travel time t_{ij} is imposed for every arc $(i, j) \in A$, with $i \in N_0 \cup N_1$. Each customer $i \in N_1$ is characterised by a demand q_i , a service or dwell time s_i and a time window $[a_i, b_i]$, where a_i is the earliest time to begin service and b_i the latest time. Accordingly, the vehicle must wait if it arrives at customer i before time a_i . It is necessary to define the set of route K to be travelled by vehicles involved.

Objective function

$$\text{Minimise} \quad \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ij}^k \quad (1)$$

Subject to :

$$\sum_{j \in N_1} x_{ij}^k = y_i^k, \quad i \in N_0 \cup N_1, \quad k \in K, \quad (2)$$

$$\sum_{k \in K} y_i^k = 1, \quad i \in N_0 \cup N_1, \quad (3)$$

$$\sum_{i \in N_1} x_{ih}^k - \sum_{j \in N_1} x_{hj}^k = 0, \quad h \in N_0 \cup N_1, \quad k \in K, \quad (4)$$

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$$\sum_{i \in N_1} x_{0i}^k = 1, \quad k \in K, \quad (5)$$

$$\sum_{i \in N_1} x_{i(i+1)}^k = 1, \quad k \in K, \quad (6)$$

$$\sum_{i \in N_0 \cup N_1} q_i x_i^k = Q, \quad k \in K, \quad (7)$$

$$t_i^k + s_i + t_{ij} - M(1 - x_{ij}^k) \leq t_j^k, \quad (i, j) \in N_0 \cup N_1, \quad k \in K, \quad (8)$$

$$a_i y_i^k \leq t_i^k \leq b_i y_i^k, \quad i \in N_1, \quad k \in K, \quad (9)$$

$$t_0^1 \geq \sigma^1, \quad (10)$$

$$t_{n+1}^k + \sigma^{k+1} \leq t_0^{k+1}, \quad k = 1, \dots, k-1 \quad (11)$$

$$\sigma^k = \beta \sum_{i \in N_1} s_i y_i^k, \quad k \in K, \quad (12)$$

$$t_i^k \leq t_0^k + t_{\max}, \quad i \in N_1, \quad k \in K \quad (13)$$

$$x_{ij}^k \in \{0, 1\} \quad (i, j) \in A, \quad k \in K, \quad (14)$$

$$y_i^k \in \{0, 1\}, \quad i \in N_1, \quad k \in K, \quad (15)$$

Define

x_{ij}^r is 1 if arc $(i, j) \in A$ is in route k , 0 otherwise; note that $x_{0,n+1}^k$ is 1 if route k is empty;

y_i^k is 1 if customer i is in route k , 0 otherwise;

t_i^k is the time of beginning of service at customer i in route k ;

t_0^k is the start time of route k ;

t_{n+1}^k is the end time of route k .

Equation (2) states that every customer should be visited exactly once. Equations (3), (4) and (5) are flow conservation constraints that describe the vehicle path. Equation (6) states that the total demand on a route should not exceed the vehicle capacity. Equations (7) to (11) ensure feasibility of the time schedule while Equation (12) defines the vehicle setup time as the sum of service times of all customers in a route, multiplied by parameter β . Finally, Equation (13) corresponds to the deadline constraint for serving a customer.

IV. MODELING VRPTW WITH CLOSED ROUTE

The problem can be described as follows : given a graph $G(V, A)$, and vertices i and j , and given a set X of closed paths in A which belongs to the set route K , it is necessary to find optimal set of route $P \subseteq K$ such that no path in X is a subpath of P . Routes in X are called exceptions, and the desired route is called a shortest exception avoiding route. We allow an exception avoiding route to be non-simple graph, i.e., to repeat vertices and edges. In fact the problem becomes hard if the solution is restricted to simple Szeider, [10]. This problem has been called the Shortest Path Problem with Forbidden Paths by Villeneuve and Desaulniers [11]; Ahmed and Lubiw [12].

Objective function

$$\text{Minimise} \quad \sum_{k \in P} \sum_{(i,j) \in A \setminus X} c_{ij} x_{ij}^k \quad (16)$$

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Subject to :

$$\sum_{j \in N_1} x_{ij}^k = y_i^k, \quad i \in N_0 \cup N_1 \setminus X, \quad k \in P, \quad (17)$$

$$\sum_{k \in P} y_i^k = 1, \quad i \in N_0 \cup N_1 \setminus X, \quad (18)$$

$$\sum_{i \in N_1 \setminus X} x_{ih}^k - \sum_{j \in N_1 \setminus X} x_{ij}^k = 0, \quad h \in N_0 \cup N_1 \setminus X, \quad k \in P, \quad (19)$$

$$\sum_{i \in N_1 \setminus X} x_{0i}^k = 1, \quad k \in P, \quad (20)$$

$$\sum_{i \in N_1 \setminus X} x_{i(i+1)}^k = 1, \quad k \in P, \quad (21)$$

$$\sum_{i \in N_0 \cup N_1 \setminus X} q_i x_i^k = Q, \quad k \in P, \quad (22)$$

$$t_i^k + s_i + t_{ij} - M(1 - x_{ij}^k) \leq t_j^k, \quad (i, j) \in N_0 \cup N_1 \setminus X, \quad k \in P, \quad (23)$$

$$a_i y_i^k \leq t_i^k \leq b_i y_i^k, \quad i \in N_1 \setminus X, \quad k \in P, \quad (24)$$

$$t_0^1 \geq \sigma^1, \quad (25)$$

$$t_{n+1}^k + \sigma^{k+1} \leq t_0^{k+1}, \quad k = 1, \dots, k-1 \quad (26)$$

$$\sigma^k = \beta \sum_{i \in N_1 \setminus X} s_i y_i^k, \quad k \in P, \quad (27)$$

$$t_i^k \leq t_0^k + t_{\max}, \quad i \in N_1 \setminus X, \quad k \in P \quad (28)$$

$$x_{ij}^k \in \{0, 1\} \quad (i, j) \in A \setminus X, \quad k \in P, \quad (29)$$

$$y_i^k \in \{0, 1\}, \quad i \in N_1 \setminus X, \quad k \in P, \quad (30)$$

The explanation of Equations (16) – (30) would be just the same as in the previous Eqs (1) to (15), except that the route taken by the vehicles involved through the route set P.

V. THE ALGORITHM

First we solve the mixed integer programming model Eqs. (16) – (30) by relaxing the integer restriction. After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let

$$x = [x] + f, \quad 0 \leq f \leq 1$$

be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable x and f is the fractional component.

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

(This step is taken in order to get a minimum deterioration of the objective function).

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T \alpha_j$

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With j corresponds to

$$\min_j \left\{ \frac{d_j}{\alpha_{ij}} \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased from LB or decreased from UB. If none go to next i^* .

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

VI. CONCLUSION

This paper presents a VRPTW model in which the vehicles involved in the distributing goods will experience some closed route. However these closed routes are assumed to be known. The framework of the model stems from VRP with time windows. We then exclude the closed route from the previous assigned route. We solve the model using a feasible neighbourhood search.

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