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Colouring of Graphs Using Colouring of Families of Disjoint Sets Technique

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ABSTRACT:In this paper we introduce some concepts in graph theory to introduce new method for colouring edges and vertices of graph. This method has two types first related to vertices of the graph and second related to cycles. This method depend on the theory in our paper "Family of Disjoint Sets and Its Applications", with suitable changes

KEYWORDS:colouring at vertex, colouring at cycle, degree of colouring, standard of colouring, maximum number of colours, minimum number of colours, local maximum colours, local minimum colours.

I. INTRODUCTION

In paper [1]we introduce the concept of colouring families of disjoint sets as topic of set theory and introduce some results related to this concept, we also provide examples and applications. The four colours problem is a problem in graph theory, in paper [1] we introduce four colours problem as an application of colouring families of disjoint sets. Colouring families of disjoint sets explain four colours problem as an example of m colours problem. Using colouring families of disjoint sets we can explain any familiar map or unfamiliar map with any number of colours used to draw these maps.

In this paper we introduce new method for colouring edges and vertices of graph, used same new concepts in paper [1] with some changes to be adequate in graph theory. Here we introduced the concept of degree of colouring, and our method for colouring edges and vertices of graph depend on this concept.

Our new method for colouring using degree of colouring, has two types of colouring. First type is colouring edges and vertices of graph using degree of vertex, second type is colouring edges and vertices of graph using degree of cycle.

II. RELATED WORK

Here we introduce the concept of family of disjoint sets and concepts related to family of disjoint sets with some examples and propositions from paper [1]. Also we list some definitions of graph theory, by graph we mean finite graph and infinite graph both without loop.

Definition 2.1. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets and $X_{i1}, X_{i2}, X_{i3}, \dots, X_{in}$ be n sets such that any two sets belong to two different families of disjoint sets, and X is nonempty set, $X = X_{i1} \cap X_{i2} \cap X_{i3} \cap \dots \cap X_{in}$, $X = \{x : x \in X_{ij}, 1 \leq i \leq r, 1 \leq j \leq n, n \leq r, X_{ij} \in \chi_i\}$ then the positive integer n is called degree of intersection of X [1].

Definition 2.2. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and X is nonempty set, $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, then the intersection X is called represent the families $\chi_1, \chi_2, \chi_3, \dots, \chi_r$, and we used the notation $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, or $X = (\chi_1, \chi_2, \chi_3, \dots, \chi_r)$ [1].

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Proposition 2.3. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections, the family of disjoint sets χ_i appear w_i times at w intersections and the family of disjoint sets χ_j appear w_j time at w intersections, then we say the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these w intersections if and only if $w + 1 \leq w_i + w_j$.

Definition 2.4. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ each of degree n , and w be the minimum number of nonempty intersections, such that any two families of disjoint sets meet each other at these w intersections, then these w intersections called standard of meeting for these m families of disjoint sets [1].

Definition 2.5. Let $X_1, X_2, X_3, \dots, X_w$ be standard of meeting for m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ with degree of intersection n , and let $Y_1, Y_2, Y_3, \dots, Y_w$ be standard of meeting for the same m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, with degree of intersection n , then we say standard of meeting $X_1, X_2, X_3, \dots, X_w$ is congruent to standard of meeting $Y_1, Y_2, Y_3, \dots, Y_w$. This means two standards of meeting are congruent if they satisfy three conditions: They are both intersections of the same m families of disjoint sets, they have same number of intersections with same degree of intersection, but appearance of families of disjoint sets can be different at the two standards of meeting [1].

Example 2.6. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_6$ be six families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be five intersections each with degree of intersection four, $X_1 = (1,2,3,4)$, $X_2 = (1,2,3,5)$, $X_3 = (1,3,4,6)$, $X_4 = (1,4,5,6)$, $X_5 = (1,2,5,6)$. These five intersections X_1, X_2, X_3, X_4, X_5 is standard of meeting for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_6$. Let Y_1, Y_2, Y_3, Y_4, Y_5 be five intersections each with degree of intersection four, $Y_1 = (1,2,3,4)$, $Y_2 = (1,2,6,5)$, $Y_3 = (1,3,4,6)$, $Y_4 = (2,3,4,5)$, $Y_5 = (1,2,5,6)$. These five intersections Y_1, Y_2, Y_3, Y_4, Y_5 is standard of meeting for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_6$. These five intersections X_1, X_2, X_3, X_4, X_5 is congruent to the five intersections Y_1, Y_2, Y_3, Y_4, Y_5 [1].

Theorem 2.7. Let w intersections each with degree of intersection equal n , m benumber of local maximum families of disjoint sets, meet each other at w intersections, then we have $m(\frac{w+1}{2}) \leq wn$ for w odd, and $m(\frac{w}{2} + 1) \leq wn$ for w even [1].

Definition 2.8. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m is called local maximum families of disjoint sets at w intersections, if any two families of disjoint sets meet each other at these w intersections, and any family of disjoint sets has appearance either equal $\frac{w+1}{2}$ or $\frac{w+3}{2}$ for w odd such that not all families of disjoint sets has appearance equal $\frac{w+3}{2}$, and for w even we have three types, first all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, second appearance equal $\frac{w}{2} + 1$ and $\frac{w}{2} + 2$, not all $\frac{w}{2} + 2$ but at least has one appearance equal $\frac{w}{2} + 2$, third all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, except only one family of disjoint sets has appearance equal $\frac{w}{2}$. Maximum of locals maximum families of disjoint sets called maximum families of disjoint sets [1].

Definition 2.9. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections each of degree n , then m families of disjoint sets is called local minimum families of disjoint sets at w intersections, if $m(w-1) \leq nw$ such that $m \neq n$. When $m = n$, then m families of disjoint sets is called local minimum families of disjoint sets at w if $m(w-k) \leq nw < m(w-k+1)$ such that $m \neq n$, where $k = 2, 3, 4, \dots$. Minimum of locals minimum families of disjoint sets called minimum families of disjoint sets [1].

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Remark2.10. If n is degree of intersection, m is number of families of disjoint sets then $n+1 \leq m \leq 2n-1$ [1].

Remark2.11. If n is degree of intersection, w is standard of meeting for m families of disjoint sets number of families of disjoint sets then $3 \leq w \leq 2n-1$ [1].

Remark2.12. If n is degree of intersection, w is standard of meeting for m families of disjoint sets number of families of disjoint sets, both w and m depend on n , also m depends on w [1].

Remark2.13. Let $X_1, X_2, X_3, \dots, X_w$ be standard of meeting for m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$

With degree of intersection n , it is trivial case $w = 1$. When $w = 1$ we have $m = n$. Also standard of meeting consists of X_1 and X_2 where $X_1 = (1, 2, 3, \dots, n-1, n)$ and $X_2 = (1, 2, 3, \dots, n-2, n-1, n+1)$, X_1, X_2 is non-trivial. For $w = 2$, if standard of meeting for $n+1$ families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n, \chi_{n+1}$, because X_1 does not meet X_2 . If X_1 is standard of meeting for n families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n$, also X_1, X_2 is standard of meeting for n families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n$. Since $X_1 = X_2$, not allowed, two intersections have same representation (repeated), then standard of meeting X_1, X_2 means standard of meeting X_1 , and standard of meeting X_1, X_2 is trivial [1].

Remark2.14. Trivial case when standard of meeting w equal one, we have two trivial cases, first degree of intersection n equal one and second $n = m$ where m is number of families of disjoint sets. When $n = 1$ this leads to $w = 1$, and when $n = m$ this also leads to $w = 1$. Let standard of meeting w equal two, X_1, X_2 each of degree n , for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n, \chi_{n+1}$. If $X_1 = (1, 2, 3, \dots, n-1, n)$ and $X_2 = (1, 2, 3, \dots, n-1, n+1)$, family of disjoint sets χ_n and family of disjoint sets χ_{n+1} not meet each other at X_1, X_2 , $w = 2$ is trivial because $X_1 = X_2$, and we can't find $w = 2$ [1].

Definition2.15. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m is called local maximum families of disjoint sets at w intersections, if any two families of disjoint sets meet each other at these w intersections, and any family of disjoint sets has appearance either equal $\frac{w+1}{2}$ or $\frac{w+3}{2}$

for w odd such that not all families of disjoint sets has appearance equal $\frac{w+3}{2}$, and for w even we have three types,

first all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, second appearance equal $\frac{w}{2} + 1$ and $\frac{w}{2} + 2$, not all $\frac{w}{2} + 2$

but at least has one appearance equal $\frac{w}{2} + 2$, third all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, except only

one family of disjoint sets has appearance equal $\frac{w}{2}$. Maximum of local maximum families of disjoint sets called

maximum families of disjoint sets [1].

Definition2.16. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections each of degree n , then m families of disjoint sets is called local minimum families of disjoint sets at w intersections with t families of disjoint sets appear at each intersection, if $m(w-1) + t = nw$ where $0 \leq t < w$ such that $m \neq n$. When $m = n$, then m families of disjoint sets is called local minimum families of disjoint sets at w intersections with t families of disjoint sets appear at each intersection if $m(w-k) + t = nw$ where $0 \leq t < w$, $k = 2, 3, 4, \dots$ and $m(w-k) \leq nw < m(w-k+1)$ such that $m \neq n$. Minimum of all local minimum families of disjoint sets called minimum families of disjoint sets [2].

Remark2.17. By colouring m families of disjoint sets we mean an assignment of colour to each family of disjoint sets so that no two different families of disjoint sets have the same colour, this means all sets belong to one family of disjoint sets will be in same colour. Two different families of disjoint sets intersect each other, so they have two different colours.

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The idea of colouring families of disjoint sets have been used in paper [1] and any family of disjoint sets labelled by number or colour.

Definition 2.18. Two vertices v and w of a graph G are said to be adjacent if there is an edge vw joining them. Two distinct edges of a graph G are said to be adjacent if they have a vertex in common [3].

Definition 2.19. The degree of vertex v of a graph G is the number of edges incident to v , and is written $d(v)$ or $\rho(v)$ [3].

Definition 2.20. A graph G is called simple if between two distinct edges there is on multiple edges and there is no any loop [3].

Definition 2.21. A vertex colouring of a graph $G = (V, E)$ is a map $c : V \rightarrow S$ such that $c(v) \neq c(w)$ whenever v and w are adjacent. The element of the set S are called the available colours. For small integer k such that G has a map k -colouring a vertex colouring $c : V \rightarrow \{1, 2, 3, \dots, k\}$. This k is chromatic number of G , it is denoted by $\chi(G)$. A graph G with $\chi(G) = k$ is called k -chromatic, if $\chi(G) \leq k$, we called G k -colourable [5].

Definition 2.22. k -colouring is a vertex partition into k independent sets called colour classes [5].

Definition 2.23. A walk in which no vertex appears more than once is called a path, a walk in which no vertex appears more than once and closed is called cycle. The length of a cycle is its number of edges (or vertices) [6].

Example 2.24. If we labelled vertices by v_1, v_2, v_3, v_4, v_5 then $v_1v_2v_3v_2v_3$ is a walk, $v_1v_2v_3v_4v_3$ is a path and $v_1v_2v_3v_4v_1$ is a cycle [6].

III. COLOURING OF EDGES AND VERTICES USING DEGREE OF VERTEX

Suppose we have a graph G and maximum degree of vertex is n , if we want to colour edges of this graph such that two adjacent edges have not the same colour, and want to colour vertices such that adjacent vertices and vertices joined adjacent edges have not the same colour. In state of colours (black, blue, green, red, ...) we used numbers (1, 2, 3, 4, ...). Many or infinite edges e.g. can be labelled by the number 1, this means all edges labelled by the number 1 have the same colour and any two of them are nonadjacent edges and same thing for adjacent vertices and vertices joined adjacent edges.

Here we prove same results in paper [1] as results in graph theory after introducing suitable definitions.

Remark 3.1. In this paper we will label a vertex using its degree and edges. If v is vertex of degree three we can use the notation $v = (e_i, e_j, e_k)$ pointing to v as common point and any two of edges e_i, e_j, e_k are adjacent. The notation $v = (i, j, k)$ pointing to colours i, j, k , where i, j, k are integers. If v of degree n we write $v = (e_1, e_2, e_3, \dots, e_n)$ for pointing to edges $e_1, e_2, e_3, \dots, e_n$, or we can use notation $v = (1, 2, 3, \dots, n)$ for pointing to colours (or colour classes) 1, 2, 3, ..., n , and we say v represented (or labelled) by 1, 2, 3, ..., n . Also we will label a vertex using its degree and vertices, suppose in graph G degree of vertex v_i equal n , and any one of vertices $v_1, v_2, v_3, \dots, v_n$ is adjacent with vertex v_i , we used the notations $v_i = (v_i, v_1, v_2, v_3, \dots, v_n)$ for simplicity or pointing to colour classes we write $v_i = (i, 1, 2, 3, \dots, n)$, and we say v_i represented by $i, 1, 2, 3, \dots, n$, where i is an integer not equal any one of $\{1, 2, 3, \dots, n\}$.

Remark 3.2. Let e_1 and e_2 be adjacent edges at vertex v and of degree n , and then there exist vertices $e_3, e_4, e_5, \dots, e_n$ such that $v = (e_1, e_2, e_3, \dots, e_n)$.

Definition 3.3. Colouring edges at vertex is defined by assignment one of m colours to each edge so that adjacent edges have not same colour.

Definition 3.4. Degree of colouring edges at vertex is maximum number of edges have different colours at vertex.

Example 3.5. Degree of colouring edges at vertex $v = (e_i, e_j, e_k)$ equal three, and degree of colouring edges at vertex

$v = (e_1, e_2, e_3, \dots, e_n)$ equal n .

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Definition 3.6. Let $1, 2, 3, \dots, m$ be m colours used for colouring edges, $v_1, v_2, v_3, \dots, v_w$ be w vertices each of degree n , $1 \leq i \leq m, 1 \leq j \leq m$, then i, j , called two different colours at w vertices if whatever we exchange the numbers $1, 2, 3, \dots, m$, (which labelled the vertices, $v_1, v_2, v_3, \dots, v_w$) there exists a vertex represented (labelled) by i, j . We called i, j same colour class (same colour) if we exchange the numbers $1, 2, 3, \dots, m$ (which labelled the vertices edges, $v_1, v_2, v_3, \dots, v_w$) there exists a vertex not represented (labelled) by i, j .

Example 3.7. Let $1, 2, 3, 4, 5$ be 5 colours (colour classes) and v_1, v_2, v_3 be 3 vertices each of degree 3, where $v_1 = (1, 2, 3), v_2 = (1, 2, 4), v_3 = (3, 4, 5)$, the colour class 5 and colour class 1 are not different colours because there is no vertex represented by both 1 and 5, also the colour class 2 and colour class 5 are not different colours. If we exchange the numbers $1, 2, 3, 4, 5$ such that $v_1 = (1, 2, 5), v_2 = (1, 3, 4), v_3 = (2, 3, 4)$, the colour class 5 and colour class 3 are not different, the colour class 5 and colour class 4 are not different. Therefore we have 4 colours, not 5 colours, suppose 1 and 5 have same colour class (same colour), v_1, v_2, v_3 will be $v_1 = (1, 2, 3), v_2 = (1, 2, 4), v_3 = (1, 3, 4)$.

Definition 3.8. Let $1, 2, 3, \dots, m$ be m colours used for colouring edges of the graph G at the vertex, and $v_1, v_2, v_3, \dots, v_w$ be w vertices each of degree n , and w be the minimum number of vertices such that any two colours be two different colours at w vertices, then these w vertices are called standard of different m colours (colour classes).

Proposition 3.9. Degree of colouring edges at vertex is the degree of vertices.

Proof: The proof directly from definitions.

Proposition 3.10. Let $1, 2, 3, \dots, m$ be m colours used for colouring edges at vertices $v_1, v_2, v_3, \dots, v_w$, the colour i appear w_i times at w vertices and the colour j appear w_j time at these w vertices where $1 \leq i \leq m, 1 \leq j \leq m$, then the colour i is different from the colour j at these w vertices if and only if $w + 1 \leq w_i + w_j$.

Proof: First suppose colour i is different from the colour j at these w vertices, this means whatever we exchange colours represent these vertices there exists at least one vertex represented by the two colours i and j . Let us arrange each of vertices $v_1, v_2, v_3, \dots, v_w$ such that each vertex represented by one of i or j not both, since i and j are different there exists at least one vertex represented by the two colours i and j , therefore $w + 1 \leq w_i + w_j$.

Conversely suppose $w + 1 \leq w_i + w_j$, if we exchange colours represent the vertices $v_1, v_2, v_3, \dots, v_w$, such that each vertex represented by one of i or j not both, since $w + 1 \leq w_i + w_j$, there exists at least one vertex represented by the two colours i and j , therefore i and j are two different colours.

Proposition 3.11. Let $1, 2, 3, \dots, m$ be m colours used to colouring edges at vertices, $v_1, v_2, v_3, \dots, v_w$ be w vertices, then any two colours i, j , where $1 \leq i \leq m, 1 \leq j \leq m$, will be two different colours at w vertices if any colour appear at these w vertices not less than $\frac{w+1}{2}$ times for w odd, and not less than $\frac{w}{2} + 1$ times for w even.

Proof: Let $1, 2, 3, \dots, m$ be m colours of edges at vertices, $v_1, v_2, v_3, \dots, v_w$ and i, j , will be two colours at w vertices $v_1, v_2, v_3, \dots, v_w$. Let colour i appear w_i times at w vertices and the colour j appear w_j times at these w vertices, then $\frac{w+1}{2} + \frac{w+1}{2} = w + 1 \leq w_i + w_j$, for w odd and $\frac{w}{2} + 1 + \frac{w}{2} + 1 = w + 2 \leq w_i + w_j$, for w even, using

Definition 3.12. Colouring of vertices at vertex is defined by assignment one of m colours to each vertex so that adjacent vertices have different colours and vertices jointed adjacent edges have different colours.

Remark 3.13. Vertices jointed two adjacent edges are three, vertices jointed three adjacent edges are four, vertices jointed four adjacent edges are five and vertices jointed n adjacent edges are of $n + 1$.

Definition 3.14. Degree of colouring vertices at vertex is maximum number of vertices have different colours at vertex.

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Remark3.15. If degree of colouring edges at vertex is n , (where n is degree of the vertex), then degree of colouring vertices at the same vertex is $n+1$.

Definition3.16. Let $1,2,3,\dots,m$ be m colours used for colouring vertices of the graph G at the vertex, and $v_1, v_2, v_3, \dots, v_w$ be w vertices each of degree n , and w be the minimum number of vertices such that any two colours be two different colours at w vertices, then these w vertices are called standard of different m colours (colour classes).

Proposition3.17. Let $1,2,3,\dots,m$ be m colours used to colouring edges (or vertices) at vertices, $v_1, v_2, v_3, \dots, v_w$, each with of degree of colouring n , and t_i is number of times the colour i appear at these w vertices, where $1 \leq i \leq m$, then $\sum_{i=1}^m t_i = wn$.

Proof: Since the appearance for m colours at w vertices, $v_1, v_2, v_3, \dots, v_w$, and each vertex with degree of colouring n , then summation of all appearance for all m colours equal wn therefore $\sum_{i=1}^m t_i = wn$.

Theorem3.18. Let $v_1, v_2, v_3, \dots, v_w$ be w vertices each with degree of colouring equal n , m be number of different colours at these w vertices, then we have $m(\frac{w+1}{2}) \leq wn$ for w odd, and $m(\frac{w}{2} + 1) \leq wn$ for w even.

Proof: Let t_i be number of times the colour i appear at these w vertices, where $1 \leq i \leq m$, then from Proposition3.17. we have $\sum_{i=1}^m t_i = wn$. Since every two colours are different at w vertices, for each i and w odd number we have $\frac{w+1}{2} \leq t_i$,

Corollary 3.19. Let $v_1, v_2, v_3, \dots, v_w$ be w vertices each with degree of colouring equal n , m be number of different colours at these w vertices, then we have $m \leq 2n - 1$.

Proof: Using Theorem3.18 we have

$$m(\frac{w+1}{2}) \leq wn \text{ for } W \text{ odd, and } m(\frac{w}{2} + 1) \leq wn \text{ for } W \text{ even.}$$

Suppose $m \geq 2n + s$ where $s = 0,1,2,3,\dots$ then we have

$$2n + ws + s \leq 0 \text{ for } w \text{ odd, and } 4n + sw + 2s < 0 \text{ for } W \text{ even.}$$

This contradiction since w, n are positive integers and $s = 0,1,2,3,\dots$. Therefore $m \leq 2n - 1$.

Definition3.20 Let $v_1, v_2, v_3, \dots, v_w$ be w vertices each with degree of colouring equal n , m be number of different colours at these w vertices, then m is called local maximum different colours at w vertices, if any two different colours at these w vertices, has appearance either equal $\frac{w+1}{2}$ or $\frac{w+3}{2}$ for w odd such that not all different colours has appearance equal

$\frac{w+3}{2}$, and for w even we have three types of different colours, first all different colours has appearance equal $\frac{w}{2} + 1$,

second appearance equal $\frac{w}{2} + 1$ and $\frac{w}{2} + 2$, not all $\frac{w}{2} + 2$ but at least has one appearance equal $\frac{w}{2} + 2$, third all different

colours has appearance equal $\frac{w}{2} + 1$, except only one colour has appearance equal $\frac{w}{2}$. Maximum of local maximum

different colours called maximum different colours.

then $m(\frac{w+1}{2}) \leq \sum_{i=1}^m t_i = wn$. For each i and w be even number, we have $\frac{w}{2} + 1 \leq t_i$, then $m(\frac{w}{2} + 1) \leq \sum_{i=1}^m t_i = wn$.

Definition3.21. Let $v_1, v_2, v_3, \dots, v_w$ be w vertices each with degree of colouring equal n , m be number of different colours at these w vertices, then m is called local minimum different colours at w vertices, if $m(w-1) + t = nw$ where $0 \leq t < w$ such that $m \neq n$. When $m = n$, then m different colours is called local minimum different colours at w if

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$m(w - k) \leq nw < m(w - k + 1)$ such that $m \neq n$, where $k = 2, 3, 4, \dots$. Minimum of local maximum different colours is called minimum different colours.

Example 3.22. The following table, Table (I), explain Definition 3.20 and Definition 3.21, where n is degree of colouring, w is number of vertices, m_1 is local maximum colours, and m_2 is local minimum colours.

n	w	m_1	m_2
3	3	4	4
3	4	4	4
3	5	5	4
4	3	6	6
4	4	5	5
4	5	6	5
4	6	6	5
4	7	7	5
5	3	7	7
5	4	6	6
5	5	8	6
5	6	7	6
5	7	8	6
5	8	8	6
5	9	9	6

Table (I)

Remark 3.23. If local minimum colours at w vertices equal local maximum colours at w vertices, we have $w - 1 = \frac{w + 1}{2}$, $w = 3$ for

w odd, and $w - 1 = \frac{w}{2} + 1$, $w = 4$ for w even. Table (1) explain this when $w = 3$ and

when $w = 4$.

Example 3.24. Let G be the graph of dodecahedron, the vertices of the graph are marked 1,2,3,...,20 and each vertex of degree three.

If we want to colour edges at vertex and vertices at vertex, number of maximum colours and minimum colours we used the rule

$m(\frac{w + 1}{2}) \leq wn$ or $m(\frac{w}{2} + 1) \leq wn$ as shown in Table (II).

	Colouring of edges at vertex	Colouring of vertices at vertex
Degree of colouring	$n = 3$	$n = 4$
Standard of colouring for maximum colours w	$w = 5$	$w = 7$
Standard of colouring for minimum colours w	$w = 3$	$w = 3$
Maximum colours m_1	$m_1 = 5$	$m_1 = 7$
Minimum colours m_2	$m_2 = 4$	$m_2 = 5$

Table (II)

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Remark 3. 25. Let $v_1, v_2, v_3, \dots, v_w$ be w vertices each with degree of colouring n , there are many local maximum different colours at w , where $3 \leq w \leq 2n-1$ and there are many local minimum different colours at w , where $3 \leq w \leq 2n-1$. There is maximum different colours at w cycles, where $3 \leq w \leq 2n-1$, for each n , this maximum is equal only one local maximum and greater than all other local maximum. The maximum of all local maximum depend only on n , it is equal $2n-1$. Table (III) explain this, where m local maximum different colours at w .

n	w	m	$m \leq 2n-1$
3	3	4	$m < 2n-1$
3	4	4	$m < 2n-1$
3	5	5	$m = 2n-1$
4	3	6	$m < 2n-1$
4	4	5	$m < 2n-1$
4	5	6	$m < 2n-1$
4	6	6	$m < 2n-1$
4	7	7	$m = 2n-1$
5	3	7	$m < 2n-1$
5	4	6	$m < 2n-1$
5	5	8	$m < 2n-1$
5	6	7	$m < 2n-1$
5	7	8	$m < 2n-1$
5	8	8	$m < 2n-1$
5	9	9	$m = 2n-1$

Table (III)

IV. COLOURING OF EDGES AND VERTICES USING LENGTH OF CYCLE

Suppose we have a graph G and for all cycle of length n we colour the n edges by n different colours, also we colour the n vertices by n different colours, for all cycle of length less than n we colour the edges and vertices by different colours, any two adjacent edges and two adjacent vertices will be indifferent colours.

Definition 4.1. Colouring of edges at cycle of length n is defined by assignment one of m colours to each edge of graph G so that adjacent edges have different colours edges of any cycle of length n have different colours (or cycle of length less than n edges and path of n edges have different colours).

Definition 4.2. Degree of colouring at cycle of length n for edges is maximum number of edges have different colours at cycle of length n .

Definition 4.3. Let $1, 2, 3, \dots, m$ be m colours used to colouring edges at cycle of length n , and $C_1, C_2, C_3, \dots, C_w$ be w cycles each of length n , and w be the minimum number of cycles such that for any two edges of m edges there exists one cycle of the cycles $C_1, C_2, C_3, \dots, C_w$ such that the two edges have different colours, then these w cycles are called standard of colouring for m edges.

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Definition 4.4. Colouring of vertices at cycle of length n is defined by assignment one of m colours to each vertex of graph G so that adjacent vertices have different colours and vertices of any cycle of length n vertices have different colours, (or length less than n vertices and any path of n vertices have different colours).

Definition 4.6. Let $1, 2, 3, \dots, m$ be m colours used to colouring vertices at cycle of length n , and $C_1, C_2, C_3, \dots, C_w$ be w cycles each of length n , and w be the minimum number of cycles such that for any two vertices of m vertices there exists one cycle of the cycles $C_1, C_2, C_3, \dots, C_w$ such that the two vertices have different colours, then these w cycles are called standard of colouring for m vertices.

Proposition 4.7. Let $1, 2, 3, \dots, m$ be m colours used to colouring edges (or vertices) at cycle. $C_1, C_2, C_3, \dots, C_w$ be w cycles each of length equal n , (each with of degree of colouring n), and t_i is number of times the colour i appear at these w cycles, where $1 \leq i \leq m$, then $\sum_{i=1}^m t_i = wn$.

Proof: Same as in Proposition 3.18.

Theorem 4.8. Let $C_1, C_2, C_3, \dots, C_w$ be w cycles each of length equal n , m be number of different colours used to colouring edges or vertices at cycle, then we have $m(\frac{w+1}{2}) \leq wn$ for w odd, and $m(\frac{w}{2} + 1) \leq wn$ for w even.

Proof: Same as in Theorem 3.19.

Definition 4.9. Let $C_1, C_2, C_3, \dots, C_w$ be w cycles each of length equal n , m be number of different colours at these w cycles, then m is called local maximum different colours at w cycles, if any different colours has appearance either equal $\frac{w+1}{2}$ or $\frac{w+3}{2}$ for w odd such that not all different colours has appearance equal $\frac{w+3}{2}$, and for w even we have three types of different colours, first all different colours has appearance equal $\frac{w}{2} + 1$, second appearance equal $\frac{w}{2} + 1$ and $\frac{w}{2} + 2$, not all $\frac{w}{2} + 2$ but at least has one appearance equal $\frac{w}{2} + 2$, third all different colours has appearance equal $\frac{w}{2} + 1$, except only one colour has appearance equal $\frac{w}{2}$. Maximum of all local maximum different colours called maximum different colours.

Definition 4.10. Let $C_1, C_2, C_3, \dots, C_w$ be w cycles each of length equal n , m be number of different colours at these w cycles, then m is called local minimum different colours at w cycles, if $m(w-1) + t = nw$ where $0 \leq t < w$ such that $m \neq n$. When $m = n$, then m is called local minimum different colours at w if $m(w-k) \leq nw < m(w-k+1)$ such that $m \neq n$, where $k = 2, 3, 4, \dots$. Minimum of all local minimum different colours called minimum different colours.

Example 4.11. Let G be the graph of dodecahedron, the vertices of the graph are marked $1, 2, 3, \dots, 20$ and each vertex of degree three. If we want to colour edges at cycle and vertices cycle, number of maximum colours and minimum colours we used the rule $m(\frac{w+1}{2}) \leq wn$ or $m(\frac{w}{2} + 1) \leq wn$ as shown in Table (IV).

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	Colouring of edges at cycle	Colouring of vertices at cycle
Degree of colouring	$n = 5$	$n = 5$
Standard of colouring for maximum colours w	$w = 9$	$w = 9$
Standard of colouring for minimum colours w	$w = 3$	$w = 3$
Maximum colours m_1	$m_1 = 9$	$m_1 = 9$
Minimum colours m_2	$m_2 = 6$	$m_2 = 6$

Table (IV)

Remark 4. 12. Let $C_1, C_2, C_3, \dots, C_w$ be w cycles each with degree of colouring n , there are many local maximum different colours at w , where $3 \leq w \leq 2n - 1$ and there are many local minimum different colours at w , where $3 \leq w \leq 2n - 1$. There is maximum different colours at w cycles, where $3 \leq w \leq 2n - 1$, for each n , this maximum is equal only one local maximum and greater than all other local maximum. The maximum of all local maximum depend only on n , it is equal $2n - 1$.

Table (V) explain this, where m local maximum different colours at w .

n	w	m	$m \leq 2n - 1$
3	3	4	$m < 2n - 1$
3	4	4	$m < 2n - 1$
3	5	5	$m = 2n - 1$
4	3	6	$m < 2n - 1$
4	4	5	$m < 2n - 1$
4	5	6	$m < 2n - 1$
4	6	6	$m < 2n - 1$
4	7	7	$m = 2n - 1$
5	3	7	$m < 2n - 1$
5	4	6	$m < 2n - 1$
5	5	8	$m < 2n - 1$
5	6	7	$m < 2n - 1$
5	7	8	$m < 2n - 1$
5	8	8	$m < 2n - 1$
5	9	9	$m = 2n - 1$

Table (V)

Remark 4. 13. Table (III) and Table (V) are same, but degree of colouring in Remark 3. 25. for colouring edges at vertex and colouring vertices at vertex are different. This means in Remark 3. 25 for each degree of vertex we have two degrees of colouring and in Remark 4. 12. We have one degrees of colouring for colouring at cycle.

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