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Some Results of Edge Colouring using Families of Disjoint Sets Colouring Technique

M.E. Hassan

Assistant Professor, Department of Mathematics, Faculty of Science, Sudan University of Science and Technology,
Khartoum, Sudan

ABSTRACT: In this paper modify the definitions of local maximum families of disjoint sets and local minimum families of disjoint sets. We define local minimum and local maximum colour classes to colour edges of the graph, and prove some results related to these two concepts. For any graph with known number of vertices, known maximum degree of vertex, and known number of edges, such that number of vertices not exceed twice maximum degree of vertex, we can determine minimum and maximum number of colour classes to colour edges of the graph, for any huge number of edges, and give many examples..

KEYWORDS: edges colouring, families of disjoint sets colouring, local maximum, local minimum.

I. INTRODUCTION

In paper [1] we introduced the concept of colouring of families of disjoint sets technique as a topic of set theory. We also introduce definitions of degree of intersection, meeting and standard of meeting, minimum number of families of disjoint sets and maximum number of families of disjoint sets. In paper [2] we introduce some concepts in graph theory to introduce new method for colouring edges and vertices of graph. This method has two types first related to vertices of the graph and second related to cycles. This method depend on the theory in our paper "Family of disjoint sets and Its Applications", with suitable changes. In paper [3] we prove minimum number of families of disjoint sets with known degree of intersection (minimum number of colours) and we introduce another proof for maximum number of families of disjoint sets (maximum number of colours). Also in that paper we introduce the concept types of families of disjoint sets (types of colouring) and prove some results related to this concept, and we introduce the concept restricted families of disjoint sets (restricted colouring), restricted colouring is colouring in which we add a condition. Suppose we have small identical cubes, we want to colour these cubes such that any two adjacent small cubes coloured by two different colours, and any big cube consists of twenty seven small identical cubes coloured by twenty seven different colours, how many colours will be used. This problem of cubes can be solved by using restricted colouring. In paper[4] we assume both sets and graphs are finite. Results in that paper use colouring of families of disjoint sets technique for finite sets and finite graphs. Using results in that paper we can compare between colouring of families of disjoint sets technique and colouring of graphs in graph theory.

In paper [5] we prove some results in graph theory using colouring of families of disjoint sets technique. We introduced the concept of set labeled by families of disjoint sets and the degree of this set, as generalization to the degree of intersection and proved some results related to this concept. We introduce sorts of families of disjoint sets (sorts of colour classes) as a concept related to set labeled by families of disjoint sets, also we compare between familiar and unfamiliar sorts of families of disjoint sets and explain that by some examples.

In paper [6] we prove some results of edge colouring (trivial colouring and nontrivial colouring). We prove some results of edge colouring for many graphs connected by bridges. In many examples we determine minimum colours to colour edges of the graphs only by calculation when we know number of the edges, number of the vertices and largest degree of vertex, whatever the graph is huge. We used the concept of trivial colouring and nontrivial colouring to prove results of edge colouring for trees, bipartite graphs and complete graphs.

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In this paper we modify the definitions of local maximum families of disjoint sets and local minimum families of disjoint sets, which we introduced in paper [1], and prove some propositions related to these two concepts.

Also in this paper we define local minimum and local maximum colour classes to colour edges of the graph, and prove some results related to these two concepts. For any graph with known number of vertices, known maximum degree of vertex, and known number of edges, such that number of vertices not exceed twice maximum degree of vertex, we can determine minimum and maximum number of colour classes to colour edges of the graph, for any huge number of edges, and give many examples

II. RELATED WORK

In this section we recall some definitions and results related to colouring in graph theory and colouring of families of disjoint sets technique.

Definition 2.1. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets and $X_{i_1}, X_{i_2}, X_{i_3}, \dots, X_{i_n}$ be n sets such that any two sets belong to two different families of disjoint sets, and X is nonempty set, $X = X_{i_1} \cap X_{i_2} \cap X_{i_3} \cap \dots \cap X_{i_n}$, $X = \{x : x \in X_{ij}, 1 \leq i \leq r, 1 \leq j \leq n, n \leq r, X_{ij} \in \chi_i\}$ then the positive integer n is called degree of intersection of X [1].

Definition 2.2. Degree of colouring edges at vertex is maximum number of edges have different colours at vertex [2].

Definition 2.3. If we have m families of disjoint sets and we have a set all element of this set represent some of these m families of disjoint sets, such that any two elements of this set represent two different families of disjoint sets and there is a relation between these elements of the set, then we call this set, set labeled by families of disjoint sets [5].

Definition 2.4. Suppose we have m families of disjoint sets and many sets labeled by families of disjoint sets, if maximum number of elements for each set labeled by families of disjoint sets is n where $n < m$, then n is called the degree of set labeled by families of disjoint sets. If $n = m$ then the degree is called trivial degree or trivial case [5].

Definition 2.5. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections of these m families of disjoint sets, each intersection of degree n . The family of disjoint sets χ_i is said meets family of disjoint sets χ_j at these W intersections, if whatever we exchange families of disjoint sets at these W intersections, there exists at least one intersection the two families of disjoint sets χ_i and χ_j appear at this intersection [1].

Proposition 2.6. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections, the family of disjoint sets χ_i appear w_i times at w intersections and the family of disjoint sets χ_j appear w_j time at w intersections, then we say the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these w intersections if and only if $w + 1 \leq w_i + w_j$ [1].

Proposition 2.7. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections each of degree n , and t_i number of times the family of disjoint sets χ_i appear at these w intersections, then we have

$$\sum_{i=1}^m t_i = wn \quad [1].$$

Definition 2.8. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ each of degree n , and W be the number of nonempty intersections, such that any two families of disjoint sets meet each other at these W intersections and $3 \leq w \leq 2n - 1$, then these W intersections called standard of meeting for these m families of disjoint sets [1].

Definition 2.9. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m is called local maximum families of disjoint sets at w intersections, if any two families of disjoint sets meet each other at these w intersections, and any family of disjoint sets has appearance either equal $\frac{w+1}{2}$ or $\frac{w+3}{2}$

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for w odd such that not all families of disjoint sets has appearance equal $\frac{w+3}{2}$, and for w even we have three types, first all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, second appearance equal $\frac{w}{2} + 1$ and $\frac{w}{2} + 2$, not all $\frac{w}{2} + 2$ but at least has one appearance equal $\frac{w}{2} + 2$, third all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, except only one family of disjoint sets has appearance equal $\frac{w}{2}$. Maximum of locals maximum families of disjoint sets called maximum families of disjoint sets [1].

Definition 2.10. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections each of degree n , then m families of disjoint sets is called local minimum families of disjoint sets at w intersections with t families of disjoint sets appear at each intersection, if $m(w-1) + t = nw$ where $0 \leq t < w$ such that $m \neq n$. When $m = n$, then m families of disjoint sets is called local minimum families of disjoint sets at w intersections with t families of disjoint sets appear at each intersection if $m(w-k) + t = nw$ where $0 \leq t < w$, $k = 2, 3, 4, \dots$ and $m(w-k) \leq nw < m(w-k+1)$ such that $m \neq n$. Minimum of all locals minimum families of disjoint sets called minimum families of disjoint sets [2**].

Proposition 2.11. Let $1, 2, 3, \dots, m$ be m colours used for colouring edges at vertices $v_1, v_2, v_3, \dots, v_w$, the colour i appear w_i times at w vertices and the colour j appear w_j time at these w vertices where $1 \leq i \leq m$, $1 \leq j \leq m$, then the colour i is different from the colour j at these w vertices if and only if $w+1 \leq w_i + w_j$ [2].

Remark 2.12. If the degree of vertex v equal n , and these edges incident with the vertex v labeled by $1, 2, 3, \dots, n$. We labeled the vertex v by $(1, 2, 3, \dots, n)$. Let G be a simple graph with largest vertex degree n , and if minimum number of colours to colour edges of the graph G equal $n+1$, (nontrivial case) any vertex labeled by n labels of the set $\{1, 2, 3, \dots, n, n+1\}$. If minimum number of colours to colour edges of the graph G equal n , (trivial case) any vertex labeled by $(1, 2, 3, \dots, n)$ or labeled by labels less than n of the set $\{1, 2, 3, \dots, n\}$ [6].

Definition 2.13. A bridge is an edge whose removal increases the number of components [11].

Theorem 2.14. The sum of degrees of vertices equals twice the number of edges [12].

Proposition 2.15. If we have k finite graphs $G_1, G_2, G_3, \dots, G_k$, each graph with largest vertex degree n , the minimum number of colour to colour edges of the graph G_i is nontrivial colouring (equal $n+1$), where $1 \leq i \leq k$, and these k finite graphs connected by edges without changing largest degree n , where $n \geq 3$, then colouring edges of this cycle of k graphs and k edges is nontrivial colouring [6].

Proposition 2.16. If we have k finite graphs $G_1, G_2, G_3, \dots, G_k$, each graph with largest vertex degree n , then number of colour to colour edges of the graph G_i is nontrivial colouring (equal $n+1$), where $1 \leq i \leq k$, and these k finite graphs connected by edges without changing largest degree n , where $n \geq 3$, then colouring edges of this path of k graphs and $k+1$ edges is nontrivial colouring [6].

Proposition 2.17. If we have k finite graphs $G_1, G_2, G_3, \dots, G_k$, each graph with largest vertex degree n , the minimum number of colours to colour edges of the graph G_i is trivial colouring (equal n), where $1 \leq i \leq k$, and these k finite graphs connected by edges changing largest degree from n to $n+1$, where $n \geq 2$, then minimum colours to colour edges of this cycle (or path) of k graphs and edges (bridges) equal $n+1$ [6].

Proposition 2.18. If we have k finite graphs $G_1, G_2, G_3, \dots, G_k$, each graph with largest vertex degree n , then number of colour to colour edges of the graph G_i is nontrivial colouring (equal $n+1$), where $1 \leq i \leq k$, and these k finite graphs

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connected by edges (bridges), changing largest degree from n to $n + 1$, where $n \geq 2$, then minimum colours to colour edges of this cycle (or path) of k graphs and edges (bridges) equal $n + 1$.

III. SOME RESULTS OF EDGE COLOURING

In this section we modify the definitions of local maximum families of disjoint sets and local minimum families of disjoint sets, which we introduced in paper [1], here also we introduce definition of local minimum and local maximum colour classes to colour edges of the graph, and prove some results related to these two concepts. For any graph with known number of vertices, known maximum degree of vertex, and known number of edges, such that number of vertices not exceed twice maximum degree of vertex, we can determine minimum and maximum number of colour classes to colour edges of the graph, for any huge number of edges, and give many examples.

Definition 3.1. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m is called local maximum families of disjoint sets at w intersections, if for any $m + 1$ families of disjoint sets there are two families of disjoint sets not meet each other at these w intersections.

Definition 3.2. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m is called local minimum families of disjoint sets at w intersections, if for any $m - 1$ families of disjoint sets there exist at least two adjacent sets both belong to one family of disjoint sets, or $m - 1$ families of disjoint sets satisfy trivial colouring (every colour class appear w times at w vertices).

The following definition analogue to Definition 2.5.

Definition 3.3. Let $c_1, c_2, c_3, \dots, c_m$ be m colour classes used to colour edges of graph G , and $v_1, v_2, v_3, \dots, v_w$ be w vertices with maximum degree of vertex equal n , the colour class c_i is said to be meets the colour class c_j at these w vertices, if whenever we exchange colour classes at w vertices, there exists at least one vertex v_l such that the two vertices c_i and c_j appear at this vertex, where $1 \leq l \leq w$. When we say the colour class c_i not meets the colour class c_j at these w vertices, means $t_i + t_j < w + 1$.

Remark 3.4. As abbreviation in state of notation $c_1, c_2, c_3, \dots, c_m$ for colour classes we use notation $1, 2, 3, \dots, m$, we labeled edges by labels $1, 2, 3, \dots, m$, such that two adjacent edges have two different labels, and we label any vertex by labels equal its degree and same labels of edges join this vertex, then any colour class appear once at an edge appear twice at vertices.

The following proposition is the same as Proposition 2.7. restate to be suitable for graph theory, and t_i even number, because t_i is number of edges belong to colour class c_i multiple by two, and each edge belongs to colour class c_i , labeled by i . As an example let G be a graph with 3 vertices and 3 edges, each vertex of degree equal 2, (triangle), the number of colour classes to colour edges of the graph equal 3, we label the edges by labels (1),(2),(3), we label the vertices by labels (1,2),(2,3),(3,1), each vertex labeled by labels equal its degree, the number t_i at w vertices is even number, $t_1 = t_2 = t_3 = 2$.

Proposition 3.5. Let $c_1, c_2, c_3, \dots, c_m$ be m colour classes to colour edges of graph G , and $v_1, v_2, v_3, \dots, v_w$ be w vertices with maximum degree of vertex equal n , and t_i number of times the colour class c_i appear at these w vertices, then we have $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j)$ where $1 \leq i \leq m$ and t_i is even for all i .

Proof: We label edges by colour classes such that two adjacent edges have two different colours, and label any vertex by labels equal its degree and same labels of edges join this vertex, then any colour class appear once at edge appear

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Vol. 8, Issue 4, April 2019

twice at vertices then each t_i is even for all i , since $\sum_{i=1}^m t_i$ equal number of edges in the graph G multiple by two, from Theorem 2.14. the sum of degrees of vertices equal twice the number of edges, therefore $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j)$.

Remark 3.6. Let G be a graph with w vertices, with maximum degree of vertex equal n , and m be the number of colour classes to colour edges of graph G , for any t_i and t_j we have $w+1 \leq t_i + t_j$. For all t_i where $1 \leq i \leq m-1$ we have $\frac{w+1}{2} \leq t_i \leq w$ where w is odd, and $\frac{w+2}{2} \leq t_i \leq w$ where w is even, and $2 \leq t_m \leq w$, such that $w+1 \leq t_i + t_m$, where $1 \leq i \leq m-1$.

Definition 3.7. Let $c_1, c_2, c_3, \dots, c_m$ be m colour classes used to colour edges of graph G , and $v_1, v_2, v_3, \dots, v_w$ be w vertices with maximum degree of vertex equal n , the colour class c_i is said to be not meets the colour class c_j at these w vertices, if $t_i + t_j < w+1$.

Definition 3.8. Let G be a graph, m is called local minimum colour classes to colour edges of graph G , at w vertices $v_1, v_2, v_3, \dots, v_w$, if we use $m-1$ colour classes to colour edges of graph G , then exists at least two adjacent edges belong to one colour class or satisfy trivial colouring (every colour class appear w times at w vertices).

Proposition 3.9. Let $c_1, c_2, c_3, \dots, c_m$ be m colour classes to colour edges of graph G , and $v_1, v_2, v_3, \dots, v_w$ be w vertices, and t_i number of times the colour class c_i appear at these w vertices, if m is local minimum colour classes to colour edges of graph G , at w vertices, from equation $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j)$ then all these t_i have maximum value, such that all t_i not equal w .

Proof: Since m is colour classes to colour edges of graph G , at w vertices, then any two colour classes meet each other at w vertices, this means for all i , we have $\frac{w+1}{2} \leq t_i \leq w$ if w is odd, or $\frac{w+2}{2} \leq t_i \leq w$ if w is even, from definition of local minimum colour classes if we divide $\sum_{j=1}^w d(v_j)$ by $m-1$ then exists at least two adjacent edges belong to one colour class or satisfy trivial colouring, means $w < t_i$ or $t_i = w$, therefore t_i have maximum value.

Definition 3.10. Let G be a graph, m is called local maximum colour classes to colour edges of graph G , at w vertices $v_1, v_2, v_3, \dots, v_w$, if we use $m+1$ colour classes to colour edges of graph G , then exists a colour class c_i does not meet colour class c_j at these w vertices, where $1 \leq i < j \leq m+1$.

Proposition 3.11. Let $c_1, c_2, c_3, \dots, c_m$ be m colour classes to colour edges of graph G , and $v_1, v_2, v_3, \dots, v_w$ be w vertices, and t_i number of times the colour class c_i appear at these w vertices, if m is local maximum colour classes to colour edges of graph G , at w vertices, from equation $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j)$ then all these t_i have minimum value.

Proof: Since m is colour classes to colour edges of graph G , at w vertices, then any two colour classes meet each other at w vertices, this means for all i , we have $\frac{w+1}{2} \leq t_i \leq w$ if w is odd, or $\frac{w+2}{2} \leq t_i \leq w$ if w is even, from definition of local maximum colour classes if we divide $\sum_{j=1}^w d(v_j)$ by $m+1$ then exists a colour class c_i does not meet colour class c_j at these w vertices, where $1 \leq i < j \leq m+1$, this means $t_i + t_j \leq w$, therefore t_i have minimum value.

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Vol. 8, Issue 4, April 2019

Proposition 3.12. Let G be a graph of w vertices, with maximum degree of vertex equal n , and m is the number of local maximum colour classes to colour edges of graph G , let $D = \sum_{j=1}^k d(v_j)$, $t = \frac{w+1}{2}$ or $t = \frac{w+3}{2}$ if w is odd, such that t is even, and $t = \frac{w+2}{2}$ or $t = \frac{w+4}{2}$ if w is even, such that t is even, then we have the following:

- (i) $D = mt + r = m\left(\frac{w+2}{2}\right) + r$, where $0 \leq r \leq \frac{w-2}{2}$, and $\frac{w+2}{2}$ is even.
- (ii) $D = mt + r = m\left(\frac{w+4}{2}\right) + r$, where $0 \leq r \leq \frac{w-4}{2}$, and $\frac{w+4}{2}$ is even.
- (iii) $D = t(m-1) + r = (m-1)\left(\frac{w+4}{2}\right) + r$, where $r = \frac{w}{2}$, and $\frac{w+4}{2}$ is even.
- (iv) $D = mt + r = m\left(\frac{w+1}{2}\right) + r$, where $0 \leq r \leq \frac{w-3}{2}$, and $\frac{w+1}{2}$ is even.
- (v) $D = mt + r = m\left(\frac{w+3}{2}\right) + r$, where $0 \leq r \leq \frac{w-5}{2}$, and $\frac{w+3}{2}$ is even.
- (vi) $D = t(m-1) + r = m\left(\frac{w+3}{2}\right) + r$, where $r = \frac{w-1}{2}$, and $\frac{w+3}{2}$ is even.

Proof: In each case of these six cases any two colour classes meet each other at w vertices, and if we used $m+1$ colour classes to colour edges of graph G , then exists a colour class c_i does not meet colour class c_j at these w vertices, where $1 \leq i < j \leq m+1$.

Proposition 3.13. Let G be a graph of w vertices, with maximum degree of vertex equal n , and m is the number of local minimum colour classes to colour edges of graph G , then we have the following:

- (i) $D = m(w-1)$, where $w-1$ is even.
- (ii) $D = (m-1)(w-1) + r$, where $2 \leq r \leq w-3$, and $w-1$ is even.
- (iii) $D = m(w-2)$, where $w-2$ is even.
- (iv) $D = (m-1)(w-2) + r$, where $2 \leq r \leq w-4$, and $w-2$ is even.
- (v) $D = w(m-1) + r$, where $2 \leq r \leq w-2$, and w is even.

Proof: In each case of these five cases any two colour classes meet each other at w vertices, if we used $m-1$ colour classes instead of m colour classes, then there exists at least one t_i such that $w < t_i$, therefore m is the number of local minimum colour classes to colour edges of graph G . If we distribute $w-1$ or $w-2$ colour classes between $m-1$ colour classes then there exists at least one t_i such that $w < t_i$, or $w = t_i$, therefore m is the number of local minimum colour classes to colour edges of graph G .

Proposition 3.14. Let G be a graph of w vertices, with maximum degree of vertex equal n , and m is the number of colour classes to colour edges of graph G , therefore $\frac{D}{w-1} \leq m \leq \frac{D}{t}$, if w is odd, and $\frac{D}{w-2} \leq m \leq \frac{D}{t}$, if w is even, where

$D = \sum_{j=1}^k d(v_j)$, $t = \frac{w+1}{2}$ or $t = \frac{w+3}{2}$ if w is odd, such that t is even, and $t = \frac{w+2}{2}$ or $t = \frac{w+4}{2}$ if w is even, such that t is even.

Proof: Let m be the number of colour classes to colour edges of graph G , from definition local minimum number of colour classes, and definition local maximum number of colour classes, m lies between local minimum number of colour

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Vol. 8, Issue 4, April 2019

classes, and local maximum number of colour classes, using Proposition 3.12. and Proposition 3.13. therefore the result holds.

Example 3.15. Let G be a graph of 7 vertices, the sum of degrees of vertices equal 18, and m be the number of colour classes to colour edges of the graph, find the values of m .

Solution: Using Proposition 3.14. we have $\frac{D}{w-1} \leq m \leq \frac{D}{t}$, where $D = 18$, $w = 7$, $t = 4$, therefore $\frac{18}{6} \leq m \leq \frac{18}{4}$, if apply the rule $\frac{D}{t} = \frac{2D}{w+1} = m+r$, where $0 \leq r \leq \frac{w-1}{2}$, and $\frac{w+1}{2}$ is even, from (Proposition 3.12.), we have $\frac{18}{6} \leq m \leq \frac{18}{4} = m+r = 4+2$, then $3 \leq m \leq 4$.

Example 3.16. Let G be a graph of 999 vertices, with maximum degree of vertex equal 700, and the sum of degrees of vertices equal 698602 and m be the number of colour classes to colour edges of the graph, find the values of m .

Solution: Using Proposition 3.14. we have $\frac{D}{w-1} \leq m \leq \frac{D}{t}$, where $D = 698602$, $w = 999$, $t = 500$, from Proposition 3.13. if we apply the rule $D = (m-1)(w-1) + r$, where $2 \leq r \leq w-3$, and $w-1$ is even, we have $698602 = (700)(998) + 2$, we have $m = 701$, minimum value of m equal 701, from (Proposition 3.12.), if we apply the rule $\frac{D}{t} = \frac{2D}{w+1} = m+r$, where $0 \leq r \leq \frac{w-1}{2}$, and $\frac{w+1}{2}$ is even, we have $\frac{D}{t} = \frac{698602}{500} = 1397 + 102$, maximum value of m equal 1397, therefore we have $701 \leq m \leq 1397$.

Example 3.17. Let G be a graph of 7 vertices, with maximum degree of vertex equal 4, and sum of degrees of vertices equal 24, local minimum number of colour classes to colour edges of graph G equal 4. We labeled the colour class c_1 by the label 1, the colour class c_2 by the label 2, the colour class c_3 by the label 3 and the colour class c_4 by the label 4. We have 12 edges, each one of the labels 1,2,3,4, labeled by 3 edges. The 7 vertices labeled by (1,2,3,4), or labeled by some of labels (1,2,3,4), such that each vertex has labels equal its degree. At these 7 vertices the colour class c_1 appear 6 times, and each of the colour classes c_2, c_3, c_4 appear 6 times. We can write $\sum_{i=1}^4 t_i = 6 + 6 + 6 + 6 = \sum_{j=1}^7 d(v_j) = 24$.

Example 3.18. Let G be a graph of 7 vertices, with maximum degree of vertex equal 4, and sum of degrees of vertices equal 24, local maximum number of colour classes to colour edges of graph G equal 6. We labeled the colour class c_1 by the label 1, the colour class c_2 by the label 2, and generally the colour class c_i by the label i , where $1 \leq i \leq 6$. We have 12 edges, 2 edges labeled by i , where $1 \leq i \leq 6$. The 7 vertices labeled by some of labels (1,2,3,4,5,6), such that each vertex has labels equal its degree. At these 7 vertices each colour class appear 4 times. We can write $\sum_{i=1}^6 t_i = 4 + 4 + 4 + 4 + 4 + 4 = \sum_{j=1}^7 d(v_j) = 24$. Also we can use 5 colour classes to colour edges of this graph, and we can write $\sum_{i=1}^5 t_i = 6 + 6 + 4 + 4 + 4 = \sum_{j=1}^7 d(v_j) = 24$.

Example 3.19. Let G be a graph of 4 vertices, with maximum degree of vertex equal 3, we have 5 edges, sum of degrees of vertices equal 10, local maximum number of colour classes to colour edges of graph G equal 3, also local minimum number of colour classes to colour edges of graph G equal 3. We labeled the colour class c_1 by the label 3, the colour class c_2 by the label 2, the colour class c_3 by the label 3. We have 2 edges labeled 1, 2 edges labeled 2, one edge labeled 3. The 4 vertices two of them labeled by (1,2,3), and two labeled by (1,2). We can write $\sum_{i=1}^3 t_i = 4 + 4 + 2 = \sum_{j=1}^4 d(v_j) = 10$.

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Remark 3.20. Example 3.17. and Example 3.18. satisfies Proposition 3.14, we have $4 \leq m \leq 6$. Example 3.15. also satisfies Proposition 3.14, we have $3 \leq m \leq 3$.

Example 3.21. Let G be a graph with 20 vertices, with maximum degree of vertex equal 11, and sum of degrees of vertices equal 166, local maximum number of colour classes to colour edges of graph G equal 14. We labeled the colour class c_1 by the label 1, the colour class c_2 by the label 2, the colour class c_3 by the label 3, till we labeled the colour class c_{14} by the label 14. We have 83 edges, each of labels 1, 2, 3, ..., 12, 13 labeled 6 edges, there are 5 edges labeled by 14. The 20 vertices labeled by some of labels 1, 2, 3, ..., 12, 13, 14 such that each vertex has labels equal its degree. At these 20 vertices each colour class appear 12 times, except the colour class c_{14} appear 10 times. We can write $\sum_{i=1}^{14} t_i = 13(12) + 1(10) = \sum_{j=1}^{20} d(v_j) = 166$.

The local minimum number of colour classes to colour edges of graph G equal 11. We labeled the colour class c_1 by the label 1, the colour class c_2 by the label 2, the colour class c_3 by the label 3, till we labeled the colour class c_{11} by the label 11. We have 83 edges, each one of labels 1, 2, 3, ..., 9, 10 labeled 8 edges, there are 3 edges labeled by 11. The 20 vertices labeled by 1, 2, 3, ..., 9, 10, 11 or some of labels 1, 2, 3, ..., 9, 10, 11 such that each vertex has labels equal its degree. At these 20 vertices each colour class appear 16 times, except the colour class c_{11} appear 6 times. We can write $\sum_{i=1}^{11} t_i = 10(16) + 1(6) = \sum_{j=1}^{20} d(v_j) = 166$. There are numbers of colour classes, between local maximum and local

minimum, for number of colour classes to colour edges of graph G equal 12, we have 83 edges, each one of labels 1, 2, 3, ..., 9, 10, 11 labeled 7 edges, there are 6 edges labeled by 12. The 20 vertices labeled by some of labels 1, 2, 3, ..., 9, 10, 11, 12 such that each vertex has labels equal its degree. At these 20 vertices each colour class appear 14 times, except the colour class c_{12} appear 12 times. We can write $\sum_{i=1}^{12} t_i = 11(14) + (1)12 = \sum_{j=1}^{20} d(v_j) = 166$. For number of colour classes to colour edges of graph G equal 13, we have 83 edges, each one of labels 1, 2, 3, 4, 5, 6, 7, 8 labeled 6 edges, and each one of labels 9, 10, 11, 12, 13 labeled 7 edges. The 20 vertices labeled by some of labels 1, 2, 3, ..., 9, 10, 11, 12, 13 such that each vertex has labels equal its degree. At these 20 vertices there are 8 colour classes appear 12 times, and there are 5 colour classes appear 14 times. We can write $\sum_{i=1}^{13} t_i = 8(12) + 5(14) = \sum_{j=1}^{20} d(v_j) = 166$.

Remark 3.22. Example 3.17. satisfies Proposition 4.10, we have $11 \leq m \leq \frac{166}{12} + 1$.

Example 3.23. From Example 3.21. we find four graphs with colour classes equal 14, 13, 12, 11. In first step we draw the graph in 14 colour, we have 83 edges. We draw 20 vertices, each of degree zero, then we draw 6 edges parallel (any two nonadjacent) each edge labeled by 1, and draw 6 edges parallel (any two nonadjacent) each edge labeled by 2, we continue till draw 6 edges parallel (any two nonadjacent) each edge labeled by 13, finally we draw 5 edges parallel (any two nonadjacent) each edge labeled by 14, such that maximum degree of vertex equal 11. In second step we draw the graph in 13 colours. We draw 20 vertices, each of degree zero, then we draw 6 edges parallel (any two nonadjacent) each edge labeled by 1, and draw 6 edges parallel (any two nonadjacent) each edge labeled by 2, we continue till draw 6 edges parallel (any two nonadjacent) each edge labeled by 8, then we draw 7 edges parallel (any two nonadjacent) each edge labeled by 9, and draw 7 edges parallel (any two nonadjacent) each edge labeled by 10, we continue till draw 7 edges parallel (any two nonadjacent) each edge labeled by 13, such that maximum degree of vertex equal 11. In step three we draw the graph in 12 colours. In step four we draw the graph in 11 colours.

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Proposition 3.24. Let G be a graph of w vertices, with maximum degree of vertex equal n , and m is the number of colour classes to colour edges of graph G . From the equation $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j)$ if we have T is maximum value of t_i for all i , and $nT < \sum_{j=1}^w d(v_j)$, then $m \leq n + 1$.

Proof: From definition of T we have $\sum_{i=1}^m t_i \leq mT$ and from equation $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j)$ we have $\sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j) \leq mT$, and from $nT < \sum_{j=1}^w d(v_j)$, we have $nT < \sum_{i=1}^m t_i = \sum_{j=1}^w d(v_j) \leq mT$, then we have $n < m$, then $m \leq n + 1$.

Example 3.25. Let G be a graph of 5 vertices, the sum of degrees of vertices equal 14, with maximum degree of vertex equal 3, if 3 is the minimum number of colour classes to colour edges of the graph, at these 5 vertices the colour class c_1 appear 6 times, and each of the colour classes c_2, c_3 appear 4 times, it is impossible the colour class c_1 6 times at these 5 vertices, therefore minimum number of colour classes to colour edges of this graph equal 4.

Example 3.26. Let G be a graph with 999 vertices, with maximum degree of vertex equal 700. Find maximum number of edges such that number of colour classes to colour edges of the graph G equal 700, and find minimum number of edges such that number of colour classes to colour edges of the graph G equal 701,

Solution: Let the sum of degrees of all vertices equal D , the number of edges equal D divided by two, therefore maximum number of edges depend on maximum number of D , maximum value of D such that number of colour classes is 700, equal 998 multiple by 700, maximum number of edges such that number of colour classes is 700, equal 349300. minimum number of edges such that number of colour classes is 701, equal 349301.

Remark 3.27. In this paper each graph G of w vertices, with maximum degree of vertex equal n , and m is the number of colour classes to colour edges of the graph, $3 \leq w \leq 2n - 1$, and $n \leq m \leq 2n - 1$. If the graph G of number of vertices greater than graph G of $2n - 1$ vertices, we try find subgraphs G_1, G_2, G_3, \dots of G such that any subgraph has number of vertices less than $2n - 1$, and then use Proposition 2.15, Proposition 2.16, Proposition 2.17, or Proposition 2.18, which is suitable.

Example 3.28. Let G_1 be a graph of 7 vertices, with maximum degree of vertex equal 4 and number of edges equal 15, G_2 be a graph of 7 vertices, with maximum degree of vertex equal 4 and number of edges equal 15, G_3 be a graph of 7 vertices, with maximum degree of vertex equal 4 and number of edges equal 13, 4 is minimum number of colour to colour edges of every one of graphs G_1, G_2, G_3 , we have three bridges, any one connect two of these three graphs, such that largest degree of vertex remain 4, using Proposition 2.18. minimum number of colour to colour edges of these 45 edges equal 5.

REFERENCES

- [1] Hassan, M.E., "Family of Disjoint Set and its Applications". IJIRSET Vol. 7. Issue 1, Jan 2018, 362-393.
- [2] Hassan, M.E., "Colouring of Graphs Using Colouring of Families of Disjoint Sets Technique". IJIRSET Vol. 7. Issue 10, October 2018, 10219-10229. October
- [3] Hassan, M.E., "Types of Colouring and Types of Families of Disjoint Sets". IJIRSET Vol. 7. Issue 11, Nov 2018, 362-393.
- [4] Hassan, M.E., "Colouring of finite Sets and Colouring of Edges finite Graphs". IJIRSET Vol. 7. Issue 12, Dec 2018, 11663-11675.
- [5] Hassan, M.E., "Sorts of Colour Classes and Sorts of Families of Disjoint Sets". IJIRSET Vol. 8. Issue 1, Jan 2019, 56-63.
- [6] Hassan, M.E., "Trivial Colouring and Non-Trivial Colouring for Graph's Edges". IJIRSET Vol. 8. Issue 2, Feb 2019, 1014-1024.
- [7] Bondy, J. A., Murty, S. R., "Graph Theory with Applications". Macmillan Press 1967 & Elsevier, 1967.
- [8] Wilson, R., "Introduction to Graph Theory". Fourth Edition, Addison Wesley Longman Limited, England, (1996).
- [9] Wilson, R., "Four Colors Suffice : How the Map Problem Was Solved". Princeton, NJ: Princeton University Press, 2004.
- [10] Reinhard Diestel, "Graph Theory". Electronic 2000, Springer-Verlag New York 1997, 2000.
- [11] Frank Harary, "Graph Theory". Addison-Wesley Publication Company, Inc. 1969.
- [12] Douglas B West, "Introduction to Graph Theory". Second Edition, Department of Mathematics Illinois University, (2001).
- [13] Thomas, R. "An Update on the Four-color Theorem" Notices Amer. Math. Soc. (45) no. 7, pp.848-859, (1998).
- [14] Robertson, N., Sanders, D., Seymour, P., and Thomas, R., "A New Proof of the Four-Colour Theorem", Electronic Research Announcements of the American Mathematical Society 2(1) pp.84-96, (1996).
- [15] Gonthier, G., "Formal Proof-The Four Color Theorem", Notices of the AMS, Vol. (55) No.11, pp.1382-1393, (2008).