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Differential Equations-A Mini Review

Meetha Lal Meena

Assistant Professor, Mathematics, SCRS Government College, Sawai Madhopur, Rajasthan, India

ABSTRACT: In Mathematics, a **differential equation** is an equation that contains one or more functions with its derivatives. The derivatives of the function define the rate of change of a function at a point. It is mainly used in fields such as physics, engineering, biology and so on. The primary purpose of the differential equation is the study of solutions that satisfy the equations and the properties of the solutions. A **differential equation** is an equation which contains one or more terms and the derivatives of one variable (i.e., dependent variable) with respect to the other variable (i.e., independent variable)

$$dy/dx = f(x)$$

Here “x” is an independent variable and “y” is a dependent variable

For example, $dy/dx = 5x$

A differential equation contains derivatives which are either partial derivatives or ordinary derivatives. The derivative represents a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying with respect to the change in another quantity. There are a lot of differential equations formulas to find the solution of the derivatives.

KEYWORDS: differential equations, mathematics, variable, formulas, derivatives, quantity, independent

I.INTRODUCTION

Order of Differential Equation

The order of the differential equation is the order of the highest order derivative present in the equation. Here some examples for different orders of the differential equation are given.¹

- $dy/dx = 3x + 2$, The order of the equation is 1
- $(d^2y/dx^2) + 2(dy/dx) + y = 0$. The order is 2
- $(dy/dt) + y = kt$. The order is 1

First Order Differential Equation

You can see in the first example, it is a first-order differential equation which has degree equal to 1. All the linear equations in the form of derivatives are in the first order. It has only the first derivative such as dy/dx , where x and y are the two variables and is represented as:

$$dy/dx = f(x, y) = y'$$

Second-Order Differential Equation

The equation which includes the second-order derivative is the second-order differential equation. It is represented as;

$$d/dx(dy/dx) = d^2y/dx^2 = f''(x) = y''$$

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Degree of Differential Equation

The degree of the differential equation is the power of the highest order derivative, where the original equation is represented in the form of a polynomial equation in derivatives such as y' , y'' , y''' , and so on.²

Suppose $(d^2y/dx^2) + 2(dy/dx) + y = 0$ is a differential equation, so the degree of this equation here is 1. See some more examples here:

- $dy/dx + 1 = 0$, degree is 1
- $(y''')^3 + 3y'' + 6y' - 12 = 0$, degree is 3
- $(dy/dx) + \cos(dy/dx) = 0$; it is not a polynomial equation in y' and the degree of such a differential equation can not be defined.

Types of Differential Equations

Differential equations can be divided into several types namely³

- Ordinary Differential Equations
- Partial Differential Equations
- Linear Differential Equations
- Nonlinear differential equations
- Homogeneous Differential Equations
- Nonhomogeneous Differential Equations

Ordinary Differential Equation

An ordinary differential equation involves function and its derivatives. It contains only one independent variable and one or more of its derivatives with respect to the variable.

The order of ordinary differential equations is defined as the order of the highest derivative that occurs in the equation. The general form of n-th order ODE is given as

$$F(x, y, y', \dots, y^n) = 0$$

Differential Equations Solutions

A function that satisfies the given differential equation is called its solution. The solution that contains as many arbitrary constants as the order of the differential equation is called a general solution. The solution free from arbitrary constants is called a particular solution. There exist two methods to find the solution of the differential equation.⁴

1. Separation of variables
2. Integrating factor

Separation of the variable is done when the differential equation can be written in the form of $dy/dx = f(y)g(x)$ where f is the function of y only and g is the function of x only. Taking an initial condition, rewrite this problem as $1/f(y)dy = g(x)dx$ and then integrate on both sides.

Also, check: [Solve Separable Differential Equations](#)

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Integrating factor technique is used when the differential equation is of the form $dy/dx + p(x)y = q(x)$ where p and q are both the functions of x only.

First-order differential equation is of the form $y' + P(x)y = Q(x)$. where P and Q are both functions of x and the first derivative of y . The higher-order differential equation is an equation that contains derivatives of an unknown function which can be either a partial or ordinary derivative. It can be represented in any order.⁵

Applications

Differential equations have several applications in different fields such as applied mathematics, science, and engineering. Apart from the technical applications, they are also used in solving many real life problems. Let us see some differential equation applications in real-time.⁶

- 1) Differential equations describe various exponential growths and decays.
- 2) They are also used to describe the change in return on investment over time.
- 3) They are used in the field of medical science for modelling cancer growth or the spread of disease in the body.
- 4) Movement of electricity can also be described with the help of it.
- 5) They help economists in finding optimum investment strategies.
- 6) The motion of waves or a pendulum can also be described using these equations.

The various other applications in engineering are: heat conduction analysis, in physics it can be used to understand the motion of waves. The ordinary differential equation can be utilized as an application in the engineering field for finding the relationship between various parts of the bridge.

Now, go through the **differential equations examples** in real-life applications .

1) Linear Differential Equations Real World Example

To understand Differential equations, let us consider this simple example. Have you ever thought about why a hot cup of coffee cools down when kept under normal conditions? According to Newton, cooling of a hot body is proportional to the temperature difference between its temperature T and the temperature T_0 of its surrounding.⁷ This statement in terms of mathematics can be written as:

$$dT/dt \propto (T - T_0) \dots \dots \dots (1)$$

This is the form of a linear differential equation.

Introducing a proportionality constant k , the above equation can be written as:

$$dT/dt = k(T - T_0) \dots \dots \dots (2)$$

Here, T is the temperature of the body and t is the time,

T_0 is the temperature of the surrounding,

dT/dt is the rate of cooling of the body⁸

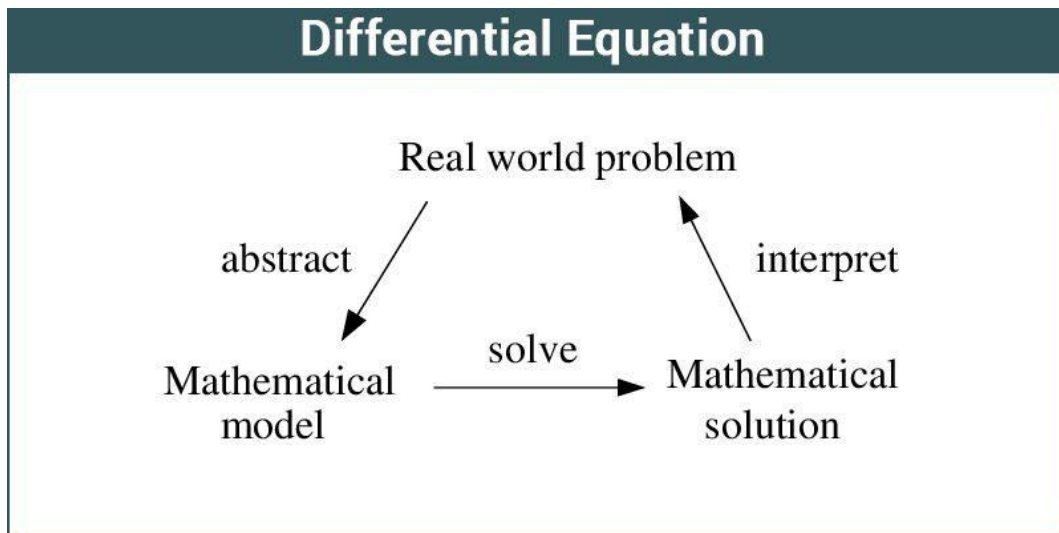
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II.DISCUSSION



Eg: $dy/dx = 3x$

Here, the differential equation contains a derivative that involves a variable (dependent variable, y) w.r.t another variable (independent variable, x). The types of differential equations are :

1. An ordinary differential equation contains one independent variable and its derivatives. It is frequently called ODE. The general definition of the ordinary differential equation is of the form: Given an F , a function of x and y and derivative of y , we have

$F(x, y, y', \dots, y^{(n)}) = y^{(n)}$ is an explicit ordinary differential equation of order n .

2. Partial differential equation that contains one or more independent variables.⁹

The main purpose of the differential equation is to compute the function over its entire domain. It is used to describe the exponential growth or decay over time. It has the ability to predict the world around us. It is widely used in various fields such as Physics, Chemistry, Biology, Economics and so on¹⁰

Mainly the study of differential equations consists of the study of their solutions (the set of functions that satisfy each equation), and of the properties of their solutions. Only the simplest differential equations are solvable by explicit formulas; however, many properties of solutions of a given differential equation may be determined without computing them exactly.

Often when a closed-form expression for the solutions is not available, solutions may be approximated numerically using computers. The theory of dynamical systems puts emphasis on qualitative analysis of systems described by differential equations, while many numerical methods have been developed to determine solutions with a given degree of accuracy.

In classical mechanics, the motion of a body is described by its position and velocity as the time value varies. Newton's laws allow these variables to be expressed dynamically (given the position, velocity, acceleration and various forces acting on the body) as a differential equation for the unknown position of the body as a function of time.¹¹

In some cases, this differential equation (called an equation of motion) may be solved explicitly.

An example of modeling a real-world problem using differential equations is the determination of the velocity of a ball falling through the air, considering only gravity and air resistance. The ball's acceleration towards the ground is the acceleration due to gravity minus the deceleration due to air resistance. Gravity is considered constant, and air

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resistance may be modeled as proportional to the ball's velocity. This means that the ball's acceleration, which is a derivative of its velocity, depends on the velocity (and the velocity depends on time). Finding the velocity as a function of time involves solving a differential equation and verifying its validity.

An ordinary differential equation (ODE) is an equation containing an unknown function of one real or complex variable x , its derivatives, and some given functions of x . The unknown function is generally represented by a variable (often denoted y), which, therefore, depends on x . Thus x is often called the independent variable of the equation. The term "ordinary" is used in contrast with the term partial differential equation, which may be with respect to more than one independent variable.¹²

Linear differential equations are the differential equations that are linear in the unknown function and its derivatives. Their theory is well developed, and in many cases one may express their solutions in terms of integrals.

Most ODEs that are encountered in physics are linear. Therefore, most special functions may be defined as solutions of linear differential equations (see Holonomic function).

As, in general, the solutions of a differential equation cannot be expressed by a closed-form expression, numerical methods are commonly used for solving differential equations on a computer.¹³

III.RESULTS

A partial differential equation (PDE) is a differential equation that contains unknown multivariable functions and their partial derivatives. (This is in contrast to ordinary differential equations, which deal with functions of a single variable and their derivatives.) PDEs are used to formulate problems involving functions of several variables, and are either solved in closed form, or used to create a relevant computer model.

PDEs can be used to describe a wide variety of phenomena in nature such as sound, heat, electrostatics, electrodynamics, fluid flow, elasticity, or quantum mechanics. These seemingly distinct physical phenomena can be formalized similarly in terms of PDEs. Just as ordinary differential equations often model one-dimensional dynamical systems, partial differential equations often model multidimensional systems. Stochastic partial differential equations generalize partial differential equations for modeling randomness.¹⁴

A non-linear differential equation is a differential equation that is not a linear equation in the unknown function and its derivatives (the linearity or non-linearity in the arguments of the function are not considered here). There are very few methods of solving nonlinear differential equations exactly; those that are known typically depend on the equation having particular symmetries. Nonlinear differential equations can exhibit very complicated behaviour over extended time intervals, characteristic of chaos. Even the fundamental questions of existence, uniqueness, and extendability of solutions for nonlinear differential equations, and well-posedness of initial and boundary value problems for nonlinear PDEs are hard problems and their resolution in special cases is considered to be a significant advance in the mathematical theory (cf. Navier–Stokes existence and smoothness). However, if the differential equation is a correctly formulated representation of a meaningful physical process, then one expects it to have a solution. Differential equations are described by their order, determined by the term with the highest derivatives. An equation containing only first derivatives is a first-order differential equation, an equation containing the second derivative is a second-order differential equation, and so on.¹⁵

- A delay differential equation (DDE) is an equation for a function of a single variable, usually called **time**, in which the derivative of the function at a certain time is given in terms of the values of the function at earlier times.
- An integro-differential equation (IDE) is an equation that combines aspects of a differential equation and an integral equation.
- A stochastic differential equation (SDE) is an equation in which the unknown quantity is a stochastic process and the equation involves some known stochastic processes, for example, the Wiener process in the case of diffusion equations.

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- A stochastic partial differential equation (SPDE) is an equation that generalizes SDEs to include space-time noise processes, with applications in quantum field theory and statistical mechanics.
- An ultrametric pseudo-differential equation is an equation which contains p-adic numbers in an ultrametric space. Mathematical models that involve ultrametric pseudo-differential equations use pseudo-differential operators instead of differential operators.
- A differential algebraic equation (DAE) is a differential equation comprising differential and algebraic terms, given in implicit form.¹⁶

IV. CONCLUSIONS

The study of differential equations is a wide field in pure and applied mathematics, physics, and engineering. All of these disciplines are concerned with the properties of differential equations of various types. Pure mathematics focuses on the existence and uniqueness of solutions, while applied mathematics emphasizes the rigorous justification of the methods for approximating solutions. Differential equations play an important role in modeling virtually every physical, technical, or biological process, from celestial motion, to bridge design, to interactions between neurons. Differential equations such as those used to solve real-life problems may not necessarily be directly solvable, i.e. do not have closed form solutions. Instead, solutions can be approximated using numerical methods.¹⁷

Many fundamental laws of physics and chemistry can be formulated as differential equations. In biology and economics, differential equations are used to model the behavior of complex systems.¹⁸ The mathematical theory of differential equations first developed together with the sciences where the equations had originated and where the results found application. However, diverse problems, sometimes originating in quite distinct scientific fields, may give rise to identical differential equations. Whenever this happens, mathematical theory behind the equations can be viewed as a unifying principle behind diverse phenomena. As an example, consider the propagation of light and sound in the atmosphere, and of waves on the surface of a pond.¹⁹ All of them may be described by the same second-order partial differential equation, the wave equation, which allows us to think of light and sound as forms of waves, much like familiar waves in the water. Conduction of heat, the theory of which was developed by Joseph Fourier, is governed by another second-order partial differential equation, the heat equation. It turns out that many diffusion processes, while seemingly different, are described by the same equation; the Black-Scholes equation in finance is, for instance, related to the heat equation.²⁰

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