

# International Journal of Innovative Research in Science, Engineering and Technology

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## Generalization of the H-Function

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**ABSTRACT:** The topics of special H-function and fractional calculus are currently undergoing rapid changes both in theory and application. Taking into account the latest research results, the authors delve into these topics as they relate to applications to problems in statistics, physics, and engineering, particularly in condensed matter physics, plasma physics, and astrophysics. There are the definitions, contours, existence conditions, and particular cases for the H-function, then explores the properties and relationships among the Laplace, Fourier, Hankel, and other transforms. From here, the H-functions are utilized for applications in statistical distribution theory, structures of random variables, generalized distributions, Mathai's pathway models, and versatile integrals. Functions of matrix argument are introduced with a focus on real-valued scalar functions when the matrices are real or Hermitian positive-definite. The text concludes with important recent applications to physical problems in reaction, diffusion, reaction-diffusion theory and statistics, and superstatistics. Generalized entropies as well as applications in astrophysics are dealt with. Over the last few years, material in this book has been added to various courses and developed to meet the needs of scholars at the PhD level

**KEYWORDS-**H-Function,Calculus,Laplace,Fourier,Hankel,Mathai's-Pathway,Reaction-Diffusion-Theory,Statistics

### I. INTRODUCTION

In order to explain the memory effects of complicated systems, the fact that fractional derivatives add a convolution integral with a power-law memory kernel or exponential-law memory kernel strengthens the value of fractional differential equations. It can also be shown that in the last few decades, very fascinating and revolutionary applications[1,2,3] of fractional calculus operators have been developed in physics, chemistry, biology, engineering, finance and other fields of research. Some of the applications include: diffusion processes, mechanics of materials, combinatorics, inequalities, signal processing, image processing, advection and dispersion of solutes in porous or fractured media, modeling of viscoelastic materials under external forces, bioengineering, relaxation and reaction kinetics of polymers, random walks, mathematical finance, modeling of combustion, control theory, heat propagation, modeling of viscoelastic materials, in biological systems and many more. The recent work [1–4], and references therein, can be referred to for further information.[5,7,8] Because of the importance in astrophysics, control systems, and mathematics, the research on the fractional kinetic equations and their solution has attracted interest from many researchers [5–12]. The fractional kinetic equation has indeed been commonly used to test various physical phenomena regulating the diffusion in porous media, reactions and relaxation mechanisms in complex structures. As a consequence, a significant number of research papers (see [13–20]) focused on the solution of these equations including generalized Mittag-Leffler function, Bessel's function, Struve function, G-function, H-function, and Aleph-function have recently been written. In this new fractional generalization of the kinetic equation, use of the incomplete special functions gives a different dimension to this study. The equation involves a family of polynomials, incomplete H-function, incomplete Fox-Wright function and incomplete generalized hypergeometric function.[9,10,11]

There is a new fractional generalization of the standard kinetic equation and to use the integral transformation approach to analyze its solution. A study of several interesting fractional kinetic equations and their solutions has been made, which include a family of polynomials and the incomplete Hfunction, incomplete Fox-Wright function and incomplete generalized hypergeometric function. The main results and their corollaries are of general nature. Analogously, various fractional kinetic equations and their solutions available in literature (see, [15–20]) can be obtained as special cases of the main results.[12,13,15] Through addition, a number of recognized polynomials are produced by the polynomials family as their specific cases on a properly specialized connected sequence  $An,s$ . As a

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consequence, by providing appropriate basic values to the arbitrary sequences and the constraints, the main results can be used to generate a set of kinetic equations and their potential solutions. We intend to continue this study of the more generalized kinetic equations and their proposed solutions in the future work.[17,18,19]

### Example 1

The functions  $f$ ,  $g$ , and  $h$  are defined for  $x \in \mathbb{R}$ , by  
 $f(x) = 2x+1$ ,  $g(x) = x^2$ ,  $h(x) = 1/x$ .  $x \neq 0$

<p>(i) <math>fg(2)</math>: <math>g(2) = (2)^2 = 4</math> <math>f(g(2)) = f(4) = 2(4)+1 = 9</math> <math>f(g(2)) = 9</math></p>	<p>(ii) <math>gf(3)</math> <math>f(3) = 2(3)+1 = 7</math> <math>g(f(3)) = g(7) = 7^2 = 49</math> <math>g(f(3)) = 49</math></p>
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<p>(iii) <math>gf(x)</math> <math>f(x) = 2x+1</math> <math>g(f(x)) = (2x+1)^2</math> <math>g(f(x)) = 4x^2 + 4x + 1</math></p>	<p>(iv) <math>fg(x)</math></p>
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## II. DISCUSSION

The hyperbolic functions have similar names to the trigonometric functions, but they are defined in terms of the exponential function. In this unit we define the three main hyperbolic functions, and sketch their graphs. We also discuss some identities relating these functions, and mention their inverse functions and reciprocal functions. In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature. After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- define the functions  $f(x) = \cosh x$  and  $f(x) = \sinh x$  in terms of the exponential function, and define the function  $f(x) = \tanh x$  in terms of  $\cosh x$  and  $\sinh x$ ,
- sketch the graphs of  $\cosh x$ ,  $\sinh x$  and  $\tanh x$ ,
- recognize the identities  $\cosh^2 x - \sinh^2 x = 1$  and  $\sinh 2x = 2 \sinh x \cosh x$ ,
- understand the meaning of the inverse functions  $\sinh^{-1} x$ ,  $\cosh^{-1} x$  and  $\tanh^{-1} x$  and specify their domains,
- define the reciprocal functions  $\operatorname{sech} x$ ,  $\operatorname{csch} x$  and  $\operatorname{coth} x$ .

If you know the trigonometric identities, you may notice that these hyperbolic identities are very similar, although sometimes plus signs have become minus signs and vice versa. In fact the hyperbolic functions are very closely related to the trigonometric functions, and  $\sinh x$  and  $\cosh x$  are sometimes called the hyperbolic sine and hyperbolic cosine

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functions. If you go on to study complex numbers then you might learn more about how these functions are related.[20,21,22]

Hyperbolic functions are defined in mathematics in a way similar to trigonometric functions. As the name suggests, the graph of a hyperbolic function represents a rectangular hyperbola, and its formula can often be seen in the formulas of a hyperbola. They are defined using a hyperbola instead of a unit circle as in the case of trigonometry. Hyperbolic functions are analogous to trigonometric functions but are derived from a hyperbola as trigonometric functions are derived from a unit circle.

Hyperbolic functions are expressed in terms of the exponential function  $e^x$ . There are six hyperbolic functions are  $\sinh x$ ,  $\cosh x$ ,  $\tanh x$ ,  $\coth x$ ,  $\operatorname{sech} x$ ,  $\operatorname{csch} x$ . In this article, we will define these hyperbolic functions and their properties, graphs, identities, derivatives, etc. along with some solved examples.[23]

As an ordinary trigonometric function is defined for or on a circle, similarly a hyperbolic function is defined for a hyperbola. In ordinary trigonometry, we were using sine, cosine, and other functions. Similarly, for hyperbolic functions, we use  $\sinh$ ,  $\cosh$ ,  $\tanh$ ,  $\coth$ ,  $\operatorname{sech}$ , and  $\operatorname{csch}$ . As in ordinary trigonometric function, we know that the coordinates of points on the unit circle are  $(\cos \theta, \sin \theta)$ , similarly in hyperbolic functions,  $(\cosh \theta, \sinh \theta)$  forms right half of equilateral hyperbola.

These functions can be solutions of various linear differential equations, cubic equations, and Laplace's equation. The six basic hyperbolic functions are:

- Hyperbolic sine or  $\sinh x$
- Hyperbolic cosine or  $\cosh x$
- Hyperbolic tangent or  $\tanh x$
- Hyperbolic cosecant or  $\operatorname{cosech} x$
- Hyperbolic secant or  $\operatorname{sech} x$
- Hyperbolic cotangent or  $\coth x$

Hyperbolic Meaning

Hyperbolic functions are defined analogously to trigonometric functions. We have main six hyperbolic functions, namely  $\sinh x$ ,  $\cosh x$ ,  $\tanh x$ ,  $\coth x$ ,  $\operatorname{sech} x$ , and  $\operatorname{cosech} x$ . They can be expressed as a combination of the exponential function. These functions are derived using the hyperbola just like trigonometric functions are derived using the unit circle.

Hyperbolic Functions Formulas[

The hyperbolic functions are defined through the algebraic expressions that include the exponential function ( $e^x$ ) and its inverse exponential functions ( $e^{-x}$ ), where  $e$  is the Euler's constant. Let us see all their formulas one by one:

- **Sinh x:** This is the odd part of the exponential functions. An algebraic expression for hyperbolic sine function is:  
$$\sinh x = (e^x - e^{-x})/2$$
- **Cosh x:** This is the even part of the exponential function. Algebraic expression for hyperbolic cosine function is:  
$$\cosh x = (e^x + e^{-x})/2$$
- **Tanh x:**  $\tanh x = \sinh x / \cosh x = (e^x - e^{-x}) / (e^x + e^{-x})$

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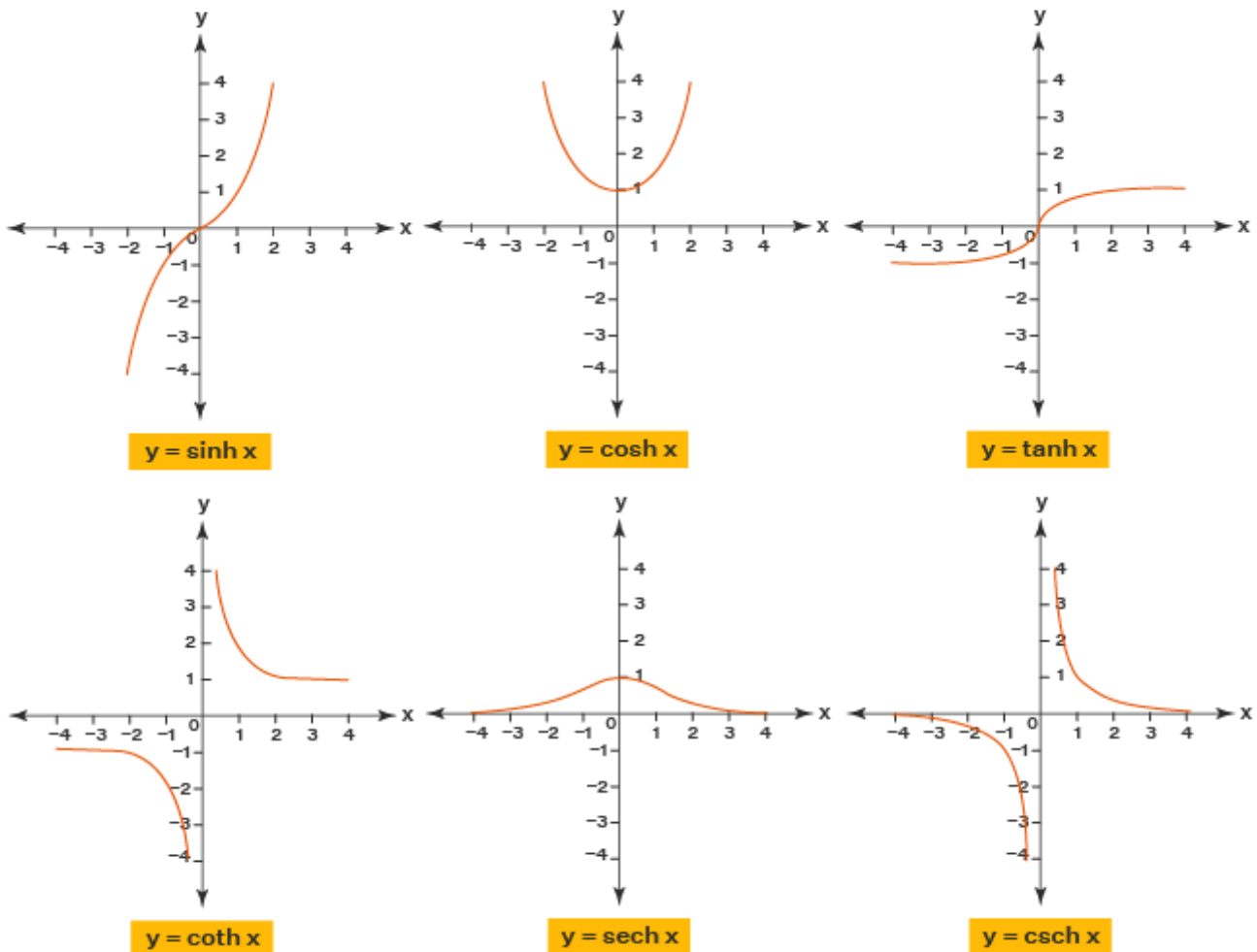
- Coth x:  $\text{coth } x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
- Sech x:  $\text{sech } x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$
- Csch x:  $\text{csch } x = \frac{2}{e^x - e^{-x}}$  [21,22]

### III. RESULTS

#### Hyperbolic Functions Graphs

The graph of a hyperbolic function synonymous with its name represents a rectangular hyperbola and its formula can often be seen in the formulas of a hyperbola. Chart of graphs of different hyperbolic functions is given below:

#### Hyperbolic Functions



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### Domain and Range of Hyperbolic Functions

Looking at the graph of a hyperbolic function, we can determine its domain and range. The domains and ranges of these functions are summarized in the following table:

Hyperbolic Function	Domain	Range
$\sinh x$	$(-\infty, +\infty)$	$(-\infty, +\infty)$
$\cosh x$	$(-\infty, +\infty)$	$[1, \infty)$
$\tanh x$	$(-\infty, +\infty)$	$(-1, 1)$
$\coth x$	$(-\infty, 0) \cup (0, +\infty)$	$(-\infty, -1) \cup (1, +\infty)$
$\operatorname{sech} x$	$(-\infty, +\infty)$	$(0, 1]$
$\operatorname{csch} x$	$(-\infty, 0) \cup (0, +\infty)$	$(-\infty, 0) \cup (0, +\infty)$

### Properties of Hyperbolic Functions

The properties of hyperbolic functions are analogous to the properties of trigonometric functions. Let us go through some of the important properties of these functions which are used to solve various problems in mathematics.

- $\sinh(-x) = -\sinh(x)$
- $\cosh(-x) = \cosh(x)$
- $\tanh(-x) = -\tanh x$
- $\coth(-x) = -\coth x$
- $\operatorname{sech}(-x) = \operatorname{sech} x$
- $\operatorname{csc}(-x) = -\operatorname{csch} x$
- $\cosh 2x = 1 + 2 \sinh^2(x) = 2 \cosh^2 x - 1$
- $\cosh 2x = \cosh^2 x + \sinh^2 x$
- $\sinh 2x = 2 \sinh x \cosh x$

Hyperbolic functions can also be deduced from trigonometric functions with complex arguments:

- $\sinh x = -i \sin(ix)$
- $\cosh x = \cos(ix)$
- $\tanh x = -i \tan(ix)$
- $\coth x = i \cot(ix)$
- $\operatorname{sech} x = \sec(ix)$

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### Hyperbolic Trig Identities

The hyperbolic trig identities are similar to trigonometric identities and can be understood better from below. Osborn's rule states that trigonometric identities can be converted into hyperbolic trig identities when expanded completely in terms of integral powers of sines and cosines, which includes changing sine to sinh, cosine to cosh. The sign of every term that contains a product of two sinh should be replaced.[19,20]

- $\sinh x - \sinh y = 2 \cosh [(x+y)/2] \sinh [(x-y)/2]$
- $\sinh x + \sinh y = 2 \sinh [(x+y)/2] \cosh [(x-y)/2]$
- $\cosh x + \cosh y = 2 \cosh [(x+y)/2] \cosh [(x-y)/2]$
- $\cosh x - \cosh y = 2 \sinh [(x+y)/2] \sinh [(x-y)/2]$
- $2 \sinh x \cosh y = \sinh (x + y) + \sinh (x - y)$
- $2 \cosh x \sinh y = \sinh (x + y) - \sinh (x - y)$
- $2 \sinh x \sinh y = \cosh (x + y) - \cosh (x - y)$
- $2 \cosh x \cosh y = \cosh (x + y) + \cosh (x - y)$
- $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
- $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
- $\tanh(x \pm y) = (\tanh x \pm \tanh y) / (1 \pm \tanh x \tanh y)$
- $\coth(x \pm y) = (\coth x \coth y \pm 1) / (\coth y \pm \coth x)$
- $\cosh^2 x - \sinh^2 x = 1$
- $\tanh^2 x + \operatorname{sech}^2 x = 1$
- $\coth^2 x - \operatorname{csch}^2 x = 1$

### Hyperbolic Function Integrals and Derivatives

The derivative and integral of a hyperbolic function are similar to the derivative and integral of a trigonometric function. Unlike the derivative of trigonometric functions, we can observe the change in sign in the derivative of the hyperbolic secant function. The derivatives and integrals of the hyperbolic functions are summarized in the following table:

Hyperbolic Function	Derivative	Integral
$\sinh x$	$\cosh x$	$\cosh x + C$
$\cosh x$	$\sinh x$	$\sinh x + C$
$\tanh x$	$\operatorname{sech}^2 x$	$\ln(\cosh x) + C$
$\coth x$	$-\operatorname{csch}^2 x$	$\ln(\sinh x) + C$
$\operatorname{sech} x$	$-\operatorname{sech} x \cdot \tanh x$	$\arctan(\sinh x) + C$

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Hyperbolic Function	Derivative	Integral
csch x	-csch x. coth x	$\ln(\tanh (x/2)) + C$

### Inverse Hyperbolic Functions

The inverse of a hyperbolic function is called an inverse hyperbolic function. For example, if  $x = \sinh y$ , then  $y = \sinh^{-1} x$  is the inverse of the hyperbolic sine function.[17,18] The inverse hyperbolic functions expressed in terms of logarithmic functions are shown below:

- $\sinh^{-1}x = \ln (x + \sqrt{x^2 + 1})$
- $\cosh^{-1}x = \ln (x + \sqrt{x^2 - 1})$
- $\tanh^{-1}x = \ln [(1 + x)/(1 - x)]$
- $\coth^{-1}x = \ln [(x + 1)/(x - 1)]$
- $\operatorname{sech}^{-1}x = \ln \{ [1 + \sqrt{1 - x^2}]/x \}$
- $\operatorname{csch}^{-1}x = \ln \{ [1 + \sqrt{1 + x^2}]/x \}$

### Important Notes on Hyperbolic Functions

- There are six hyperbolic functions, namely  $\sinh x$ ,  $\cosh x$ ,  $\tanh x$ ,  $\coth x$ ,  $\operatorname{sech} x$ ,  $\operatorname{csch} x$ .
- A hyperbolic function is defined for a hyperbola.
- The hyperbolic identities are analogous to trigonometric identities.

## IV. CONCLUSIONS

The two basic hyperbolic functions are "sinh" and "cosh":

Hyperbolic Sine:

$$\sinh(x) = e^x - e^{-x} / 2$$

(pronounced "shine")

Hyperbolic Cosine:

$$\cosh(x) = e^x + e^{-x} / 2$$

(pronounced "cosh")

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They use the natural exponential function  $e^x$

And are not the same as  $\sin(x)$  and  $\cos(x)$ , but a little bit similar:

$\sinh$  vs  $\sin$

$\cosh$  vs  $\cos$

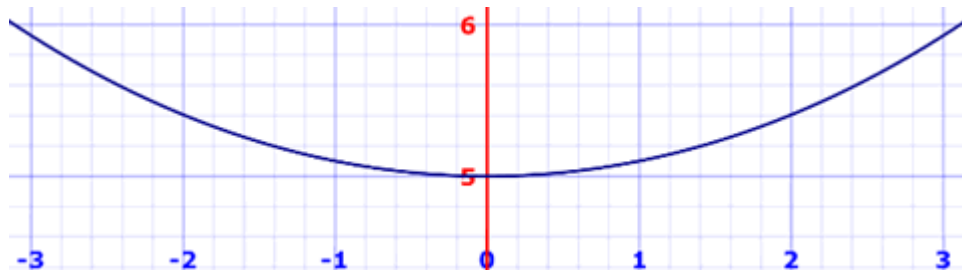
Catenary

One of the interesting uses of Hyperbolic Functions is the curve made by suspended cables or chains.

A hanging cable forms a curve called a catenary defined using the  $\cosh$  function:

$$f(x) = a \cosh(x/a) \dots [15]$$

Like in this example from the page arc length :



Other Hyperbolic Functions

From  $\sinh$  and  $\cosh$  we can create:

Hyperbolic tangent "tanh" (pronounced "than"):

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



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tanh vs tan

Hyperbolic cotangent:

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Hyperbolic secant:

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$$

Hyperbolic cosecant "csch" or "cosech":

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$$

Because it comes from measurements made on a Hyperbola:

So, just like the trigonometric functions relate to a circle, the hyperbolic functions relate to a hyperbola.

Identities

- $\sinh(-x) = -\sinh(x)$
- $\cosh(-x) = \cosh(x)$

And

- $\tanh(-x) = -\tanh(x)$
- $\coth(-x) = -\coth(x)$
- $\operatorname{sech}(-x) = \operatorname{sech}(x)$
- $\operatorname{csch}(-x) = -\operatorname{csch}(x)$

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Odd and Even

Both cosh and sech are Even Functions, the rest are Odd Functions.[9,10,11]

Derivatives

Derivatives are:

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

$$\frac{d}{dx} \tanh(x) = 1 - \tanh^2(x) \dots [12,13,15]$$

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