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# **Two Body Calculation in Harmonic Oscillator**

### Dr. Shad Husain

Associate Professor, Dept. of Physics, Shia PG College, Lucknow, Uttar Pradesh, India

**ABSTRACT:** In classical mechanics, a **harmonic oscillator** is a system that, when displaced from its equilibrium position, experiences a restoring force F proportional to the displacement x:

F= -kx

where *k* is a positive constant.

If F is the only force acting on the system, the system is called a **simple harmonic oscillator**, and it undergoes simple harmonic motion: sinusoidal oscillations about the equilibrium point, with a constant amplitude and a constant frequency (which does not depend on the amplitude).

If a frictional force (damping) proportional to the velocity is also present, the harmonic oscillator is described as a **damped oscillator**. Depending on the friction coefficient, the system can:

- Oscillate with a frequency lower than in the undamped case, and an amplitude decreasing with time (underdamped oscillator).
- Decay to the equilibrium position, without oscillations (overdamped oscillator).

The boundary solution between an underdamped oscillator and an overdamped oscillator occurs at a particular value of the friction coefficient and is called critically damped.

If an external time-dependent force is present, the harmonic oscillator is described as a *driven oscillator*.

Mechanical examples include pendulums (with small angles of displacement), masses connected to springs, and acoustical systems. Other analogous systems include electrical harmonic oscillators such as RLC circuits. The harmonic oscillator model is very important in physics, because any mass subject to a force in stable equilibrium acts as a harmonic oscillator for small vibrations. Harmonic oscillators occur widely in nature and are exploited in many manmade devices, such as clocks and radio circuits. They are the source of virtually all sinusoidal vibrations and waves.

**KEYWORDS:** oscillator, harmonic, calculation, two body, amplitude, waves, clocks, damped, friction

#### I. INTRODUCTION

In terms of energy, all systems have two types of energy: potential energy and kinetic energy. When a spring is stretched or compressed, it stores elastic potential energy, which is then transferred into kinetic energy. The potential

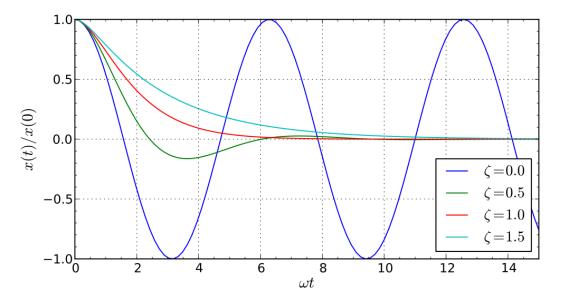
energy within a spring is determined by the equation

When the spring is stretched or compressed, kinetic energy of the mass gets converted into potential energy of the spring. By conservation of energy, assuming the datum is defined at the equilibrium position, when the spring reaches its maximal potential energy, the kinetic energy of the mass is zero. When the spring is released, it tries to return to equilibrium, and all its potential energy converts to kinetic energy of the mass.[1,2]



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A parametric oscillator is a driven harmonic oscillator in which the drive energy is provided by varying the parameters of the oscillator, such as the damping or restoring force. A familiar example of parametric oscillation is "pumping" on a playground swing.<sup>[4][5][6]</sup> A person on a moving swing can increase the amplitude of the swing's oscillations without any external drive force (pushes) being applied, by changing the moment of inertia of the swing by rocking back and forth ("pumping") or alternately standing and squatting, in rhythm with the swing's oscillations. The varying of the

parameters drives the system. Examples of parameters that may be varied are its resonance frequency .

Parametric oscillators are used in many applications. The classical varactor parametric oscillator oscillates when the diode's capacitance is varied periodically. The circuit that varies the diode's capacitance is called the "pump" or "driver". In microwave electronics, waveguide/YAG based parametric oscillators operate in the same fashion. The designer varies a parameter periodically to induce oscillations.[3,4]

Parametric oscillators have been developed as low-noise amplifiers, especially in the radio and microwave frequency range. Thermal noise is minimal, since a reactance (not a resistance) is varied. Another common use is frequency conversion, e.g., conversion from audio to radio frequencies. For example, the Optical parametric oscillator converts an input laser wave into two output waves of lower frequency

#### **II. DISCUSSION**

The **quantum harmonic oscillator** is the quantum-mechanical analog of the classical harmonic oscillator. Because an arbitrary smooth potential can usually be approximated as a harmonic potential at the vicinity of a stable equilibrium point, it is one of the most important model systems in quantum mechanics. Furthermore, it is one of the few quantum-mechanical systems for which an exact, analytical solution is known.[5,6]

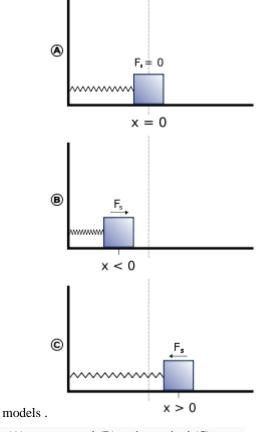


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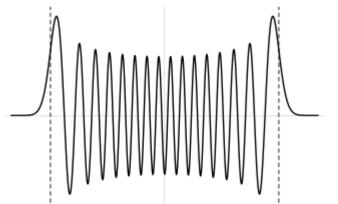
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Harmonic oscillators occurring in a number of areas of engineering are equivalent in the sense that their mathematical



Spring-mass system in equilibrium (A), compressed (B) and stretched (C) states



Vertical dashed lines indicate the classical turning points, while the dotted line represents the classical probability density.



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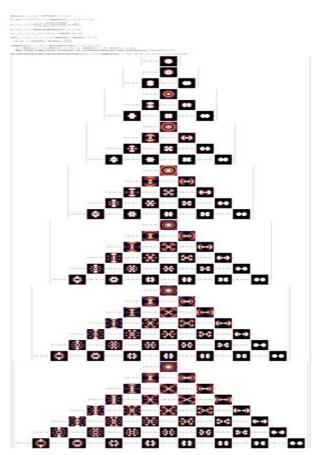
As the form of this Hamiltonian makes clear, the *N*-dimensional harmonic oscillator is exactly analogous to *N* independent one-dimensional harmonic oscillators with the same mass and spring constant. In this case, the quantities  $x_1, ..., x_N$  would refer to the positions of each of the *N* particles. This is a convenient property of the  $r^2$  potential, which allows the potential energy to be separated into terms depending on one coordinate each.

This observation makes the solution straightforward. For a particular set of quantum numbers the energy eigenfunctions for the *N*-dimensional oscillator are expressed in terms of the 1-dimensional eigenfunctions

As in the one-dimensional case, the energy is quantized. The ground state energy is N times the one-dimensional ground energy, as we would expect using the analogy to N independent one-dimensional oscillators. There is one further difference: in the one-dimensional case, each energy level corresponds to a unique quantum state. In N-dimensions, except for the ground state, the energy levels are *degenerate*, meaning there are several states with the same energy.[7,8]

The degeneracy can be calculated relatively easily. As an example, consider the 3-dimensional case: Define  $n = n_1 + n_2 + n_3$ . All states with the same *n* will have the same energy. For a given *n*, we choose a particular  $n_1$ . Then  $n_2 + n_3 = n - n_1$ . There are  $n - n_1 + 1$  possible pairs  $\{n_2, n_3\}$ .  $n_2$  can take on the values 0 to  $n - n_1$ , and for each  $n_2$  the value of  $n_3$  is fixed.







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Schrödinger 3D spherical harmonic orbital solutions in 2D density plots; the Mathematica source code that used for generating the plots

In physics, specifically in quantum mechanics, a coherent state is the specific quantum state of the quantum harmonic oscillator, often described as a state which has dynamics most closely resembling the oscillatory behavior of a classical harmonic oscillator. It was the first example of quantum dynamics when Erwin Schrödinger derived it in 1926, while searching for solutions of the Schrödinger equation that satisfy the correspondence principle. The quantum harmonic oscillator (and hence the coherent states) arise in the quantum theory of a wide range of physical systems. For instance, a coherent state describes the oscillating motion of a particle confined in a quadratic potential well (for an early reference, see e.g. Schiff's textbook)The coherent state describes a state in a system for which the ground-state wavepacket is displaced from the origin of the system. This state can be related to classical solutions by a particle oscillating with an amplitude equivalent to the displacement.[9,10]

These states, expressed as eigenvectors of the lowering operator and forming an overcomplete family, were introduced in the early papers of John R. Klauder, e. g. In the quantum theory of light (quantum electrodynamics) and other bosonic quantum field theories, coherent states were introduced by the work of Roy J. Glauber in 1963 and are also known as Glauber states.

The concept of coherent states has been considerably abstracted; it has become a major topic in mathematical physics and in applied mathematics, with applications ranging from quantization to signal processing and image processing (see Coherent states in mathematical physics). For this reason, the coherent states associated to the quantum harmonic oscillator are sometimes referred to as canonical coherent states (CCS), standard coherent states, Gaussian states, or oscillator states.

#### **IV. CONCLUSIONS**

A Bose–Einstein condensate (BEC) is a collection of boson atoms that are all in the same quantum state. In a thermodynamic system, the ground state becomes macroscopically occupied below a critical temperature — roughly when the thermal de Broglie wavelength is longer than the interatomic spacing. Superfluidity in liquid Helium-4 is believed to be associated with the Bose–Einstein condensation in an ideal gas. But 4He has strong interactions, and the liquid structure factor (a 2nd-order statistic) plays an important role. The use of a coherent state to represent the superfluid component of 4He provided a good estimate of the condensate / non-condensate fractions in superfluidity, consistent with results of slow neutron scattering. Most of the special superfluid properties follow directly from the use of a coherent state to represent the superfluid component — that acts as a macroscopically occupied single-body state with well-defined amplitude and phase over the entire volume. (The superfluid component of 4He goes from zero at the transition temperature to 100% at absolute zero. But the condensate fraction is about 6%[8] at absolute zero temperature, T=0K.)[11,12]

Early in the study of superfluidity, Penrose and Onsager proposed a metric ("order parameter") for superfluidity.[2] It was represented by a macroscopic factored component (a macroscopic eigenvalue) in the first-order reduced density matrix. Later, C. N. Yang[30] proposed a more generalized measure of macroscopic quantum coherence, called "Off-Diagonal Long-Range Order" (ODLRO),[3] that included fermion as well as boson systems. ODLRO exists whenever there is a macroscopically large factored component (eigenvalue) in a reduced density matrix of any order. Superfluidity corresponds to a large factored component in the first-order reduced density matrix. (And, all higher order



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reduced density matrices behave similarly.) Superconductivity involves a large factored component in the 2nd-order ("Cooper electron-pair") reduced density matrix.

The reduced density matrices used to describe macroscopic quantum coherence in superfluids are formally the same as the correlation functions used to describe orders of coherence in radiation. Both are examples of macroscopic quantum coherence. The macroscopically large coherent component, plus noise, in the electromagnetic field, as given by Glauber's description of signal-plus-noise, is formally the same as the macroscopically large superfluid component plus normal fluid component in the two-fluid model of superfluidity.[13,14]

Every-day electromagnetic radiation, such as radio and TV waves, is also an example of near coherent states (macroscopic quantum coherence). That should "give one pause" regarding the conventional demarcation between quantum and classical.

The coherence in superfluidity should not be attributed to any subset of helium atoms; it is a kind of collective phenomena in which all the atoms are involved [15]

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