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Mumford-Shah model for image segmentation

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ABSTRACT: Image segmentation is an important pre-processing task for many computer vision problems. Till date generalized segmentation i.e. segmentation without a priori knowledge of number of segments, features like colour, texture, shadows or overlapping, is an open problem. There are many approaches to obtain image segmentation like thresholding, clustering or classification, region-based methods, and edge-based methods. In this paper we describe the Mumford-Shah model which considers segmentation as an energy minimization problem.

KEYWORDS: Image segmentation, Mumford-Shah model, Douglas-Rachford algorithm, proximal splitting techniques, variational methods

I. INTRODUCTION

Image segmentation is the most critical pre-processing task in many computer vision problems. Hence, research in the area of image segmentation has gained a lot of attention since many years. Image segmentation deals with the problem of isolating portions or regions of an image that possess similar properties or are spatially connected. Later these isolated regions are used to identify objects, recognize various patterns in biometric images, recognize various parts of land in satellite images, or detect infected areas in medical images (MRI, X-ray images). Numerous image segmentation techniques are available in literature but a universal method applied to any type of image, related to any image processing problem is not yet known [19].

A. Image segmentation

Let the function $g(x, y)$ be an image where g represents intensity at point (x, y) of a domain $\Omega \in \mathbb{R}^2$ for a gray scale image or domain $\Omega \in \mathbb{R}^3$ for a colour image. As a result of segmentation we want to divide the image into regions $\Omega_1, \Omega_2, \dots, \Omega_n$, $n \in \mathbb{N}$, such that

$$\Omega = \bigcup_{i=1}^n \Omega_i$$

and g varies smoothly over the regions Ω_i ; $i = 1, 2, \dots, n$ whereas it is discontinuous between the regions, creating the boundaries.

B. Classical segmentation techniques:

- **Edge detection techniques**
Detecting discontinuities partitions an image based on abrupt changes in intensity. When applied on complex and noisy images they may produce missing edges or extra edges.
- **Detecting similarities**
Thresholding, region growing, region splitting and merging are based on similarity feature in an image. It works for images having light objects on dark background, but the number of partitions is limited to two. Also it does not take into account the spatial characteristics of an image and is sensitive to noise.
Region based methods like region growing, region splitting & merging, partition an image into regions that are similar according to a set of predefined criteria.

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- **Clustering**

The grouping of pixels into clusters is based on the principle of the intra class similarity minimization and the inter class similarity maximization. Clustering can be categorized into hard and soft clustering depending on the large difference or small difference between the clusters respectively. One of the most popular hard clustering methods is K-means clustering algorithm. See figure (1). Groundtruth of the images is also given in Fig.(1) (c) to readily compare the result. Groundtruth of an image is the segmentation done by humans. The drawback is the number of clusters must be determined or known a priori and it may lead to different results for each execution based on selection of initial cluster centroids.

Fuzzy clustering techniques like Fuzzy C means, Gustafson-Kessel, Gaussian mixture decomposition are used when there are no crisp boundaries between objects in an image. Fuzzy C means is the most popular soft clustering method.



Fig.1 Result of K-means segmentation method

- **Segmentation Methods Based on PDE (Partial Differential Equation)**

Kass et al. in 1987 first introduced partial differential equations and its numerical scheme to segment a noisy image [20]. Active contours or snakes are computer generated curves obtained using PDE which are placed by user interaction around the object which then move within the image to find object boundaries under the influence of internal and external forces. The level set method which was highly useful was developed by Osher and Sethian [6] used PDEs based on moving curves and surfaces with curvature based velocities.

C. Recent Technique

- **Energy minimization**

The segmentation problem has also been considered as an energy minimization problem such as the Mumford-Shah model (section II). It was introduced by Mumford and Shah in 1989 for solving the image segmentation problem [1]. This model is closely related to a functional introduced by D. and S. Geman [3]. Since then it has been also applied to the various fields of image processing like segmentation, denoising, reconstruction and inpainting.

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II. THE MUMFORD-SHAH MODEL FOR IMAGE SEGMENTATION

The Mumford-Shah model is one of the widely used and extensively studied energy minimization image segmentation models. The Mumford-Shah model can be implemented using various proximal splitting optimization techniques which we further detail in section III.

The Mumford-Shah approach to segment an image into nearly homogeneous regions, separated by smooth boundaries, consists in finding the minimum of a functional which is formed by combining the length of the hypothetical boundaries, the gradient of a smoothed version of the image and the difference of the true and smoothed images [2].

The Mumford-Shah functional is defined as follows:

$$E(f, \tau) = \mu^2 \int_{\Omega} \|f - g\|^2 dx dy + \int_{\Omega - \tau} \|\nabla f\|^2 dx dy + \lambda L(\tau) \quad (1)$$

where function f approximates image g which is differentiable over the regions Ω_i and

$$\Omega = \bigcup_{i=1}^n \Omega_i \cup \tau$$

τ be the set of arcs forming the boundaries, μ and λ are positive weighting factors that control the quality of approximation and coarseness of the segmentation respectively. $L(\tau)$ denote the total length of all the arcs belonging to τ . A large value of λ will lead to fewer boundaries. The goal is to minimize E i.e. find f and τ so that E is minimized, because smaller the E better the segmentation.

The first term in equation (1) ensures f approximates image g , the second term gives us homogeneous regions and third term prevents boundaries from growing very large. It be noted that dropping any of the terms in equation (1), we have trivial solution $\inf E = 0$ [1].

- Remove the first term, taking $f = 0$ makes $\tau = \emptyset$.
- Remove the second term, taking $f = g$, make $\tau = \emptyset$.
- Removing the third term means τ can be fine grid of N horizontal and vertical lines, each square block being infinitesimally small such that $f(x, y) = g(x, y)$ and $\|\nabla f\|^2 = 0$.

The resulting (f, τ) ends up becoming a simplified cartoon version of the original image. The cartoon has smooth regions without texture and precise edges but its main features still exist.

This model allows the extraction of features at different scales i.e. different level of details. The functions f and g can be scalar or vector valued implying it works for grayscale as well as colour images like RGB. It can be used to process data captured from various sensors and hue channels. Hence the Mumford-Shah model has been extensively used by researchers for segmenting MRI images, satellite images, biometric images and many more. See [5] for an overview on the wide applicability of the Mumford-Shah functional.

III. RELATED WORK

There is a lot of literature available about both the explicit solutions of this problem and about algorithms available to solve it numerically. Without making assumptions about result, it is very difficult to study the model as in their paper Mumford and Shah only conjecture that the problem is well posed. A problem is said to be *well posed* if:

- a solution exists
- the solution is unique and

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- the solution depends continuously on the data.

An assumption often made is that the image is constant, or at least has small variation, in each of the regions of the segmentation. This makes the second term vanish from equation (1) and we obtain the simplified or the piece-wise constant Mumford-Shah model as equation (2).

$$E(f, \tau) = \mu^2 \int_{\Omega} \|f - g\|^2 dx dy + \lambda L(\tau) \quad (2)$$

An exact minimization of the equation (1) is a non-trivial task. The Mumford-Shah model is often solved using the level set framework [6,7,8]. The improvement of local minima was achieved by graph-based methods but these methods do not allow for open boundaries [9]. A convex relaxation approach was presented in [10] using functional lifting. This approach guarantees a globally optimal solution, is independent of initialization, and takes care of open boundaries.

In [14], the method of active contours without edges (Chan-Vese model) was introduced which actually solves the piecewise constant Mumford-Shah model but restricting the solution to be a piecewise constant solution with only two regions. This model uses a regularization of the Heaviside function to write the Euler-Lagrange equation for the energy functional in equation (1). Multiphase Chan-Vese segmentation methods were also proposed later [7, 12]. The main drawback of these methods is that they can easily get stuck in local minima.

Later Song and Chan proposed an improvement to the Chan-Vese algorithm [14, 15]. It eliminates the Euler-Lagrange equations. There is no need for regularization of the Heaviside function because it deals with the energy functional directly. The method has lesser processing time than its predecessor algorithm as at each pixel it easily computes the change in the energy functional value if the pixel is moved from one partition to the other. Other approximations of the Mumford-Shah functional include the model by [13] which allows for open boundaries, making it a good approximation of the equation (1).

Also the functional in equation (1) and equation (2) are structured in such a way that it encourages the use of algorithms that decompose the problem into easier parts. For example, the constraints mentioned are often very simple sets and the projection onto them is cheap. Hence, starting from simple gradient descent algorithm for the dual problem of the ROF model, proximal splitting methods like forward-backward splitting, Douglas-Rachford splitting, Split-Bregman, alternating direction method of multipliers etc. have become very popular to solve variational problems in image processing. This can be seen in the work done in [16,17, 18].

IV. EXPERIMENTAL RESULTS USING DOUGLAS-RACHFORD ALGORITHM

Proximal algorithms are used for solving non-smooth, constrained, large-scale, or distributed optimization problems [21,22]. Many image processing problems can be formulated as

$$\min_{x \in \mathbb{R}^n} f_1(x) + \dots + f_m(x) \quad (3)$$

where $f_1(x), \dots, f_m(x)$ are convex functions from \mathbb{R}^n to $]-\infty, +\infty]$. These algorithms use splitting technique such that the functions $f_1(x), \dots, f_m(x)$ in equation (3) are used individually to yield an easily implementable algorithm. Each non-smooth function in equation (3) is involved via its proximity operator. Hence the algorithms are called proximal.

We have implemented the Douglas-Rachford algorithm to minimize the Mumford-Shah functional using Java. We have segmented the image into two regions, foreground and background. Initially we obtain the segmentation by k-means method, result shown in Fig.(2) (b) and then we minimize it using the DR algorithm. Figure (2) (c) & (d) shows

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the results using $\mu = 1$ and different values of λ . Minimization of E is achieved when the difference of E between two iterations is less than 0.1 tolerance.

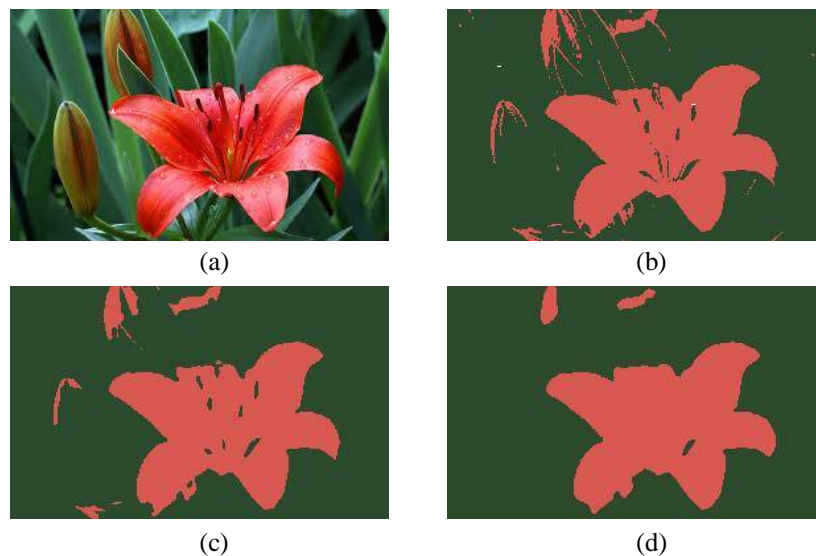


Fig.2 (a) Original image (b) K-means result (c) & (d) Result of DR algorithm using $\lambda = 0.1$ and 0.5 respectively.

V. CONCLUSION

The energy functional in Mumford-Shah model takes care that the segmented image approximates the original image, the segments do not vary much in terms of content and length of the segment boundaries is as small as possible. The results show that when λ is large, the minimization of Mumford-Shah functional E leads to fewer boundaries and coarser segmentation and when λ is small, we obtain a finer segmentation.

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