

Integral Solutions of Non- Homogeneous Ternary Cubic Equation $[2(x+y)]^2 - 15xy = 64z^3$

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ABSTRACT: The non-homogeneous cubic equation $[2(x+y)]^2 - 15xy = 64z^3$ is considered for determining its non-zero distinct integral solutions. Employing the linear transformations $x=u+v$, $y=u-v$ ($u \neq v \neq 0$) and applying the method of factorization in complex conjugates, different patterns of integral solutions to the ternary cubic equation under consideration are obtained. In each pattern, interesting relations among the solutions, some special polygonal, pyramidal numbers and central pyramidal numbers are presented.

KEYWORDS: Non- homogeneous cubic, ternary cubic equation with three unknowns, integral solutions, polygonal number, pyramidal number.

I. INTRODUCTION

The diophantine equation offers an unlimited field for research due to their variety [1-4]. In particular, one may refer [5-10] for cubic equation with three unknowns. This communication concerns with yet another interesting for ternary cubic equation $[2(x+y)]^2 - 15xy = 64z^3$ for determining its infinitely many non-zero integral solutions. Also a few interesting relations among the solutions have been presented.

Need and Importance

Notations used

- $t_{m,n}$ - polygonal number of rank 'n' with size 'm'.
- OH_n - octahedral number of rank 'n'.
- SO_n - stella octangular number of rank 'n'.
- Ky_n - kynea number.
- S_n - star number of rank 'n'.
- Pr_n - pronic number of rank 'n'.
- Obl_n - oblong number of rank 'n'.
- Cs_n - centered square number.
- J_n - Jacobsthal number of rank 'n'
- J_n - Jacobsthal-Lucas number of rank 'n'
- M_n - Mersenne number.
- P_n^5 - pentagonal pyramidal number.

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II. METHOD OF ANALYSIS

The non-homogeneous cubic equation to be solved for its non-zero distinct integral solutions is given by

$$[2(x+y)]^2 - 15xy = 64z^3 \quad (1)$$

Introducing the transformations

$$x=u+v, y=u-v, u \neq v \neq 0 \quad (2)$$

in (1), it leads to

$$u^2 + 15v^2 = 64z^3 \quad (3)$$

$$\text{Assume } z=z(a,b)=a^2+15b^2 \quad (4)$$

Pattern 1:

$$\text{Write } 64 \text{ as } 64 = (7+i\sqrt{15})(7-i\sqrt{15}) \quad (5)$$

Substituting (4) and (5) in (3) and applying the method of factorization, we have

$$u+i\sqrt{15}v = (7+i\sqrt{15})(a+i\sqrt{15}b)^3 \quad (6)$$

Equating the real and imaginary parts, we have

$$u = 7a^3 - 45a^2b - 315ab^2 + 225b^3$$

$$v = a^3 + 21a^2b - 45ab^2 - 105b^3$$

substituting the values of u and v in (2), we get

$$x=x(a,b)=8a^3-24a^2b-360ab^2+120b^3 \quad (7)$$

$$y=y(a,b)=6a^3-66a^2b-270ab^2+330b^3 \quad (8)$$

Thus (4),(7) and (8) represent the non-zero distinct integer solutions to (1)

Properties:

- ❖ $6z(a,a)$ is a nasty number.
- ❖ $z(a,a)$ is a perfect square.
- ❖ $x(a,2)-y(a,2)-SO_a-t_{90,a}-t_{82,a} \equiv -18 \pmod{277}$
- ❖ $z(a+1,a)-t_{22,a}-t_{14,a} \equiv 1 \pmod{16}$
- ❖ $y(2,b)-495 OH_b+t_{1002,b}+t_{82,b} \equiv 48 \pmod{967}$
- ❖ $x(a,3)+z(a,3)-4SO_a+t_{82,a}+t_{64,a} \equiv 1 \pmod{69}$.
- ❖ $-2 [x(a,a)]$ is a cubical integer.

Pattern 2:

64 can also be written as

$$64 = (2+i2\sqrt{15})(2-i2\sqrt{15}) \quad (9)$$

Substituting (4) and (9) in (3) and applying the same procedure as mentioned in the pattern 1, we have

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$$u+i\sqrt{15}v=(2+i\sqrt{15})(a+i\sqrt{15}b)^3$$

Equating the real and imaginary parts, we have

$$u=2a^3-90a^2b-90ab^2+450b^3$$

$$v=2a^3+6a^2b-90ab^2-30b^3$$

Substituting the values of u and v in (2), we get

$$x=x(a,b)=4a^3-84a^2b-180ab^2+420b^3 \tag{10}$$

$$y=y(a,b)=-96a^2b+480b^3 \tag{11}$$

Thus (4),(10) and (11) represent the non – zero distinct integer solutions to (1)

Properties:

- ❖ $z(2^{n+1}, 2^n)=j_{2n+2}+45j_{2n}+15$
- ❖ $z[(2^n+1), 2]-62=ky_n$
- ❖ $y(3,b)-192SO_b-192CP_b^3 \equiv 0 \pmod{768}$.
- ❖ $x(1,b)-840P_b^5+t_{802,b} \equiv 4 \pmod{682}$.
- ❖ $x(a,1)-y(a,1)-2S_a \equiv -58 \pmod{168}$.
- ❖ $z(2^n, 2^{n+1})= 3j_{2n}+15j_{2n+2}-14$

Pattern 3:

Equation (3) can be written as

$$u^2+15v^2=64z^3*1 \tag{12}$$

Assume $1 = \frac{(1+i\sqrt{15})(1-i\sqrt{15})}{16}$

$$16 \tag{13}$$

Using (4), (5) and (13) in (12) and applying the method of factorization, define

$$u+i\sqrt{15}v = 1/4[(7+i\sqrt{15})(1+i\sqrt{15})(a+i\sqrt{15}b)^3]$$

Equating real and imaginary parts, we have

$$u=1/4 [-8a^3-360a^2b+360ab^2+1800b^3]$$

$$v=1/4 [8a^3-24a^2b-360ab^2+120b^3]$$

As our aim is to find integer solutions, choose a=4A,b=4B in the above equations we get,

$$u=16 (-8A^3-360A^2B+360AB^2+1800B^3)$$

$$v=16 (8A^3-24A^2B-360AB^2+120B^3)$$

In view of (2), the integer solutions of (1) are

$$x=x(A,B)=16 (-384A^2B+1920B^3) \tag{14}$$

$$y=y(A,B)=16 (-16A^3-336A^2B+720AB^2+1680B^3) \tag{15}$$

$$z=z(A,B)=16 (A^2+15B^2) \tag{16}$$

Thus (14),(15),(16) represent the non-zero distinct integer solutions to (1).

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Properties:

- ❖ $y(1,b)-16128cp_n^6-16t_{1402,b}-16t_{42,b} \equiv -256 \pmod{736}$
- ❖ $z(n,n-1)-16cs_n-t_{202,n}-t_{250,n} \equiv 224 \pmod{226}$
- ❖ $x(2,B)+y(2,B)-69120cp_B^6-23000Obl_B-t_{82,B} \equiv -2048 \pmod{34639}$
- ❖ $z(1,B)+x(1,B)-36864cp_B^6-t_{202,B}-t_{282,B} \equiv 6 \pmod{12050}$
- ❖ $z(2,B+1)-240Pr_B \equiv 64 \pmod{240}$
- ❖ $x(A+1,1)+6096Obl_A \equiv -6000 \pmod{6192}$
- ❖ $z(2^n3)-2176=16M_{2n}$

Pattern 4:

$$\text{Write 1 as } 1 = \frac{(7+i\sqrt{15})(7-i\sqrt{15})}{64} \tag{17}$$

Using (4),(9) and (17) in (12) and applying the method of factorization, define

$$u+i\sqrt{15}v = 1/8 [(7+i\sqrt{15})(2+i2\sqrt{15})(a+i\sqrt{15}b)^3]$$

Equating real and imaginary parts, we have

$$u=1/8 [-16a^3-720a^2b+720ab^2+3600b^3]$$

$$v=1/8 [16a^3-48a^2b-720ab^2+240b^3]$$

As our aim is to find integer solutions, take $a=8A$, $b=8B$ in the above equations we get,

$$u= 64 [-16A^3-720A^2B+720AB^2+3600B^3]$$

$$v= 64 [16A^3-48A^2B-720AB^2+240B^3]$$

From (2), the integer solutions of (1) are

$$x= x(A,B)= 64 (-768A^2B+3840B^3) \tag{18}$$

$$y= y(A,B)= 64 (-32A^3-672A^2B+1440AB^2+3360B^3) \tag{19}$$

$$z= z(A,B)= 64 (A^2+15B^2) \tag{20}$$

Properties:3

- ❖ $z[(2^n-1),1]-1088 = 64 \text{ carol}_n$.
- ❖ $y(2,B)+z(2,B)-258048Cp_B^6.t_{370002,B}-t_{562,B} \equiv -16128 \pmod{29762}$
- ❖ $x(1,B)-294912Cp_B^6 \equiv 0 \pmod{98304}$
- ❖ $x(a,a+1)+2150P_a^5-3840Pr_a-t_{6002,a}-t_{1682,a} \equiv 3840 \pmod{11518}$
- ❖ $z(a+1,2^n+1)-44Obl_a-15ky_n-t_{42,a} \equiv 94 \pmod{103}$

Note:

We can write 64 as

$$64 = (-7+i\sqrt{15})(-7-i\sqrt{15}) \tag{21}$$

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$$64 = (-2+i2\sqrt{15}) (-2+i2\sqrt{15}) \quad (22)$$

Choice 1:

Considering (21) with (13), the integer solutions to (1) are given by

$$x = x(A,B) = 16 (-28A^3 - 204A^2B + 1260AB^2 - 1020B^3)$$

$$y = y(A,B) = 16 (-16A^3 + 336A^2B + 720AB^2 - 1680B^3)$$

$$z = z(A,B) = 16 (A^2 + 15B^2)$$

Choice 2:

Considering (22) with (13), the integer solutions to (1) are given by

$$x = x(A,B) = 16 (-32A^3 + 1440AB^2 - 96A^2B + 480B^3)$$

$$y = y(A,B) = 16 (-32A^3 + 1440AB^2 + 96A^2B - 480B^3)$$

$$z = z(A,B) = 16 (A^2 + 15B^2)$$

Choice 3:

By using (4),(17),(22) and (12) we get the integer solutions to (1) as

$$x = x(A,B) = 64 (-32A^3 - 672A^2B + 1440AB^2 + 3360B^3)$$

$$y = y(A,B) = 64 (-56A^3 - 408A^2B + 2520AB^2 + 2040B^3)$$

$$z = z(A,B) = 64 (A^2 + 15B^2)$$

Choice 4:

Again by substituting (4),(9) and (13) in (12), we get the following integer solutions to (1)

$$x = x(A,B) = 16 [-24A^3 - 264A^2B + 1080AB^2 + 1320B^3]$$

$$y = y(A,B) = 16 [-32A^3 - 98A^2B + 1440AB^2 + 480B^3]$$

$$z = z(A,B) = 16 [A^2 + 15B^2]$$

III. CONCLUSION

We found four different patterns and some other choices for the cubic diophantine equation with three unknowns $[2(x+y)]^2 - 15xy = 64z^3$. To conclude, one may find some other patterns to get different integer solutions.

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