

Dynamic Behaviour of a Partially Filled Cylinder Oscillating in Pendular Motion in a Confined Flow Beneath a Free Surface

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ABSTRACT: The dynamical behaviour of an elastically mounted hollow cylinder is studied. The cylinder ($m=50\text{g}$, $D=4\text{ cm}$, $L=25\text{ cm}$) oscillates as a pendulum in a fluid domain limited by the Free Surface (SL), the bottom of a water channel ($l=2\text{m}$, $b=0.30\text{ m}$, $H=0.15\text{ m}$) and two immersed vertical walls the so called ripples ($h=0, 10\text{ m}$, $e=0.01\text{ m}$) distant to each other by $d=25\text{ cm}$. The system shows oscillating or pseudo periodic motions depending upon the flow velocity U , the spring support orientation α , the cylinder-ripple distance DI , the reduced mass m^* . Indeed, the velocity field in the near-wake is measured by using a Particle Image Velocimetry technique and allows the quantification of the velocity profile above the cylinder for which the floatability value is adjusted by injection of water. A 3D force transducer is used to measure the forces acting onto the spring support and therefore the cylinder displacement. At last a numerical method inspired by the wake oscillator model as described by Yiming (2016) is used accounting for the cylinder confinement due to the bottom proximity, the mass of additional water, the ratio $D1/d$ and the fluid cylinder interaction.

KEYWORDS: Free Surface, Vortex, Oscillator, PIV, Cylinder.

I. INTRODUCTION

The flow around a circular cylinder have been extensively studied in the literature. The paradox between the simplicity of cylinder geometry and the complexity of the phenomena generated was one of the reasons for this great interest. From a more pragmatic point of view, the cylinder is an element of structure that is very frequently encountered in oceanic engineering, Bearman (1978), Fredsoe (1987). The complexity of the flow is first illustrated by the nature of the pressure and velocity field around the object. The speed field changes in nature according to the speed of the flow. The configurations of the flow depend on the Reynolds number, $R_e = VD / \nu$, where V is the velocity of the external flow in m / s , D the diameter of the cylinder in m , ν the kinematic viscosity which is the ratio between the dynamic viscosity μ and the density ρ of the fluid. The Reynolds number R_e defines the ratio between the inertia and viscosity forces. As the speed of the flow increases, this ratio increases, and the configuration of the near-wake varies until the appearance of two recirculation zones, as a consequence of the delamination of the boundary layer. For a R_e number greater than 40, alternating detachment is observed on one side and then on the other. The frequency f_0 in Hz of the vortex shedding is managed by the second parameter which is the Strouhal number $S_t = f_0 D / V$. The development of these vortices shedding in the wake of the cylinder is Von Karman's alley. The two parameters R_e and S_t make it possible to classify the configurations around the cylinder. We distinguish the laminar, subcritical, critical and supercritical regimes. The

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subcritical regime is characterised by a Strouhal number which varies around 0.2 for a range of R_e between 40 and 300000. In the two-dimensional case, a free shear layer separates the flow into two zones: the one that is closer to the flow of a fluid with a very low viscosity, and therefore a velocity potential, and the other characterised by the length L which separates the base of the cylinder from a point of instability where the shear layers coming from each side of the cylinder. It is shown that this point is a saddle point, it belongs to 3 zones, those of the recirculation coming from both sides of the cylinder and that of the free surface. The length L of circulation increases with the Reynolds number and its upper limit indicates the moment of the unsteady shedding described above. Its variation is often used to characterise the rate of instationnarity of the alternate shedding, as in the case of a stationary flow, uniformly accelerated or impulsive. The presence of a free surface also substantially modifies the wake and the unsteady charactersuch asthe appearance of an anticipation of the shedding of vortices in the presence of a free surface for a cylinder in translation, anticipation attributed to a flow of jet type, strongly sheared between SL and the side of the object. Finally, it should be remembered that theshedding of vortices causes a fluctuation of the pressure field around the cylinder and can cause vibrations of the obstacle, especially if it is mounted on flexible supports, Blevins (1990). Numerous studies have also been conducted to study the interaction of a vibratory cylinder with external flow, Williamson (1996) (2000), Newman (1997), Pantazopoulos (1994), Fachinetti (2004), Ogink (2010). The phenomenon known as VIV (Vortex Induced Vibrations) is exploited in some techniques of recovery of the energy injected by the flow on the cylinder or others that will prevent the rupture of the structure if the vibrations are close to the frequency of resonance.

The object of the study presented here consists of a cylinder mounted on flexible supports placed in a compartment, separated by two vertical fixed obstacles called "riffles", in the middle of a channel with free surface. This compartmentalised open channel is usually used by miners to increase the yield of the operation. Our study is conducted in collaboration with a team from the laboratory of the University of Antananarivo working on the transport of particles in this device. This paper is therefore structured as follows: in section 2, we present thethe problem and then we describe the experimental technique in section 3 as well as some results in section 4, a numerical method based on the model of the oscillator of the wake is presented in section 5, the model presented takes into account the effects of subdivision which generates a non-uniform velocity profile and the presence of a liquid injected into the cylinder which regulates its buoyancy but which generates a free surface effect. Finally, the gap between the cylinder and the bottom of the channel is taken into account as have been done by several authors, Yang (2009), Wang (2013). The presence of the bottom changes the mode of shedding and new models are proposed in the model of the wake oscillator, Yiming (2016).

II. POSITION OF THE PROBLEM

A hydrodynamic channel, of length 2 m, width 20 cm and depth 15 cm, contains a free surface flow generated by an adjustable flow supply. The channel is partitioned by vertical partitions with a thickness of 10 mm and a height of 7 cm separated from each other by 25 cm. A honeycomb conditioner is placed at the inlet of the channel to obtain a weak turbulent flow. A cylinder with a mass of 50 grams, a length of 15 cm, a diameter of 4 cm is placed between two compartments and is capable of oscillating in a pendulum movement. The horizontal polystyrene hollow cylinder is connected at its two ends to an axis of rotation by two rods of 30 cm long. The axis is itself rigidly connected to the side walls of the channel by an adjustable link in vertical and horizontal translation. This set is placed in a display tank that continuously feeds the hydrodynamic channel. The objective of the work is first to experimentally characterise the velocity field around the cylinder and its dynamics by measuring its displacement as well as the applied forces. In a second step, we propose a numerical model simulating the fluid-structure interaction resulting from the wake oscillator model. The originality of the work lies in the fact that the cylinder is placed in a confined flow limited by the bottom, the two vertical walls and the free surface and that its initial position is regulated by a quantity of water inside. The block diagram of the entire device is shown in FIG. 1

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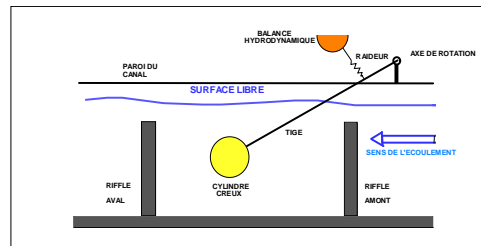


Figure 1: Schematic diagram of the entire device in so-called "normal" mode (axis of rotation upstream of the cylinder). The cylinder "mounted on flexible supports" is in pendular motion about the axis of rotation and placed in the average velocity flow V_0 . The two upstream and downstream riffles perpendicular to the bottom delimit the compartment studied in the PIV device

III. EXPERIMENTAL METHOD

The experimental method gives the possibility to measure the movement of the cylinder in two ways. On the one hand the support of the cylinder is connected to a hydrodynamic balance that is positioned by means of a three-axis robot. The extension of the spring is proportional to the displacement. The forces are transmitted by the spring to the scale, itself constituted by a bar equipped with a KISTLER brand three-component force sensor positioned by a 3-axis robot. On the other hand, a visualisation system makes it possible to reconstitute the displacement of the cylinder. The camera is integrated in a device that allows the implementation of a PIV technique to measure the velocity field around the cylinder. Image pairs are processed using DIRECTPIV. Data reduction is done using the POD method. To adjust the initial conditions, particularly the buoyancy of the cylinder, a known quantity of water is injected via a syringe. During the movement the internal liquid oscillates (liquid hull) and generates internal forces that interact with the movement of the cylinder itself.

EXPERIMENTAL DEVICE

A submerged pump draws water from the display tank. This tank 2m long, 1m wide and 1m high is made of glass. The pump has a diameter of 0.019m and a power of 0.3KW. The suction capacity is 3m. A floater adjusts the water level. The pump injects the water sucked into a small compartmentalised channel of length 2m, width 0.20m and depth 0.15m built in Plexiglas and placed in the tank. The flow rate is regulated by an adjustable valve. The small channel, placed in the display tank, is compartmentalised by three vertical walls of a height of 7 cm distant 0.25m. Circular straws 18 cm long are stacked in the direction of flow at the inlet of the channel to reduce the intensity of turbulence of the upstream flow. The hollow cylinder of length 0.15m and 4cm diameter in Polyvinyl chloride is placed horizontally in a compartment. The two ends of the cylinder are connected respectively by two steel rods of length 0.21m which pivot about a horizontal axis. The position of the pivot axis on the upper edges of the channel wall can be varied by means of a sliding system. This system also makes it possible to modify the slope of the two preceding rods for positioning the cylinder relative to the depth of the compartment at the start of an experiment.

The PIV device consists of a PULNIX DualTapAccuPIXEL CCD camera, RM-6740, with a lens with a 25 cm focal length lens; the resolution of the image is 640 (H) x 480 (V) pixels, and the capture rate is 30Hz. A box synchronises the emission frequency of the laser beam and allows the capture of successions of images by the camera placed perpendicularly to the laser plane. The time step of acquiring a pair of images is 0.033s.

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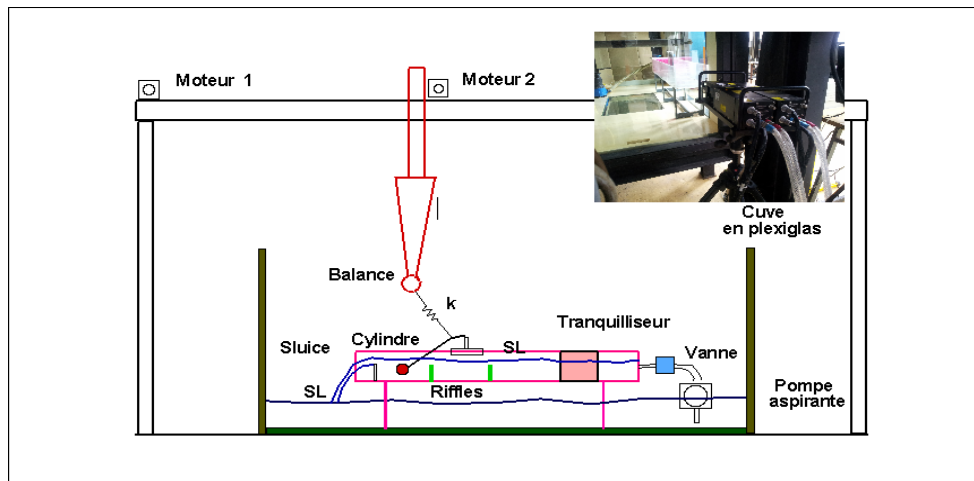


Figure 2: Overview of the system; motors 1 and 2 allow adjustment of the hydrodynamic balance in position; the free-flowing channel in pink is placed in a visualisation tank; the inset photograph represents the orientation of the lasers in the axis of the canal and the tank

A 0.04m diameter bar acts as a hydrodynamic balance and measures the forces. It is equipped with a three-axis KISTLER piezometric force sensor at its end. It records and delivers the applied forces in the form of a voltage amplified by a KISTLER charge amplifier. A 4-channel TEKTRONICS electronic oscilloscope enables live analysis of measured forces.

PROCESSING OF MEASURES

The PIV technique used makes it possible to acquire 400 pairs of images of the instantaneous velocity field. The post-processing of velocity fields uses a multi-variable POD statistical method often used in fluid mechanics. This method consists in building up from the 400 pairs of images relating to each step of acquisition time the matrix used for the analysis and then breaking it down into three matrices by decomposition into singular values. The most significant singular values from which the data reduction matrix can be reconstructed, and the overall configuration of the flow velocity field are identified, and the most significant recirculation zones highlighted. A numerical code written in Fortran was developed for the implementation of the method.

The raw signal of the hydrodynamic forces undergone by the cylinder consists of 2500 points sampled at a time step of 0.1s. The reconstruction procedure consists at first in correcting the possible drift of the charge amplifier. For this purpose, the signal representing drift by linear regression is subtracted from the original raw signal $x(t)$. The corrected signal is then modelled as a series of pseudo periodic components as reported in Rajaona (1985). This technique implements a spectral interpolation method that identifies the most significant peaks of the discrete spectrum calculated by a Cooley-Tuckey algorithm.

IV. EXPERIMENTAL RESULTS

We propose to illustrate some characteristic experimental results. They concern the study of the spectra of the vertical movement via the vertical forces and the velocity field in the compartment between the ripples. The Fig.3 illustrates the performance of the signal model by a sum of damped sinusoids; the method allows the possibility to reconstitute all the characteristics of each component retained after a thresholding, in this case at 10% of the predominant peak. A zoom of the framed part is shown below. On the right is represented, at the top, the gross spectrum, in the middle, the amplitudes of the components down their phases.

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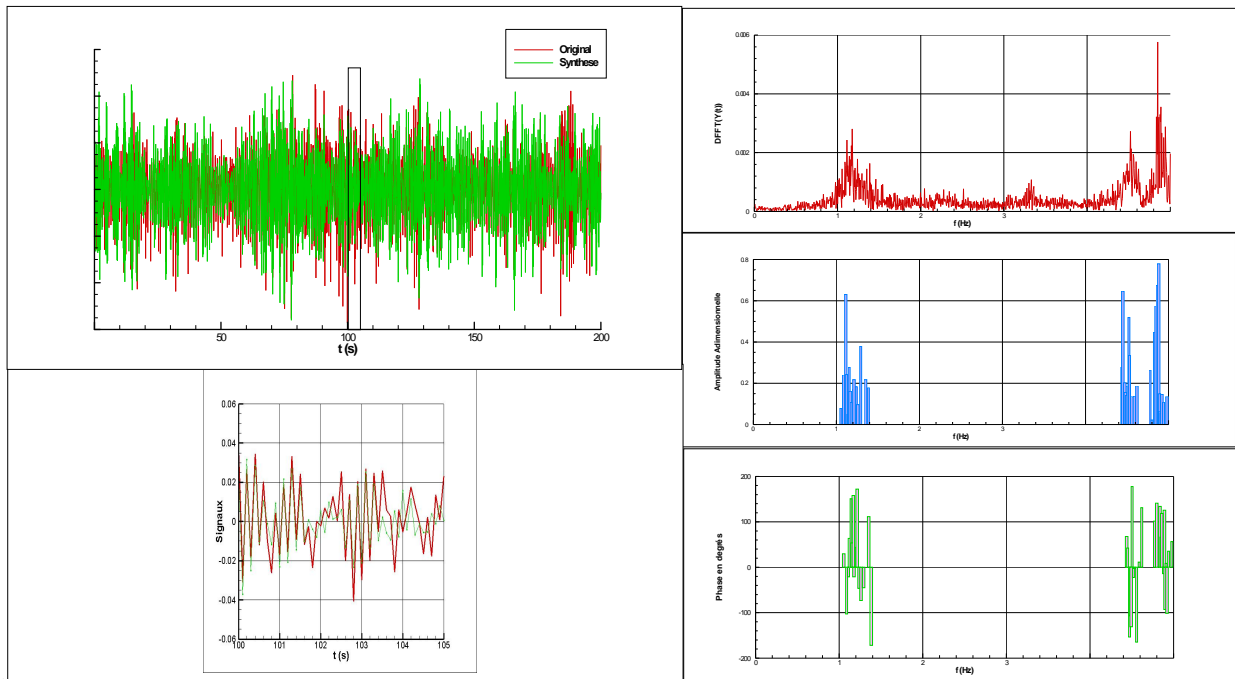


Figure 3: Approximation of the intermittent motion by a sum of damped sinusoids obtained by spectral interpolation and thresholding Rajaona (1985). The components of the reconstituted signal are represented by their amplitude, frequency and phase. 39 components are used for the case studied here. A zoom of the framed part illustrates the differences between the original signal and the synthesised signal.

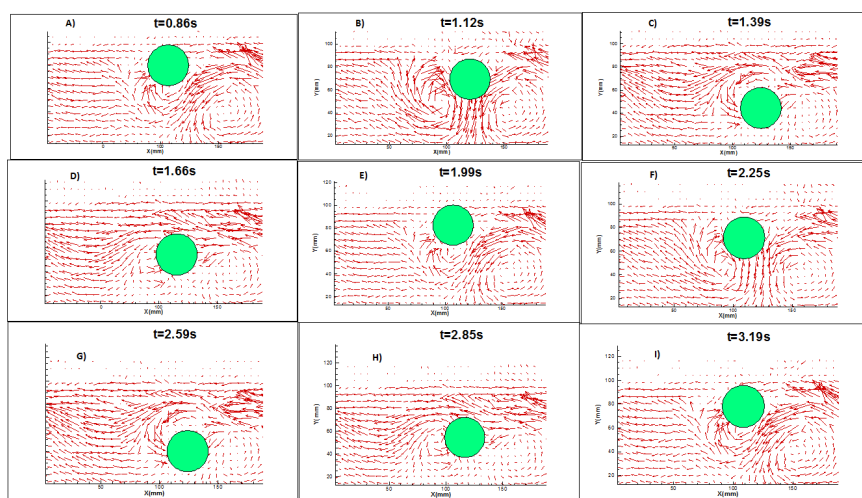


Figure 4: Velocity field for the characteristic positions of the cylinder on its trajectory. The 6 letters A, B, C, D, E, F correspond to the moments that can be seen in FIG. 3. Fields G), H), I) complete the illustration for the end of a pseudo period.

The complexity of the signal is due to the proximity of the lines of the spectrum. A localised resonance effect can be observed when the phase goes from $+90^\circ$ to -90° .

Fig. 4 illustrates the variation of the velocity field in the compartment during a cycle. The vortex shedding only takes place on the lower part of the cylinder. This is attributed to the presence of the free surface as has been observed by the

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numerical simulations of Chung (2016). It is clear that an alley of Von Karman cannot develop due to the downstream riffle. This justifies the fact of considering the velocity profile towards the cylinder in the wake oscillator model.

V. NUMERICAL MODEL OF FLUID-CYLINDER COUPLING

In view of the geometry of the fastening system, the vertical movement is predominant with respect to the horizontal movement. Thus, in this numerical modelling, we have reduced the field of investigation of the dynamics of the pendulum cylinder to the contributions of the vertical oscillations.

Numerical modelling is based on an extension of the wake oscillator model. This extension consists in considering the velocity profile obtained by PIV at the vertical of the cylinder. A second extension takes into account the presence of the rigid background and its effects modify the evolution of the amplification term. Considering the action of stiffness k , friction r is conventional. The modelling of the effect of the embedded water body is more delicate.

The study of sloshing in a circular cylindrical horizontal tank was presented in particular by MacIver (1989). In the context of a perfect isovolumic fluid, the method consists in using a conformal transform which transforms the fluid domain into a semi-infinite band. The free surface condition is subsequently displayed in the transformed plane on a horizontal plane parallel to the bottom of the system. From a solution that satisfies the bottom condition written as an integral, the free-surface condition takes the form of a homogeneous Fredholm integral equation whose nucleus is symmetric. The continuous integral is then approximated by a numerical series in which the oscillation frequencies appear as the eigenvalues of the matrix of the coefficients of the series. It is clear that the dispersion relation is more complex than in the case of a rectangular tank. Another approach is to consider that the body of water moves like a film of water along the wall of the cylinder. This film then behaves like a mass of water oscillating in a tube. In this case the length of the wet wall alone is involved in the calculation of the oscillation frequency. The selected oscillation mode is the antisymmetric mode. As part of the approximations, we adopted the second technique. We have specifically admitted that the amplitude of the forces induced by the movement of the embedded water body depends on the amplitude of the movement of the cylinder and the frequency is inversely proportional to the square root of the length of the "fluid film".

We thus have the two equations coupled between the movement of the cylinder, the movement of the onboard liquid and the effect of the vortex shedding and the speed of the current:

$$\left(m_s + \frac{\pi}{4} L \rho D^2 C_{m0}\right) \frac{\partial^2 y}{\partial t^2} + \left(r_s + \frac{\rho L D}{2} C_d U(y)\right) \frac{\partial y}{\partial t} + h y = \frac{\rho L D}{4} U^2(y) C_{l0} q + B \cos\left(\sqrt{\frac{2g}{l}} t + \varphi\right) \quad (1)$$

$$\frac{\partial^2 q}{\partial t^2} - \varepsilon \left[2\pi S_t \frac{U(y)}{D}\right] (1 - q^2) \frac{\partial q}{\partial t} + \left[2\pi S_t \frac{U(y)}{D}\right]^2 q + \alpha \left[2\pi S_t \frac{U(y)}{D}\right] f_n q^2 = \beta \frac{A}{D} \frac{\partial^2 y}{\partial t^2} \quad (2)$$

Where:

y is the position of the cylinder; h is the stiffness of the spring;

m_s , the mass of the structure; C_{m0} the added mass of the cylinder; r_s , structural depreciation

C_d the drag coefficient; $U(y)$ the horizontal velocity at the cylinder;

C_{l0} the lift coefficient in the stationary case;

q the amplification factor ratio between the instantaneous lift coefficient $C_l(t)$ and C_{l0} ;

l the wet length occupied by the onboard water body;

φ the phase of the movement; B is a parameter that represents the magnitude of the internal force;

S_t the Strouhal number, close to 0.2 for a Reynolds number of 10000;

ε is a parameter close to 0.3 which considers the effect of self-limitation of the movement due to vortex shedding;

f_n is the natural oscillation frequency of the mechanical system in water.

α is the coefficient that takes into account the presence of the supposed rigid free surface as shown in Yiming;

A takes the value 12 in the absence of free surface;

β takes the value 1 in the absence of free surface, otherwise it considers the presence of the free surface.

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The values of the different parameters of the model are given in Table 1

Parameters	Symbol	Definition	Value	Wang (2013)	Unit
Reduced amplitude	A^*	A/D			Without dimension
Drag coefficient	C_d	$2F_x / (\rho U^2 D L)$			Without dimension
Cylinder diameter	D		40	20	mm
Natural frequency	f_N		3.34	4.91	Hz
Vibration frequency	f_{osc}				Hz
Vortex shedding frequency	f_v		1.04		Hz
Frequency ratio	f^*	f_{osc}/f_N			Without dimension
Mass ratio	m^*	$4m / (\pi \rho D^2 L)$	0.2122	1.	Without dimension
Reynolds	Re	UD/ν	9000	3000-13000	Without dimension
Gap ratio	e/D			0.05-2.5	Without dimension
Strouhal	S_t	fD/U	0.176		Without dimension
Froude Number	F_r	U/\sqrt{gD}	0.48		Without dimension
Reduced Velocity	U^*	$U/(f_N D)$	3.90-5.02	1.53-6.62	Without dimension
Reduced damping	ζ	$c/2\sqrt{k/(m+mA)}$		0.0173	Without dimension
Bottom Effect(Yimin2016)	α	$-1.66e/D - 0.218m^*e/+1.04$			Without dimension
Bottom Effect(Yimin2016)	β	$-0.044e/D - 0.581^*e/+1.34$			Without dimension

Table 1: The main parameters used in the paper presented. The Wang column takes the values of the experiments carried out by Wang (2013), and the values of α and β taking into account the Bottom effect proposed by Yiming (2016)

VALIDATION OF THE MODEL

To validate the model, we first compared our experimental results corresponding to the parameters shown in the table on the right of Fig.5 with those of the model. Secondly, we modelled the experiments carried out by Wang 2013, Govardhan 2000 and Yiming 2016 and compared the results by varying the reduced velocity and observing the amplitude and frequency behaviour of the system.. The Fig.5 shows a comparison of the results of the current model with the LOMC experiment for maximum velocity values of 0.27 m / s. There is a good representation of the f_{osc} oscillation frequency attributed to the vortex shedding and those attributed to the drag effects. The experiments carried out for different positions of the cylinder between the two riffles show that the two peaks between 4 and 5 Hz are due to free surface effects.

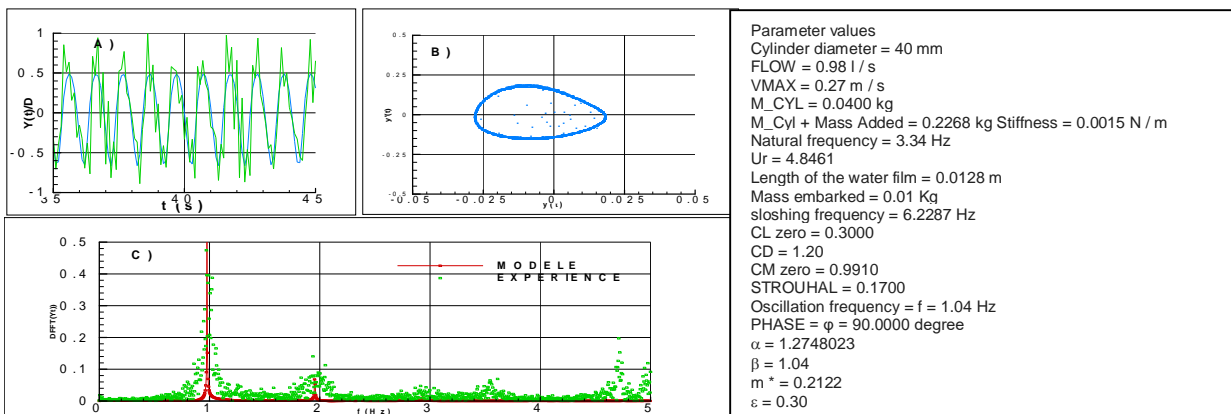


Figure 5: Comparison experiments (in green) vs. model (in red) for the parameter values shown in the table on the right. The reduced amplitude $Y(t)/D$ is well represented in the same way the two main peaks. In B, the displacement $Y(t)$ as a function of the velocity $\dot{Y}(t)$ is represented in the phase plane.

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Fig. 6 aims to compare current numerical results with experimental results given by Wang (2013). The model predicts synchronization by representing A/D as a function of the reduced speed. Similar behaviour is noted for the so-called initial branch. However, synchronization takes place for lower values of the reduced speed 3.5 to 4 compared to the Govardhan experiments. The right figure shows the reduced frequency f_{osc}/f_n as a function of the reduced speed. The absence of lock-in is attributed to confinement. The discontinuity observed seems to show a hysteresis-type behaviour. Note that the experimental results given by Govardhan and Williamson (2000) and Yang (2009) are also represented.

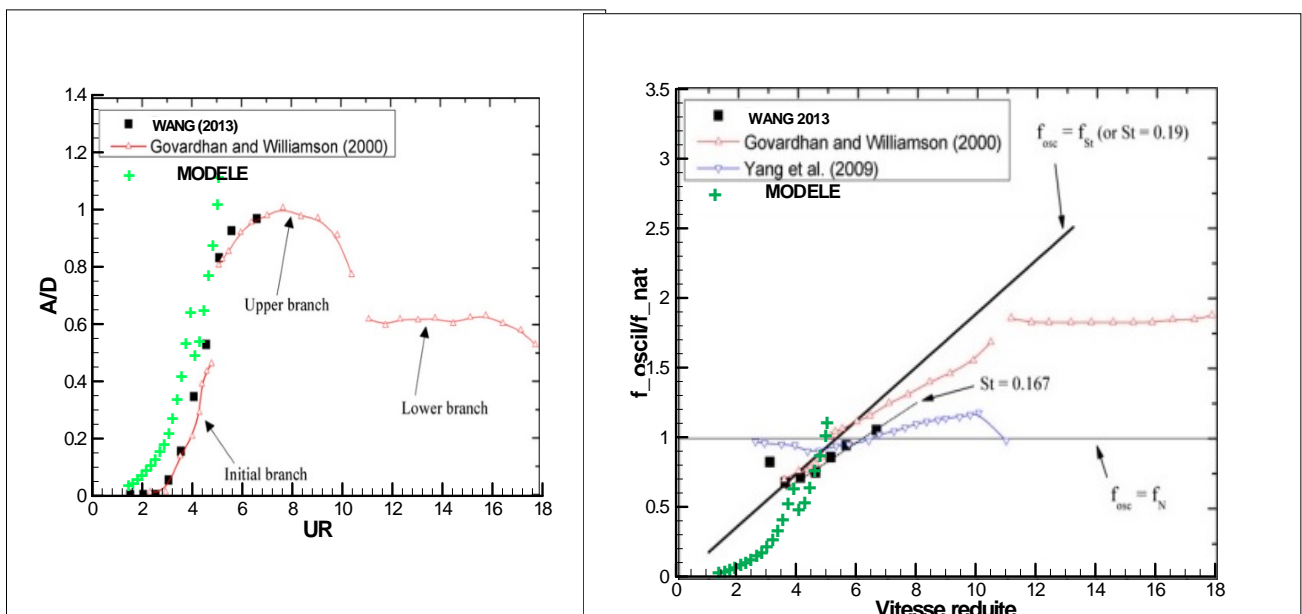


Figure 6: Comparison of numerical results with experimental results from Wang 2013 and Govardhan 2000. There is a reasonable correlation of model results for the initial branch. The results of Yang (2009) are also represented. The synchronization takes place for lower values of the reduced velocity U_r . The variation of the ratio of the f_{osc}/f_n frequency increases with U_r , and does not exhibit lock-in behaviour. The discontinuity of the amplitude and frequency curves shows a hysteresis-type behaviour.

VI. CONCLUSION

We have studied the dynamic behaviour of a cylinder mounted on flexible supports placed under a free surface and in the presence of a base and ripples. The initial position of the cylinder and its reduced mass were fixed by injecting water into the cylinder. The experimental method used implements the PIV technique supplemented by a reduction of the data by POD to analyse the evolution of the velocity field near the synchronization. In general, we observed pseudo periodic or intermittent movements representing the trajectory of the cylinder in the phase plane. In order to highlight the significant parameters of the cylinder dynamics, we proposed a wake oscillator model taking into account the bottom by introducing the parameters proposed by Yiming (2016). The presence of the free surface and the side walls have been taken into account by introducing a variable speed profile with the cylinder position. The presence of the bottom has been considered by introducing the complementary terms into the coupling equations, terms introduced by Yiming, among others. The main results are reported below:

- The deformation of the free surface is very strongly dependent on the distance between the cylinder and the ripples, likewise it depends on the pendular operation in "normal" mode (axis of rotation upstream of the cylinder) or "inverted" mode (axis of rotation downstream of the cylinder).
- The movement of the cylinder causes a blockage for large values of buoyancy (cylinder moving near the free surface). For the lower values, a jet-like flow takes place alternately above and below the cylinder. The lower stream is less intense than the upper one.

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- At large amplitudes reduced A/D , the free surface causes a rebound of the cylinder downwards.
- The movement of the cylinder is pseudo periodic (almost periodic, chaotic and / or intermittent). By analysing the velocity fields corresponding to the characteristic positions of the cylinder on its trajectory, the organised structures of the near-wake and the recirculation zones in the associated compartment are shown in FIG. 5.
- The dominant peaks of the spectrum come from a) the oscillation motion of the onboard water body, b) the interaction of the cylinder with the external flow, c) the profile of the vertical velocity and d) the geometric characteristics and mechanical assembly, Fig. 6.
- The numerical model in the form of a fourth order differential system (derived from a modified Van der Pol equation coupled with the differential equation of cylinder motion) is solved using a method of Petzold-Gear BDF including the Newton and Runge-Kutta method. It makes it possible to satisfactorily represent the behaviour of the cylinder in the proximity and during the synchronization. The results are comparable to those made by Wang for a cylinder in the presence of a bottom surface. However, the peaks of the spectrum of experimental results corresponding to the highest frequencies can only be observed by using parameter values that are quite different from those of the experimental model.

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