

Anisotropic Dark Energy Cosmological Model in Lyra Geometry

V P Kadam¹, S.N. Bayaskar²,

Department of Mathematics, Gopikabai Sitaram Gawande College, Umarched, India¹

Department of Mathematics, Adarsha Science, J.B.Arts and Birla Commerce Mahavidyalaya, Dhamangaon, India²

ABSTRACT: An anisotropic and homogeneous new class of Bianchi type cosmological model in the presence and absence of magnetism in Lyra Geometry have been investigated in the present paper. By using a constant deceleration parameter, field equations have been solved. The behaviors of anisotropy of the dark energy and some important features of the models have been discussed.

KEYWORDS: New class of Bianchi type universe, Dark energy, Lyra geometry.

I. INTRODUCTION

Recently so many researchers are interested in cosmological models with dark energy in general theory of relativity as well as in modified theories of gravity because the most of remarkable observational discoveries have shown that our universe is currently undergoing an accelerated expansion. On the basis of these astronomical observations cosmologists have accepted that existence of dark energy which account more than 70% of the total energetic content of the universe to be mostly responsible for the accelerated expansion of the universe due to repulsive gravitations [1,2]. Cosmological observations of supernovae of the type Ia (SNIa) [3, 4], cosmic microwave background radiation (CMBR) [5-11], large-scale structure (LSS) [12,13] and other cosmic phenomena have firmly established the picture of the accelerated expansion of the contemporary universe [14,15]. The present accelerating phase of the expansion of the universe and its onset at a relatively low redshift ($z \approx 1$) represent one of the most intriguing and most studied problems in modern cosmology. A lucid introduction and nice review of the work on dark energy model in general relativity is given by Farooq et al. [16] and Pradhan et al. [17].

Soleng [18] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include as creation field and be equal to Hoyle's creation field cosmology [19–21] or contain a special vacuum field, which together with the gauge vector term may be considered as a cosmological term. Sri Ram and Singh [22] have obtained exact solutions of the field equations in vacuum and in the presence of stiff-matter for anisotropic Bianchi type V cosmological models in the normal gauge with a time-dependent displacement vector field. Singh [23] considered flat FRW model in Lyra's geometry by using varying adiabatic equation of state and solved the field equations for the early phases of evolution of universe.

Several authors [24–29] have been investigated cosmological models which are based on Lyra's manifold with constant displacement field vector. Singh et al. [30–36] had studied Bianchi-type I , III , Kantowski-Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson-Walker models with constant deceleration parameter in Einstein's theory with cosmological term in the cosmological theory based on Lyra's geometry. Bianchi type - I models in Lyra's geometry have been presented by Kumar and Singh [37]. A new class of Bianchi type- I cosmological models in Lyra's geometry have obtained by Singh et al. [38]. Ram et al. [39] have obtained exact solutions for anisotropic Bianchi type V perfect fluid cosmological models in Lyra's geometry. Recently, Chaubey [40] has obtained exact solutions for Kantowski-Sachs cosmological model in Lyra's geometry.

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Recently, Adhav [41] studied LRS Bianchi type – I universe with anisotropic dark energy in Lyra Geometry. Shchigolev et al. [42] investigated Cosmological models with a varying Λ term in Lyra's geometry. Scalar Field Cosmology in Lyra's Geometry studied by Shchigolev et al. [43]. HoavoHova [44, 45] obtained a dark energy model in Lyra manifold and Vacuum expansion in arbitrary gauge Lyra geometry. Quintessence Cosmology with an Effective Λ Term in Lyra Manifold investigated by Khurshudyan et al. [46]. Also Toy models of Universe with an Effective varying Λ Term in Lyra Manifold investigated by Khurshudyan [47]. Very recently, Chirde et al. [48] studied Bianchi Type VI_0 space-time under the assumption of anisotropy of the fluid within the frame work of Lyra geometry.

In this paper we have investigated anisotropic and homogeneous new class of cosmological model under the assumption of the fluid in the presence and absence of magnetism in the framework of Lyra geometry. Some physical and geometrical behaviors of the model are also discussed.

II. METRIC AND FIELD EQUATIONS

An anisotropic and homogeneous new class of Bianchi type metric is in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 e^{-2mx} dz^2 \quad (1)$$

Where A, B, C are functions of cosmic time t only. The metric (1) corresponds to a Bianchi type III model for $m = 0$, Bianchi type V model for $m = 1$ and Bianchi type VI_0 model for $m = -1$.

The Einstein's modified field equation in normal gauge for Lyra's manifold as obtained by Sen [44] is given by (In geometrized unit where $8\pi G = 1, C = 1$)

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_\alpha \phi^\alpha = -T_{ij} \quad (2)$$

Where ϕ_i is the displacement field vector defined as $\phi_i = (0, 0, 0, \lambda(t))$ & the symbols have their usual meaning i.e. R_{ij} is the Ricci tensor, R is the Ricci scalar and T_{ij} is the stress energy momentum tensor of the matter.

The energy momentum tensor of the fluid can be written most generally in anisotropic diagonal form as follows,

$$T_j^i = \text{diag}[T_1^1, T_2^2, T_3^3, T_4^4] \quad (3)$$

Allowing for anisotropy in the pressure of the fluid and thus in its EoS parameter gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in (3) as follows:

$$\begin{aligned} T_j^i &= \text{diag}[-p_x, -p_y, -p_z, \rho] \\ &= \text{diag}[-\omega_x, -\omega_y, -\omega_z, 1]\rho \\ &= \text{diag}[-\omega\rho, -(\omega + \delta)\rho, -(\omega + \gamma)\rho, \rho] \end{aligned} \quad (4)$$

Where ρ is the energy density of the fluid p_x, p_y, p_z are the pressure and $\omega_x, \omega_y, \omega_z$ are the directional EoS parameter along the x,y,z axis respectively δ & γ are the deviations from the free EoS parameter (hence the deviation free pressure) on y-axis and z-axis respectively. Here ω, δ & γ are not necessarily constants and can be functions of the cosmic time t .

Now assuming the co-moving co-ordinate system, with the help of equations (2) and (4), the field equations for the metric (1) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B} \frac{C_4}{C} - \frac{m}{A^2} + \frac{3}{4} \lambda^2 = -\omega\rho \quad (5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A} \frac{C_4}{C} - \frac{m^2}{A^2} + \frac{3}{4} \lambda^2 = -(\omega + \delta)\rho \quad (6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A} \frac{B_4}{B} - \frac{1}{A^2} + \frac{3}{4} \lambda^2 = -(\omega + \gamma)\rho \quad (7)$$

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$$\frac{A_4}{A} \frac{B_4}{B} + \frac{A_4}{A} \frac{C_4}{C} + \frac{B_4}{B} \frac{C_4}{C} - \frac{(1+m+m^2)}{A^2} - \frac{3}{4} \lambda^2 = \rho \quad (8)$$

$$(1+m) \frac{A_4}{A} - \frac{B_4}{B} - m \frac{C_4}{C} = 0 \quad (9)$$

The energy conservation equation $T_{;j}^{ij} = 0$ leads to

$$\rho_4 + (1+\omega)\rho \left[\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right] + \frac{\rho}{A^2} (\delta + m\gamma) + \rho \left(\delta \frac{B_4}{B} + \gamma \frac{C_4}{C} \right) = 0 \quad (10)$$

Also from the field equation (2),

$$\left(R_{ij} - \frac{1}{2} R g_{ij} \right)_{;j} + \left(\frac{3}{2} \phi_i \phi_j \right)_{;j} - \left(\frac{3}{4} g_{ij} \phi_\alpha \phi^\alpha \right)_{;j} = - (T_{ij})_{;j}$$

Above equation leads to

$$\frac{3}{2} \phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^l \Gamma_{lj}^j \right] + \frac{3}{2} \phi^j \left[\frac{\partial \phi_i}{\partial x^j} + \phi_l \Gamma_{ij}^l \right] - \frac{3}{4} g_i^j \phi_k \left[\frac{\partial \phi^k}{\partial x^j} + \phi^l \Gamma_{lj}^j \right] - \frac{3}{4} \phi^k \left[\frac{\partial \phi_k}{\partial x^j} + \phi_l \Gamma_{ij}^l \right] = 0$$

Above equation is identically satisfied for $i = 1, 2, 3$. For $i = 4$, it reduces to

$$\begin{aligned} & \frac{3}{2} \lambda \left[\frac{\partial (g^{44} \phi_4)}{\partial x^4} + \phi^4 \Gamma_{4j}^j \right] + \frac{3}{2} g^{44} \phi_4 \left[\frac{\partial \phi_4}{\partial x^4} - \phi_4 \Gamma_{44}^4 \right] - \frac{3}{4} g_4^4 \phi_4 \left[\frac{\partial (g^{44} \phi_4)}{\partial x^4} + \phi^4 \Gamma_{44}^4 \right] \\ & - \frac{3}{4} (g^{44} \phi_4) \left[\frac{\partial \phi_4}{\partial x^4} - \phi_4 \Gamma_{44}^4 \right] = 0 \end{aligned}$$

which leads to

$$\frac{3}{2} \lambda \lambda_4 + \frac{3}{2} \lambda^2 \left[\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right] = 0 \quad (11)$$

Where suffix denotes derivative with respect to cosmic time t .

On integrating equation (9), we get

$$A^{(1+m)} = \gamma BC^m \quad (12)$$

Where γ is integration constant.

Without loss of generality, we consider $\gamma = 1$. We obtain

$$A^{(1+m)} = BC^m \quad (13)$$

For the new class of Bianchi type model which are important in cosmological observations, we define mean Hubble parameter as follows

$$H = (\ln a)^\bullet = \frac{a_4}{a} = \frac{1}{3} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) \quad (14)$$

The average scale factor of new class of Bianchi type model can be expressed as

$$a = (ABC)^{\frac{1}{3}} \quad (15)$$

The spatial volume (V) is given by

$$V = a^3 = (ABC) \quad (16)$$

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III. SOLUTION OF FIELD EQUATIONS

Since the field equations (5) – (8) and (11) are highly non-linear, supply only five equations in eight unknowns $A, B, C, \omega, \rho, \delta, \gamma$ and λ . In order to obtain exact solutions of the Einstein's field equations, we normally assume a form for the matter content or suppose that the space-time admits killings vectors symmetries. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter, which was first proposed by Berman [49] in FRW models and that yields a constant value of deceleration parameter. Cunha and Lima [50] favors recent acceleration and past deceleration with high degree of statistical confidence level by analyzing three SNe type Ia samples. In order to match this observation, Singh and Debnath [51], Adhav et al. [41] has defined a spatial form of deceleration parameter for FRW metric as

$$q = -\frac{aa_{44}}{a_4^2} = -1 + \frac{\alpha}{1+a^\alpha} \tag{17}$$

Where $\alpha > 0$ is a constant and a is mean scale factor of the universe.

After solving (17) one can obtain the mean Hubble parameter H as

$$H = \frac{a_4}{a} = k(1+a^{-\alpha}) \tag{18}$$

Where k is a constant of integration.

On integrating (18), we obtain the mean scale factor as

$$a = (e^{\alpha kt} - 1)^{\frac{1}{\alpha}} \tag{19}$$

Here we discussed three cases:

3.1 Case I For $B = V^2$ (20)

Using equations (11), (16), (19) and (20), we get

$$A = (e^{\alpha kt} - 1)^{\frac{3(2-m)}{(2m+1)\alpha}} \tag{21}$$

$$B = (e^{\alpha kt} - 1)^{\frac{6}{\alpha}} \tag{22}$$

$$C = (e^{\alpha kt} - 1)^{\frac{-3(m+3)}{(2m+1)\alpha}} \tag{23}$$

$$\lambda = \frac{c_1}{(e^{\alpha kt} - 1)^{\frac{3}{\alpha}}} \tag{24}$$

Where c_1 is constant of integration.

Therefore the metric (1) reduces to

$$ds^2 = dt^2 - (e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}} dx^2 - (e^{\alpha kt} - 1)^{\frac{12}{\alpha}} e^{-2x} dy^2 - (e^{\alpha kt} - 1)^{\frac{-6(m+3)}{(2m+1)\alpha}} e^{-2mx} dz^2 \tag{25}$$

Using equations (8) & (21)-(24), we get

Energy density as

$$\rho = \left\{ \frac{-9(7m^2 + 7m + 8)k^2 e^{2\alpha kt}}{(2m+1)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \tag{26}$$

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Using equations (5) & (21)-(24), we get
EoS parameter as

$$\omega = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left[2 \left[\left(\frac{6}{\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] - \left(\frac{m+3}{2m+1} \right) \left[\left(\frac{-3(m+3)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\}}{\left\{ \frac{18k^2(m+3)}{(2m+1)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} - \frac{m}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}}$$

$$\left\{ \frac{9(7m^2 + 7m + 8)k^2 e^{2\alpha kt}}{(2m+1)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \quad (27)$$

The skewness parameters δ, γ (i.e. deviation from ω along y-axis and z-axis) are computed as

$$\delta = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left[\left(\frac{2-m}{2m+1} \right) \left[\left(\frac{3(2-m)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] - 2 \left[\left(\frac{6}{\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\}}{\left\{ \frac{45m(m+3)k^2}{(2m+1)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-m^2)}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} \right\}}$$

$$\left\{ \frac{9(7m^2 + 7m + 8)k^2 e^{2\alpha kt}}{(2m+1)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \quad (28)$$

$$\gamma = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left[\left(\frac{2-m}{2m+1} \right) \left[\left(\frac{3(2-m)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] + \frac{(m+3)}{(2m+1)} \left[\left(\frac{-3(m+3)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right\}}{\left\{ \frac{90k^2}{(2m+1)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-1)}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} \right\}}$$

$$\left\{ \frac{9(7m^2 + 7m + 8)k^2 e^{2\alpha kt}}{(2m+1)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \quad (29)$$

3.2 Case II For $C = V^{\frac{1}{2}}$ (30)

Using equations (11), (16), (19) and (30), we get

$$A = (e^{\alpha kt} - 1)^{\frac{3(m+1)}{2(m+2)\alpha}} \quad (31)$$

$$B = (e^{\alpha kt} - 1)^{\frac{3}{2(m+2)\alpha}} \quad (32)$$

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$$C = (e^{\alpha kt} - 1)^{\frac{3}{2\alpha}} \tag{33}$$

$$\lambda = \frac{c_1}{(e^{\alpha kt} - 1)^{\frac{3}{\alpha}}} \tag{34}$$

Where c_1 is constant of integration.

Therefore the metric (1) reduces to

$$ds^2 = dt^2 - (e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}} dx^2 - (e^{\alpha kt} - 1)^{\frac{3}{(m+2)\alpha}} e^{-2x} dy^2 - (e^{\alpha kt} - 1)^{\frac{3}{\alpha}} e^{-2mx} dz^2 \tag{35}$$

Using equations (8) & (31)-(34), we get

Energy density as

$$\rho = \left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \tag{36}$$

Using equations (5) & (31)-(34), we get

EoS parameter as

$$\omega = - \left\{ \frac{3\alpha k^2 e^{\alpha kt}}{2(e^{\alpha kt} - 1)} \left[\left(\frac{3}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \frac{1}{(m+2)} + \left[\left(\frac{3}{2\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right\} + \frac{9k^2}{4(m+2)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} - \frac{m}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \tag{37}$$

$$\left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

The skewness parameters δ, γ (i.e. deviation from ω along y-axis and z-axis) are computed as

$$\delta = - \left\{ \frac{3\alpha k^2 e^{\alpha kt}}{2(e^{\alpha kt} - 1)} \left[\left(\frac{-1}{m+2} \right) \left[\left(\frac{3}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\} + \frac{9mk^2}{4(m+2)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} \tag{38}$$

$$\left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

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$$\gamma = - \left\{ \frac{3\alpha k^2 e^{\alpha kt}}{2(m+2)(e^{\alpha kt} - 1)} \left[- (m+2) \left[\left(\frac{3}{2\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] + \frac{(m+1)}{(m+2)} \left[\left(\frac{3(m+1)}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right. \\ \left. - \frac{9k^2}{4(m+2)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-1)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} \right\} \tag{39}$$

$$\left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \tag{40}$$

3.3 Case III For $C = V$

Using equations (11), (16), (19) and (40), we get

$$A = (e^{\alpha kt} - 1)^{\frac{3m}{(m+2)\alpha}} \tag{41}$$

$$B = (e^{\alpha kt} - 1)^{\frac{-3m}{(m+2)\alpha}} \tag{42}$$

$$C = (e^{\alpha kt} - 1)^{\frac{3}{\alpha}} \tag{43}$$

$$\lambda = \frac{c_1}{(e^{\alpha kt} - 1)^{\frac{3}{\alpha}}} \tag{44}$$

Where c_1 is constant of integration.

Therefore the metric (1) reduces to

$$ds^2 = dt^2 - (e^{\alpha kt} - 1)^{\frac{6}{(m+2)\alpha}} dx^2 - (e^{\alpha kt} - 1)^{\frac{-6m}{(m+2)\alpha}} e^{-2x} dy^2 - (e^{\alpha kt} - 1)^{\frac{6}{\alpha}} e^{-2mx} dz^2 \tag{45}$$

Using equations (8) & (41)-(44), we get

Energy density as

$$\rho = \left\{ \frac{-9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \tag{46}$$

Using equations (5) & (41)-(44), we get

EoS parameter as

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$$\omega = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left[\left[\left(\frac{3}{\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] - \left(\frac{m}{m+2} \right) \left[\left(\frac{-3m}{(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\}}{\left\{ \frac{9mk^2 e^{2\alpha kt}}{(m+2)(e^{\alpha kt} - 1)^2} - \frac{m}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}}$$

$$\left\{ \frac{9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$
(47)

The skewness parameters δ, γ (i.e. deviation from ω along y-axis and z-axis) are computed as

$$\delta = \frac{\left\{ \frac{6m(3-\alpha)k^2 e^{2\alpha kt}}{(m+2)(e^{\alpha kt} - 1)^2} + \frac{(m-m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} \right\}}{\left\{ \frac{9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}}$$
(48)

$$\gamma = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(m+2)(e^{\alpha kt} - 1)} \left[\left[\left(\frac{3m}{(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] - \frac{(m+2)}{m} \left[\left(\frac{3}{\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\}}{\left\{ \frac{18mk^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(m-1)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} \right\}}$$

$$\left\{ \frac{9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$
(49)

IV. FIELD EQUATION WITH MAGNETIZED ANISOTROPIC DARK ENERGY

Allowing for anisotropy in the presence of the fluid and thus in EoS parameter gives rise to new possibilities for the evolution of the energy source. At first, we parameterize the energy momentum tensor for magnetized dark energy in (3) as follows,

$$T_j^i = \text{diag}[\rho + \rho_B, -p_x + \rho_B, -p_y - \rho_B, -p_z - \rho_B]$$

$$= \text{diag}[-\omega\rho - \rho_B, -(\omega + \delta)\rho - \rho_B, -(\omega + \gamma)\rho - \rho_B, \rho + \rho_B]$$
(50)

Where ρ is the energy density of the fluid p_x, p_y, p_z are the pressure and $\omega_x, \omega_y, \omega_z$ are the directional EoS parameter along the x, y, z axis respectively δ & γ are the deviations from the free EoS parameter (hence the deviation free pressure) on y-axis and z-axis respectively and ρ_B are stands for energy density of magnetic field.

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Now assuming the co-moving co-ordinate system, with the help of equations (2) and (50), the field equations for the metric (1) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B} \frac{C_4}{C} - \frac{m}{A^2} + \frac{3}{4} \lambda^2 = -\omega\rho - \rho_B \quad (51)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A} \frac{C_4}{C} - \frac{m^2}{A^2} + \frac{3}{4} \lambda^2 = -(\omega + \delta)\rho - \rho_B \quad (52)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A} \frac{B_4}{B} - \frac{1}{A^2} + \frac{3}{4} \lambda^2 = -(\omega + \gamma)\rho - \rho_B \quad (53)$$

$$\frac{A_4}{A} \frac{B_4}{B} + \frac{A_4}{A} \frac{C_4}{C} + \frac{B_4}{B} \frac{C_4}{C} - \frac{(1+m+m^2)}{A^2} - \frac{3}{4} \lambda^2 = \rho + \rho_B \quad (54)$$

$$(1+m) \frac{A_4}{A} - \frac{B_4}{B} - m \frac{C_4}{C} = 0 \quad (55)$$

$$\frac{3}{2} \lambda \lambda_4 + \frac{3}{4} \lambda^2 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \quad (56)$$

The energy conservation equation $T_{;j}^{ij} = 0$ leads to

$$\rho_4 + (1+\omega)\rho \left[\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right] + \frac{\rho}{A^2} (\delta + m\gamma) + \rho \left(\delta \frac{B_4}{B} + \gamma \frac{C_4}{C} \right) = 0 \quad (57)$$

$$(\rho_B)_4 + 2\rho_B \left[\frac{B_4}{B} + \frac{C_4}{C} + \frac{A_4}{A} \right] = 0 \quad (58)$$

V. SOLUTION OF FIELD EQUATION

5.1 Case I For $B = V^2$ (59)

We obtained the expression for model (1) with $\rho_B, \rho, \omega, \delta$ & γ as

$$\rho_B = \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \quad (60)$$

Energy Density is

$$\rho = \left\{ \frac{-9(7m^2 + 7m + 8)k^2 e^{2\alpha k t}}{(2m+1)^2 (e^{\alpha k t} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} - \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \right\} \quad (61)$$

EoS parameter as

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$$\omega = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha k t}}{(e^{\alpha k t} - 1)} \left\{ 2 \left[\left(\frac{6}{\alpha} - 1 \right) \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} + 1 \right] - \left(\frac{m+3}{2m+1} \right) \left[\left(\frac{-3(m+3)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} + 1 \right] \right\} \right.}{\left. - \frac{18k^2(m+3)}{(2m+1)} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2} - \frac{m}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \right\}}$$

$$\frac{\left\{ \frac{9(7m^2 + 7m + 8)k^2 e^{2\alpha k t}}{(2m+1)^2 (e^{\alpha k t} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \right\}}$$
(62)

The skewness parameters δ, γ (i.e. deviation from ω along y-axis and z-axis) are computed as

$$\delta = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha k t}}{(e^{\alpha k t} - 1)} \left\{ \left(\frac{2-m}{2m+1} \right) \left[\left(\frac{3(2-m)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} + 1 \right] - 2 \left[\left(\frac{6}{\alpha} - 1 \right) \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} + 1 \right] \right\} \right.}{\left. + \frac{45m(m+3)k^2}{(2m+1)^2} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2} + \frac{(m-m^2)}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \right\}}$$

$$\frac{\left\{ \frac{9(7m^2 + 7m + 8)k^2 e^{2\alpha k t}}{(2m+1)^2 (e^{\alpha k t} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \right\}}$$
(63)

$$\gamma = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha k t}}{(e^{\alpha k t} - 1)} \left\{ \left(\frac{2-m}{2m+1} \right) \left[\left(\frac{3(2-m)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} + 1 \right] \right\} \right.}{\left. + \frac{(m+3)}{(2m+1)} \left[\left(\frac{-3(m+3)}{(2m+1)\alpha} - 1 \right) \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} + 1 \right] \right\}}$$

$$+ \frac{90k^2}{(2m+1)} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2} + \frac{(m-1)}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}}$$

$$\frac{\left\{ \frac{9(7m^2 + 7m + 8)k^2 e^{2\alpha k t}}{(2m+1)^2 (e^{\alpha k t} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha k t} - 1)^{\frac{6(2-m)}{(2m+1)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \right\}}$$
(64)

5.2 Case II For $C = V^{\frac{1}{2}}$ (65)

We obtained the expression for model (1) with $\rho_B, \rho, \omega, \delta$ & γ as

$$\rho_B = \frac{\beta}{(e^{\alpha k t} - 1)^{\frac{6}{\alpha}}} \quad (66)$$

Energy Density is

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$$\rho = \left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} - \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \quad (67)$$

EoS parameter as

$$\omega = - \left\{ \frac{3\alpha k^2 e^{\alpha kt}}{2(e^{\alpha kt} - 1)} \left\{ \left[\left(\frac{3}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \left(\frac{1}{m+2} \right) + \left[\left(\frac{3}{2\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right\} \right. \\ \left. + \frac{9k^2}{4(m+2)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} - \frac{m}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \quad (68)$$

$$\left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} - \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

The skewness parameters δ, γ (i.e. deviation from ω along y-axis and z-axis) are computed as

$$\delta = - \left\{ \frac{3\alpha k^2 e^{\alpha kt}}{2(e^{\alpha kt} - 1)} \left\{ \left(\frac{-1}{m+2} \right) \left[\left(\frac{3}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right\} \right. \\ \left. + \left(\frac{m+1}{m+2} \right) \left[\left(\frac{3(m+1)}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right\} \\ + \frac{9mk^2}{4(m+2)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \quad (69)$$

$$\left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} - \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

$$\gamma = - \left\{ \frac{3\alpha k^2 e^{\alpha kt}}{2(m+2)(e^{\alpha kt} - 1)} \left\{ -(m+2) \left[\left(\frac{3}{2\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right. \right. \\ \left. \left. + \frac{(m+1)}{(m+2)} \left[\left(\frac{3(m+1)}{2(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right\} \right. \\ \left. - \frac{9k^2}{4(m+2)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-1)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \quad (70)$$

$$\left\{ \frac{9(m^2 + 5m + 5)k^2 e^{2\alpha kt}}{4(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{3(m+1)}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} - \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

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5.3 Case III For $C = V$ (71)

We obtained the expression for model (1) with $\rho_B, \rho, \omega, \delta$ & γ as

$$\rho_B = \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \tag{72}$$

Energy density as

$$\rho = \left\{ \frac{-9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} - \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} - \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} - \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\} \tag{73}$$

EoS parameter as

$$\omega = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left[\left[\left(\frac{3}{\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] - \left(\frac{m}{m+2} \right) \left[\left(\frac{-3m}{(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\}}{\left\{ \frac{9mk^2}{(m+2)} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} - \frac{m}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}} \tag{74}$$

$$\left\{ \frac{9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

The skewness parameters δ, γ (i.e. deviation from ω along y-axis and z-axis) are computed as

$$\delta = \frac{\left\{ \frac{6m(3-\alpha)k^2 e^{2\alpha kt}}{(m+2)(e^{\alpha kt} - 1)^2} + \frac{(m-m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^6} \right\}}{\left\{ \frac{9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}} \tag{75}$$

$$\gamma = \frac{\left\{ \frac{3\alpha k^2 e^{\alpha kt}}{(m+2)(e^{\alpha kt} - 1)} \left[\left[\left(\frac{3m}{(m+2)\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] - \left(\frac{m+2}{m} \right) \left[\left(\frac{3}{\alpha} - 1 \right) \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} + 1 \right] \right] \right\}}{\left\{ \frac{18mk^2}{(m+2)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{(m-1)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}} \tag{76}$$

$$\left\{ \frac{9m^2 k^2 e^{2\alpha kt}}{(m+2)^2 (e^{\alpha kt} - 1)^2} + \frac{(1+m+m^2)}{(e^{\alpha kt} - 1)^{\frac{6m}{(m+2)\alpha}}} + \frac{3}{4} \frac{c_1^2}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} + \frac{\beta}{(e^{\alpha kt} - 1)^{\frac{6}{\alpha}}} \right\}$$

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VI. SOME PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

The expansion scalar θ , the shear scalar σ^2 and the mean anisotropy parameter A_m are defined as

$$\theta = 3H = u_{;i}^i = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) \tag{77}$$

$$\sigma^2 = \frac{3}{2} A_m H^2 \tag{78}$$

$$A_m = \frac{1}{3} \sum_1^3 \left(\frac{\Delta H_i}{H} \right)^2 \tag{79}$$

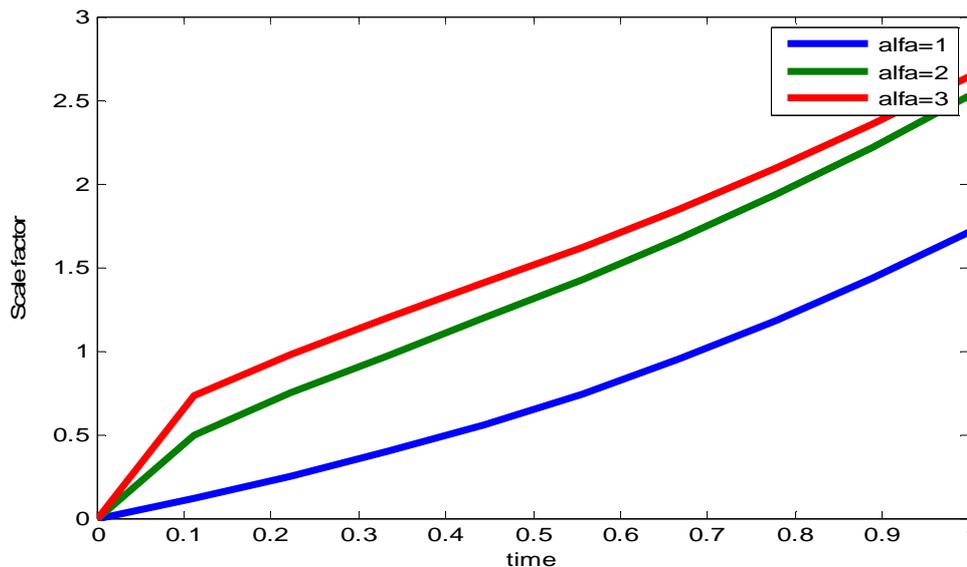
Where the mean Hubble parameter and $H_i (i=1,2,3)$ represent the Hubble parameters in the directions of x,y,z axes respectively.

The deceleration parameter q , defined as

$$q = -\frac{aa_{44}}{a_4^2} = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 \tag{80}$$

The Physical properties that are important in cosmology are average scale factor a , spatial volume V , anisotropy expansion parameter A_m , Hubble parameter H , expansion scalar θ , shear scalar σ^2 and the deceleration parameter q are as follows

The average scale factor, $a = (e^{\alpha kt} - 1)^{\frac{1}{\alpha}}$



Hence fig. shows that the average scale factor increases with time i.e. When $t \rightarrow 0$ then scalar factor is zero expand with time.

The spatial volume $V = (e^{\alpha kt} - 1)^{\frac{3}{\alpha}}$

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Hence spatial volume increases with time i.e. When $t \rightarrow 0$ then spatial volume $V \rightarrow 0$ & expand with time when $t \rightarrow \infty$ then spatial volume $V \rightarrow \infty$.

$$\text{Hubble parameter } H = \frac{ke^{\alpha kt}}{(e^{\alpha kt} - 1)}$$

$$\text{Expansion scalar } \theta = \frac{3ke^{\alpha kt}}{(e^{\alpha kt} - 1)}$$

Anisotropy expansion parameter : A_m :

$$\text{For case-I } A_m = \frac{50(m^2 + m + 1)}{(2m + 1)^2}$$

$$\text{For case-II } A_m = \frac{(m^2 + m + 1)}{2(m + 2)^2}, \quad \text{For case-III } A_m = \frac{8(m^2 + m + 1)}{(m + 2)^2}$$

shear scalar σ^2

$$\text{For case-I } \sigma^2 = \frac{75(m^2 + m + 1)k^2 e^{2\alpha kt}}{(2m + 1)^2 (e^{\alpha kt} - 1)^2}, \quad \text{For case-II } \sigma^2 = \frac{3(m^2 + m + 1)k^2 e^{2\alpha kt}}{4(m + 2)^2 (e^{\alpha kt} - 1)^2}$$

$$\text{For case-III } \sigma^2 = \frac{12(m^2 + m + 1)k^2 e^{2\alpha kt}}{(m + 2)^2 (e^{\alpha kt} - 1)^2}$$

The shear scalar σ^2 diverges at $t \rightarrow 0$ while they becomes constant at $t \rightarrow \infty$

$$\text{The deceleration parameter } q = -1 + \frac{\alpha}{e^{\alpha kt}}$$

Deceleration parameter verses time

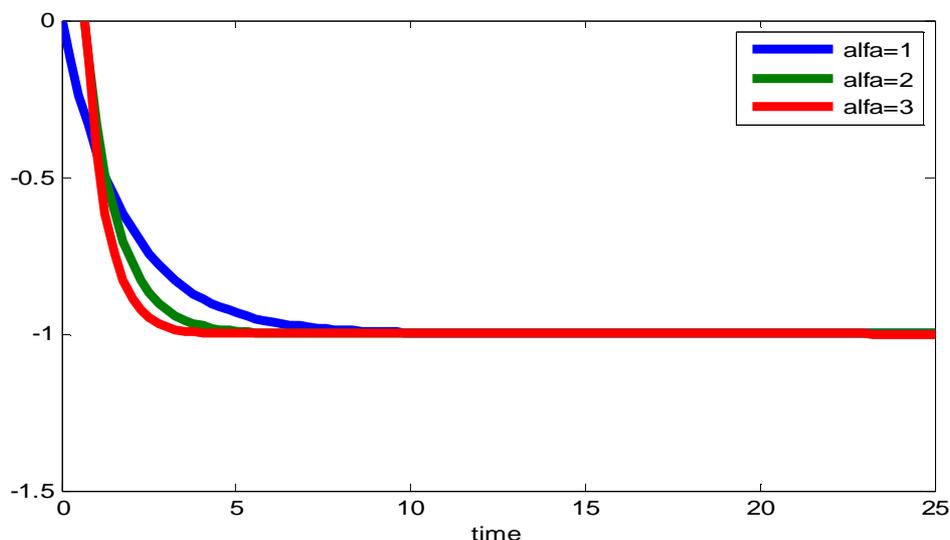


Fig shows that the deceleration parameter lies between 0 and -1.

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VII. CONCLUSION

In present paper, we have investigated the homogeneous and anisotropic new form of Bianchi type cosmological model within the framework of Lyra geometry. In order to determine the exact solution, we have used a new spatial form of the deceleration parameter proposed by Akarsu and Dereli. We observed that for $\alpha = 1, 2, 3$, the value of deceleration parameter lies between -1 and 0 i.e. hence $-1 < q < 0$, hence this value is very nearer to the observed value of DP. Also we observed that the average scale factor and spatial volume increases with time. It is observed that Hubble parameter and expansion scalar are constant when $t \rightarrow \infty$.

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