

# **Symmetrization and Numerical Study of a One-Dimensional System of Equations of Two- Speed Hydrodynamics with Single Pressure**

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**ABSTRACT.** The work is devoted to the symmetrization of the system of equations of two-speed one-dimensional hydrodynamics with a single-pressure in a reversible approximation neglecting mass forces. The obtained symmetrized system of equations is solved numerically using the HLL method.

**KEYWORDS.** Two-speed hydrodynamics, t-Friedrichs hyperbolic equations, symmetrization of a system of equations, numerical methods, HLL method.

## **I. INTRODUCTION**

Mathematical modeling of flows of multiphase media is of interest from the point of view of many scientific disciplines and has a great use in various applied fields.

To date, investigations in the construction and study of models of two-phase media are widely developed. The founder of these investigations is Kh.A. Rakhmatulin, who introduced the concept of interpenetrating continua (phases). This approach has been further developed in numerous works, both Russian and foreign authors. The behavior of each phase is described by the laws of conservation of mass, momentum and energy, and the interfacial interaction is taken into account by adding special algebraic or differential terms to the right-hand parts of the conservation laws [3].

A common approach is to use single-pressure models to describe two-phase fluid flows, which still underlie many analytical and numerical studies. Calculations carried out for similar models on coarse grids or using dissipative numerical schemes can provide plausible results, but when grinding the computational grid and using more accurate methods, the calculation results demonstrate the lack of convergence of the numerical solutions. To overcome the difficulties associated with the incorrectness of initial-boundary problems, various modifications of the model with single-pressure were proposed [1–4]. The hyperbolicity of the system of defining equations in such models is usually achieved by introducing additional differential terms to the right-hand sides of the laws of conservation of phases momentum, which are called virtual mass forces, interfacial pressure, etc.

As a result, the system of equations of the model is hyperbolic, but its reduction to a symmetric form, like the representation of all equations of the system in a divergent form, is impossible, which complicates the use of a modified model to study discontinuous solutions.

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Note the work of C.K. Godunov [5, 6], where he first identified a special class of differential equations and showed that this system can be written as a symmetric  $t$ -hyperbolic (Friedrichs) system, i.e. can be symmetrized. This paper shows that the system of equations of a two-fluid medium can be reduced to a symmetric form. Then, using the HLL method, the symmetrized system of equations of two-speed hydrodynamics in the reversible approximation is solved numerically in the one-dimensional case without taking into account mass forces.

## II. SYMMETRIC FORM OF ONE-DIMENSIONAL SYSTEM OF EQUATIONS OF TWO-SPEED HYDRODYNAMICS WITH ONE PRESSURE

The system of equations of two-speed hydrodynamics in the one-dimensional case without taking into account mass forces in the reversible approximation takes the form [1-4]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial(\sigma v)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\bar{\rho}} \frac{\partial P}{\partial x} - \frac{\sigma}{2\bar{\rho}} \frac{\partial}{\partial x} (u - v)^2, \tag{3}$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{1}{\bar{\rho}} \frac{\partial P}{\partial x} + \frac{\rho}{2\bar{\rho}} \frac{\partial}{\partial x} (u - v)^2, \tag{4}$$

where  $u$  and  $v$  are the velocity vectors of the subsystems that make up the two-velocity continuum with the corresponding partial densities  $\rho$  and  $\sigma$ ,  $\bar{\rho} = \rho + \sigma$ , and  $P$  is the common pressure. System (1) – (4) is closed by the equation of state

$$P = P(\bar{\rho}). \tag{5}$$

It can be showed that the system (1) – (4) is reduced to a symmetric (divergent) form by passing from the set of variables  $\{\rho, \sigma, u, v\}$  to  $\{\rho, \sigma, w, j\}$ , by formulas

$$w = u - v,$$

$$j = \rho u + \sigma v.$$

Thus, the expression of velocities in the new set of variables is represented as

$$u = \frac{j + \sigma w}{\bar{\rho}}, \quad v = \frac{j - \rho w}{\bar{\rho}}. \tag{6}$$

Given these expressions, equations (1) – (2) take the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho \frac{j + \sigma w}{\bar{\rho}} \right) = 0,$$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial}{\partial x} \left( \sigma \frac{j - \rho w}{\bar{\rho}} \right) = 0.$$

Subtracting from equation (3) equation (4), we obtain the equation for the velocity difference  $w$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x} \left( w \frac{j - \rho w}{\bar{\rho}} \right) = 0.$$

The sum of equation (1) multiplied by  $u$  and equation (3) multiplied by  $\rho$  is given as

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\rho}{\bar{\rho}} \frac{\partial P}{\partial x} - \frac{\rho \sigma}{2\bar{\rho}} \frac{\partial w^2}{\partial x} = 0. \tag{7}$$

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The sum of equation (2) multiplied by  $v$  and equation (4) multiplied by  $\sigma$  is given as

$$\frac{\partial(\sigma v)}{\partial t} + \frac{\partial(\sigma v^2)}{\partial x} + \frac{\sigma}{\bar{\rho}} \frac{\partial P}{\partial x} + \frac{\rho \sigma}{2\bar{\rho}} \frac{\partial w^2}{\partial x} = 0. \quad (8)$$

Taking into account formulas (6), the sum of equations (7) and (8) is

$$\frac{\partial j}{\partial t} + \frac{\partial}{\partial x} \left( \frac{j^2}{\bar{\rho}} + \frac{\rho \sigma w^2}{\bar{\rho}} + P \right) = 0.$$

Thus, we obtained the following symmetric (divergent) system of equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho \frac{j + \sigma w}{\bar{\rho}} \right) = 0, \quad (9)$$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial}{\partial x} \left( \sigma \frac{j - \rho w}{\bar{\rho}} \right) = 0, \quad (10)$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x} \left( w \frac{j - \rho w}{\bar{\rho}} \right) = 0, \quad (11)$$

$$\frac{\partial j}{\partial t} + \frac{\partial}{\partial x} \left( \frac{j^2}{\bar{\rho}} + \frac{\rho \sigma w^2}{\bar{\rho}} + P \right) = 0. \quad (12)$$

The system (9) –(12) can be written in matrix form [5-8]

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0, \quad (13)$$

where  $U = (\rho, \sigma, w, j)^T$  ( $T$  stands for transposing),

$$F(U) = \begin{pmatrix} \rho \frac{j + \sigma w}{\bar{\rho}} \\ \sigma \frac{j - \rho w}{\bar{\rho}} \\ w \frac{j - \rho w}{\bar{\rho}} \\ \frac{j^2}{\bar{\rho}} + \frac{\rho \sigma w^2}{\bar{\rho}} + P \end{pmatrix}.$$

The equation (13) is also reduced to the following matrix equation

$$\frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} = 0, \quad (14)$$

where

$$A = \frac{1}{\bar{\rho}^2} \begin{pmatrix} j\sigma + w\sigma^2 & -j\rho + w\rho^2 & \bar{\rho}\rho & \bar{\rho}\rho\sigma \\ -j\sigma - w\sigma^2 & j\rho - w\rho^2 & \bar{\rho}\sigma & -\bar{\rho}\rho\sigma \\ -jw - w^2\sigma & -jw + w^2\rho & \bar{\rho}w & \bar{\rho}j - 2\bar{\rho}w\rho \\ \bar{\rho}^2 P_{\bar{\rho}} - j^2 + w^2\sigma^2 & \bar{\rho}^2 P_{\bar{\rho}} - j^2 + w^2\rho^2 & 2\bar{\rho}j & 2\bar{\rho}^2 P_{\bar{w}} w + 2\bar{\rho}w\rho\sigma \end{pmatrix}. \quad (15)$$

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### III. SPECTRAL ANALYSIS OF THE SYSTEM OF THE EQUATIONS

In a non-conservative form, the system of equations (1) – (4) has the form

$$\begin{aligned} \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} &= 0, \\ \frac{\partial \sigma}{\partial t} + v \frac{\partial \sigma}{\partial x} + \sigma \frac{\partial v}{\partial x} &= 0, \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{P_{\bar{\rho}}}{\bar{\rho}} \left( \frac{\partial \rho}{\partial x} + \frac{\partial \sigma}{\partial x} \right) - \frac{\sigma w}{\bar{\rho}} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \right) &= 0, \\ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{P_{\bar{\rho}}}{\bar{\rho}} \left( \frac{\partial \rho}{\partial x} + \frac{\partial \sigma}{\partial x} \right) + \frac{\rho w}{\bar{\rho}} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \right) &= 0, \end{aligned}$$

or in matrix notations [5,6]

$$\frac{\partial Q}{\partial t} + B \frac{\partial Q}{\partial x} = 0, \quad (16)$$

where  $Q = (\rho, \sigma, u, v)^T$ ,

$$B = \begin{pmatrix} u & 0 & \rho & 0 \\ 0 & v & 0 & \sigma \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{P_{\bar{\rho}}}{\bar{\rho}} & u - \frac{\sigma w}{\bar{\rho}} & \frac{\sigma w}{\bar{\rho}} \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{\rho w}{\bar{\rho}} & v - \frac{\rho w}{\bar{\rho}} \end{pmatrix}. \quad (17)$$

From the equation  $\|B - \lambda E\| = 0$  for the eigenvalues of the matrix  $B$  we have

$$\begin{aligned} \|B - \lambda E\| &= \begin{vmatrix} v - \lambda & 0 & \rho & 0 \\ 0 & \tilde{v} - \lambda & 0 & \tilde{\rho} \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{P_{\bar{\rho}}}{\bar{\rho}} & v - \frac{\tilde{\rho} w}{\bar{\rho}} - \lambda & \frac{\tilde{\rho} w}{\bar{\rho}} \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{\rho w}{\bar{\rho}} & \tilde{v} - \frac{\rho w}{\bar{\rho}} - \lambda \end{vmatrix} = \\ &= (-1)^{2+2} (\tilde{v} - \lambda) \begin{vmatrix} v - \lambda & \rho & 0 \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & v - \frac{\tilde{\rho} w}{\bar{\rho}} - \lambda & \frac{\tilde{\rho} w}{\bar{\rho}} \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{\rho w}{\bar{\rho}} & \tilde{v} - \frac{\rho w}{\bar{\rho}} - \lambda \end{vmatrix} + \\ &\quad + (-1)^{2+4} \tilde{\rho} \begin{vmatrix} v - \lambda & 0 & \rho \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{P_{\bar{\rho}}}{\bar{\rho}} & v - \frac{\tilde{\rho} w}{\bar{\rho}} - \lambda \\ \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{P_{\bar{\rho}}}{\bar{\rho}} & \frac{\rho w}{\bar{\rho}} \end{vmatrix} = \\ &= (\tilde{v} - \lambda) [(v - \lambda)(v - \bar{\rho}^{-1} \tilde{\rho} w - \lambda)(\tilde{v} - \bar{\rho}^{-1} \rho w - \lambda) + \bar{\rho}^{-2} \rho \tilde{\rho} P_{\bar{\rho}} w - \end{aligned}$$

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$$\begin{aligned}
 & -\bar{\rho}^{-1}\rho P_{\bar{\rho}}(\tilde{v} - \bar{\rho}^{-1}\rho w - \lambda) - (v - \lambda)\bar{\rho}^{-2}\rho\tilde{\rho}w^2] + \\
 & +\tilde{\rho}[(v - \lambda)\bar{\rho}^{-2}\rho P_{\bar{\rho}}w + \bar{\rho}^{-2}\rho P_{\bar{\rho}}^2 - \bar{\rho}^{-2}\rho P_{\bar{\rho}}^2 - \bar{\rho}^{-1}P_{\bar{\rho}}(v - \lambda)(v - \bar{\rho}^{-1}\tilde{\rho}w - \lambda)] = \\
 & = (\tilde{v} - \lambda)[(v - \lambda)(v - \bar{\rho}^{-1}\tilde{\rho}w - \lambda)(\tilde{v} - \bar{\rho}^{-1}\rho w - \lambda + v - v) + \bar{\rho}^{-2}\rho\tilde{\rho}P_{\bar{\rho}}w - \\
 & -\bar{\rho}^{-1}\rho P_{\bar{\rho}}(\tilde{v} - \bar{\rho}^{-1}\rho w - \lambda + v - v) - (v - \lambda)\bar{\rho}^{-2}\rho\tilde{\rho}w^2] + \\
 & +\bar{\rho}^{-1}\tilde{\rho}P_{\bar{\rho}}(v - \lambda)(\bar{\rho}^{-1}\rho w - v + \bar{\rho}^{-1}\tilde{\rho}w + \lambda) = \\
 & = (\tilde{v} - \lambda)[(v - \lambda)\{(v - \bar{\rho}^{-1}\tilde{\rho}w - \lambda)(v - \lambda - w(1 + \bar{\rho}^{-1}\rho)) - \bar{\rho}^{-2}\rho\tilde{\rho}w^2 - \\
 & -\bar{\rho}^{-1}\tilde{\rho}P_{\bar{\rho}}\} + \bar{\rho}^{-2}\rho\tilde{\rho}P_{\bar{\rho}}w - \bar{\rho}^{-1}\rho P_{\bar{\rho}}(v - \lambda) - \bar{\rho}^{-1}\rho P_{\bar{\rho}}(-w(1 + \bar{\rho}^{-1}\rho))] = \\
 & = (\tilde{v} - \lambda)[(v - \lambda)\{(v - \lambda)(v - \lambda - w(1 + \bar{\rho}^{-1}\rho)) - \\
 & -\bar{\rho}^{-1}\tilde{\rho}w(v - \lambda - w(1 + \bar{\rho}^{-1}\rho)) - \bar{\rho}^{-2}\rho\tilde{\rho}w^2 - \\
 & -\bar{\rho}^{-1}\tilde{\rho}P_{\bar{\rho}} - \bar{\rho}^{-1}\rho P_{\bar{\rho}}\} + \bar{\rho}^{-1}\rho P_{\bar{\rho}}(\bar{\rho}^{-1}\tilde{\rho}w + w + \bar{\rho}^{-1}\rho w)] = \\
 & = (\tilde{v} - \lambda)[(v - \lambda)\{(v - \lambda)^2 - w(1 + \bar{\rho}^{-1}\rho)(v - \lambda) - \\
 & -\bar{\rho}^{-1}\tilde{\rho}w(v - \lambda) + \bar{\rho}^{-1}\tilde{\rho}w^2(1 + \bar{\rho}^{-1}\rho) - \bar{\rho}^{-2}\rho\tilde{\rho}w^2 - P_{\bar{\rho}}\} + 2\bar{\rho}^{-1}\rho P_{\bar{\rho}}w] = \\
 & = (\tilde{v} - \lambda)[(v - \lambda)\{(v - \lambda)^2 - 2w(v - \lambda) + \bar{\rho}^{-1}\tilde{\rho}w^2 - P_{\bar{\rho}}\} + 2\bar{\rho}^{-1}\rho P_{\bar{\rho}}w] = 0.
 \end{aligned}$$

Thus, one of the eigenvalue of the matrix  $B$  is  $\lambda_1 = v$  and the remaining eigenvalues satisfy the cubic equation

$$(u - \lambda)^3 - 2w(u - \lambda)^2 + (u - \lambda)(\bar{\rho}^{-1}\sigma w^2 - P_{\bar{\rho}}) + 2\bar{\rho}^{-1}\rho P_{\bar{\rho}}w = 0. \quad (18)$$

Multiplying the resulting equation by  $\frac{1}{(\sqrt{P_{\bar{\rho}}})^3}$ , we get

$$\frac{(u-\lambda)^3}{(\sqrt{P_{\bar{\rho}}})^3} - \frac{2w(u-\lambda)^2}{\sqrt{P_{\bar{\rho}}}(\sqrt{P_{\bar{\rho}}})^2} + \frac{(u-\lambda)(\bar{\rho}^{-1}\sigma w^2 - P_{\bar{\rho}})}{\sqrt{P_{\bar{\rho}}}(\sqrt{P_{\bar{\rho}}})^2} + \frac{2\bar{\rho}^{-1}\rho P_{\bar{\rho}}w}{\sqrt{P_{\bar{\rho}}}(\sqrt{P_{\bar{\rho}}})^2} = 0.$$

Changing the variables in this equation by the formulas

$$z = \frac{u-\lambda}{\sqrt{P_{\bar{\rho}}}}, \alpha = \frac{w}{\sqrt{P_{\bar{\rho}}}} \quad (19)$$

we have

$$f(z) \equiv z^3 - 2\alpha z^2 - z(1 - \bar{\rho}^{-1}\sigma\alpha^2) + 2\bar{\rho}^{-1}\rho\alpha = 0. \quad (20)$$

Figure 1 shows the  $f(z)$  graph of the function for values  $\frac{u}{\sqrt{P_{\bar{\rho}}}} = 0.1, \frac{v}{\sqrt{P_{\bar{\rho}}}} = 0.3, \frac{\rho}{\bar{\rho}} = 0.2$  and  $\frac{\sigma}{\bar{\rho}} = 0.8$ . We can see that the function  $f(z)$  has 3 real roots.

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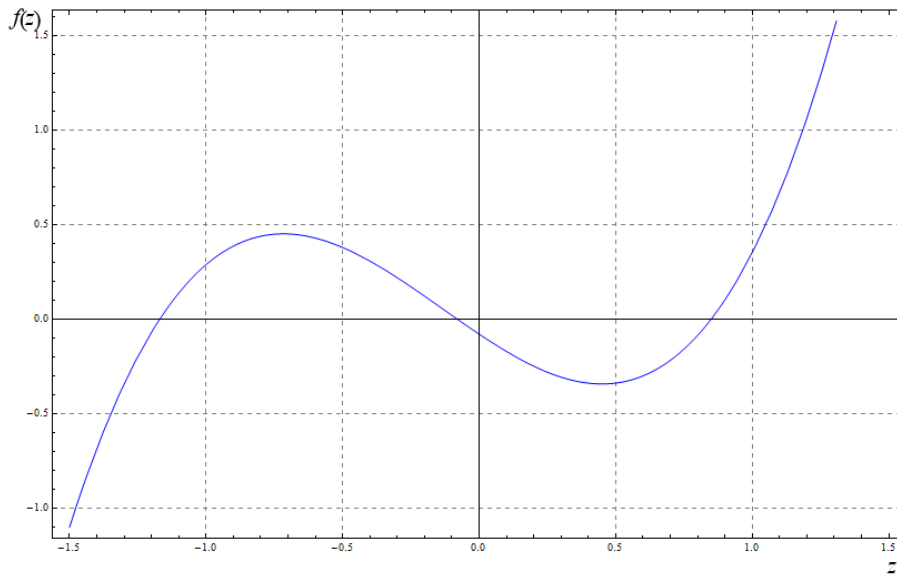


Fig.1: The graph of the function  $f(z)$

For small values of  $\alpha$ , we find the roots of the equation  $f(z) = 0$  by expanding  $z(\alpha)$  in a series in powers of  $\alpha$ .  
Let

$$z(\alpha) = z_0 + \alpha\beta. \tag{21}$$

Substituting expression (21) into equation (20), assuming  $\alpha$  is sufficiently small, we find the following equation

$$\alpha^0(z_0^3 - z_0) + \alpha^1(3z_0^2\beta - 2z_0^2 - \beta + 2\bar{\rho}^{-1}\rho) = 0.$$

The identity is achieved if the coefficients of the powers  $\alpha$  are equal to zero. The coefficient at  $\alpha^0$  is zero for the following values

$$z_{0,1} = 0, z_{0,2} = 1, z_{0,3} = -1.$$

Accordingly, we obtain the conditions for the coefficient of  $\alpha^1$  to be zero

$$\beta_1 = 2\bar{\rho}^{-1}\rho, \beta_2 = \beta_3 = 1 - \bar{\rho}^{-1}\rho.$$

Substituting these values in (21) we get

$$z_1(\alpha) = 2\bar{\rho}^{-1}\rho\alpha, z_2(\alpha) = 1 + \bar{\rho}^{-1}\sigma\alpha, z_3(\alpha) = -1 + \bar{\rho}^{-1}\sigma\alpha.$$

From (19) it follows that

$$\lambda = u - z\sqrt{P_{\bar{\rho}}}.$$

So we have got

$$\lambda_1 = v, \lambda_2 = u - 2\bar{\rho}^{-1}\rho w,$$

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$$\lambda_3 = u - \sqrt{P_{\bar{\rho}}} - \bar{\rho}^{-1}\sigma w, \lambda_4 = u + \sqrt{P_{\bar{\rho}}} - \bar{\rho}^{-1}\sigma w.$$

The eigenvalues of the matrix  $B$  up to a replacement accuracy, provided

$$P = P(\bar{\rho}),$$

are the eigenvalues of the matrix  $A$ , corresponding to the matrix representation of equations (1) - (4) for the more general equation of state. In terms of divergent variables, the eigenvalues are represented as

$$\lambda_1 = \frac{j-\rho w}{\bar{\rho}}, \lambda_2 = \frac{j+\sigma w-2\rho w}{\bar{\rho}},$$

$$\lambda_3 = \frac{j}{\bar{\rho}} - \sqrt{P_{\bar{\rho}}}, \lambda_4 = \frac{j}{\bar{\rho}} + \sqrt{P_{\bar{\rho}}}.$$

## IV. NUMERICAL SOLUTION OF THE SYSTEM OF EQUATIONS

The calculations performed in the preceding paragraph allow us to apply for the system of equations (13) the HLL-method [9-13]

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0;$$

$$U_i^{n+1} = U_i^n - \frac{\tau}{h} (\mathcal{F}_{i+1/2}^n - \mathcal{F}_{i-1/2}^n) = 0, i = \overline{0, I}, n = \overline{0, N};$$

$$\mathcal{F}_{i+1/2}^n = \frac{M_{i+1}^n F_i^n - m_i^n F_{i+1}^n + M_{i+1}^n m_i^n (U_{i+1}^n - U_i^n)}{M_{i+1}^n - m_i^n};$$

$$\mathcal{F}_{i-1/2}^n = \frac{M_i^n F_{i-1}^n - m_{i-1}^n F_i^n + M_i^n m_{i-1}^n (U_i^n - U_{i-1}^n)}{M_i^n - m_{i-1}^n};$$

$$M_{i+1}^n = \max\left(\lambda_{k, i+\frac{1}{2}}^n, 0\right), k = \overline{1, 4};$$

$$m_i^n = \min\left(\lambda_{k, i+\frac{1}{2}}^n, 0\right), k = \overline{1, 4}.$$

Consider the case when the equation of state has the following form

$$P = \bar{\rho}^3.$$

Then, respectively, the following formulas hold

$$P_{\bar{\rho}} = \frac{\partial}{\partial \bar{\rho}} (\bar{\rho}^3) = 3\bar{\rho}^2, \quad \sqrt{P_{\bar{\rho}}} = \bar{\rho}\sqrt{3}.$$

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The calculation of eigenvalues is made according to the following formulas

$$\lambda_{1,i+\frac{1}{2}}^n = \frac{j_{i+1/2}^n - \rho_{i+1/2}^n w_{i+1/2}^n}{\rho_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n},$$

$$\lambda_{2,i+\frac{1}{2}}^n = \frac{j_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n w_{i+1/2}^n - 2\rho_{i+1/2}^n w_{i+1/2}^n}{\rho_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n},$$

$$\lambda_{3,i+\frac{1}{2}}^n = \frac{j_{i+1/2}^n - \sqrt{3}(\rho_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n)^2}{\rho_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n},$$

$$\lambda_{4,i+\frac{1}{2}}^n = \frac{j_{i+1/2}^n + \sqrt{3}(\rho_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n)^2}{\rho_{i+1/2}^n + \tilde{\rho}_{i+1/2}^n},$$

where

$$\rho_{i+1/2}^n = \frac{\rho_i^n + \rho_{i+1}^n}{2}, \tilde{\rho}_{1/2}^n = \frac{\tilde{\rho}_i^n + \tilde{\rho}_{i+1}^n}{2},$$

$$w_{1/2}^n = \frac{w_i^n + w_{i+1}^n}{2}, j_{1/2}^n = \frac{j_i^n + j_{i+1}^n}{2}.$$

For  $i = 0$  we have

$$U_0^{n+1} = U_0^n - \frac{\tau}{h} (\mathcal{F}_{1/2}^n - \mathcal{F}_{-1/2}^n) = 0,$$

$$\mathcal{F}_{1/2}^n = \frac{M_1^n F_0^n - m_0^n F_1^n + M_1^n m_0^n (U_1^n - U_0^n)}{M_1^n - m_0^n},$$

$$\mathcal{F}_{i-1/2}^n = \frac{M_0^n F_{i-1}^n - m_{i-1}^n F_i^n + M_0^n m_{i-1}^n (U_i^n - U_{i-1}^n)}{M_0^n - m_{i-1}^n}.$$

Redenote the index  $-1$  as  $0$ . In this case

$$\mathcal{F}_{i-1/2}^n = F_0^n,$$

$$U_0^{n+1} = U_0^n - \frac{\tau}{h} (\mathcal{F}_{1/2}^n - F_0^n) = 0.$$

If  $i = I$  we have

$$U_I^{n+1} = U_I^n - \frac{\tau}{h} (\mathcal{F}_{I+1/2}^n - \mathcal{F}_{I-1/2}^n) = 0,$$

$$\mathcal{F}_{I+1/2}^n = \frac{M_{I+1}^n F_I^n - m_I^n F_{I+1}^n + M_{I+1}^n m_I^n (U_{I+1}^n - U_I^n)}{M_{I+1}^n - m_I^n},$$

$$\mathcal{F}_{I-1/2}^n = \frac{M_I^n F_{I-1}^n - m_{I-1}^n F_I^n + M_I^n m_{I-1}^n (U_I^n - U_{I-1}^n)}{M_I^n - m_{I-1}^n}.$$

Reassign the index  $I + 1$  as  $I$ . Then

$$\mathcal{F}_{I+1/2}^n = F_I^n,$$



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$$U_I^{n+1} = U_I^n - \frac{\tau}{h} (F_I^n - \mathcal{F}_{I-\frac{1}{2}}^n) = 0,$$

$$\rho_{I+1/2}^n = \rho_I^n, \tilde{\rho}_{I+1/2}^n = \tilde{\rho}_I^n,$$

$$w_{I+1/2}^n = w_I^n, j_{I+1/2}^n = j_I^n.$$

It is easy to verify that for constant values of densities and velocities, the application of this method will keep the values constant. This fact was verified on a test calculation.

A numerical calculation was made for the following initial data

$$I = 21, N = 200, h = 0.1,$$

$$\rho_i^0 = 0.7\bar{\rho}_i^0, \tilde{\rho}_i^0 = 0.3\bar{\rho}_i^0,$$

$$v_i^0 = 0, \tilde{v}_i^0 = 0,$$

$$\bar{\rho}_i^0 = \begin{cases} -2(ih - 1)^3 - 3(ih - 1)^2 + 1.1, & i < \frac{I}{2}, \\ 2\left(i - \frac{I}{2}\right)^3 h^3 - 3\left(i - \frac{I}{2}\right)^2 h^2 + 1.1, & i \geq \frac{I}{2}, \end{cases}$$

$$\tau_i^n = 0.25h/M_{I+1}^n.$$

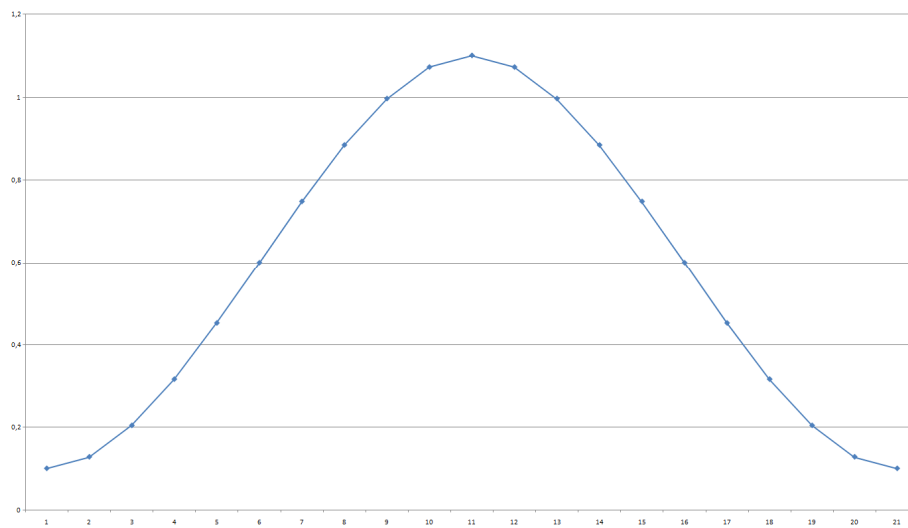


Fig.2: Density  $\bar{\rho}^0$

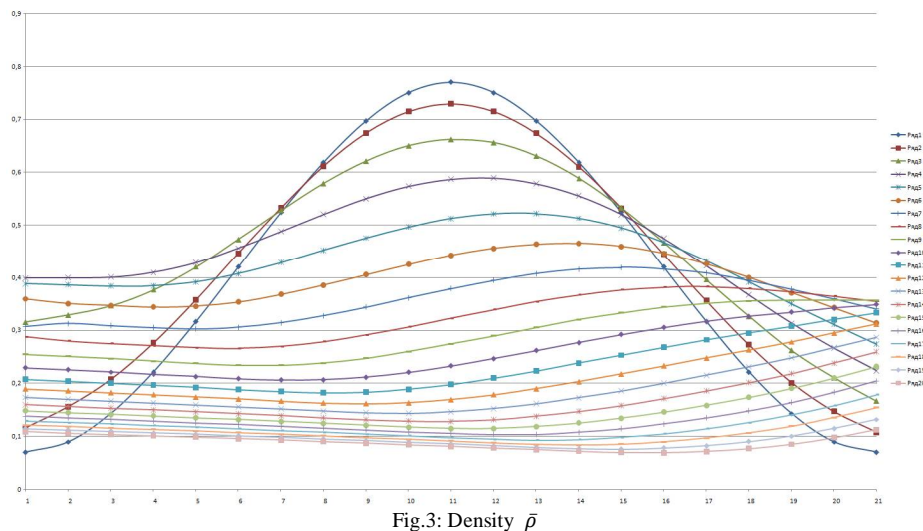
Figure 3 shows a graph representing the dynamics of changes of  $\bar{\rho}$  as a function on one spatial interval.

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## V. CONCLUSION

A symmetrized system of dynamic equations describing the propagation of nonlinear waves in a two-fluid medium without taking into account mass forces in a reversible approximation in the case when the equation of state is a function of the density of a two-velocity continuum is obtained. The spectral analysis of this system is carried out. Using the HLL method, the system of equations under consideration is solved numerically.

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