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New Exponentiated Exponential Extension Distribution with Properties and Applications

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ABSTRACT: In this paper, we have presented a three-parameter univariate continuous distribution known as new exponentiated exponential extension distribution. Some statistical and mathematical distribution properties such as the cumulative distribution function, probability density function and hazard rate function, survival function, quantile function, the skewness, and kurtosis measures. The model parameters of the proposed distribution are estimated with the help of three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods. The goodness of fit of the proposed distribution is also evaluated by fitting it in comparison with some other existing distributions using a real data set.

KEYWORDS: Exponential extension distribution, Hazard function, MLE, LSE, CVM.

I. INTRODUCTION

Lifetime distributions are generally used to study the lifespan of components of a system, and in general, reliability and survival analysis. Lifetime distributions are frequently used in fields like life science, medicine, biology, engineering, insurance, etc. In fields including life science, medicine, genetics, engineering, insurance, etc., lifetime distributions are commonly used. In statistical literature, for evaluating lifetime data, many probability continuous distributions, like exponential, Weibull, gamma have been used. For a few years, most of the researchers are attracted towards the exponential distribution for its potential in modeling lifetime data, and it has been observed that this distribution has performed excellently in many applications due to the existence of closed form solutions to many survival analyses. It can easily be justified under the supposition of constant failure rate but in the practice, the failure rates are not always constant. Hence, haphazard use of exponential lifetime model seems to be inappropriate and unrealistic. In recent years, new classes of models have been introduced based on modifications of the existing classical probability models, Marshall and Olkin(2007)[10]. In recent times, some efforts have been seen to develop alternative distributions to broaden the capacities of well-known distributions while simultaneously providing great versatility in the practice of modeling data. With the incorporation of extra parameters, many procedures may be used to shape an extended family from an existing distribution. Therefore several groups were proposed in the statistical literature by introducing one or more parameters to create new models Rinne(2009) and Pham and Lai(2007)[14][16].

A random variable Y is said to follow exponential distribution with parameter λ if its cumulative probability distribution (CDF) function is given by

$$F_Y(y; \lambda) = 1 - e^{-\lambda y} ; y > 0, \lambda > 0 \quad (1.1)$$

Many generalizations exponential distribution have been found in statistical literatures to generate more flexible lifetime models. Some of them well known generalizations are as follows,

The modification of exponential distribution was introduced by Smith and Bain (1975) called exponential power distribution[17]. The generalized exponential distribution was introduced by (Gupta & Kundu, 1999) which is flexible than exponential distribution, having increasing and decreasing failure rate hazard function[4]. Lan and Leemis (2008) has proposed the logistic-exponential distribution[8]. It has increasing, de-creasing, bathtub (BT)-shaped, and upside-down bathtub(UBT)-shaped failure rates for various values of the parameters. Another generalization of exponential distribution was introduced by Nadarajah and Haghghi (2011) and called it as an extension of exponential distribution[13]. Its density can have decreasing and unimodal shapes, and the hazard rate exhibits increasing and decreasing shapes. Joshi (2015) has proposed another extension of exponential distribution called new extended exponential (EEN) distribution having monotonically increasing and constant hazard rate shapes[5]. The continuous random variable X follows EEN distribution with parameters α and λ if its CDF is given by



$$F(x) = 1 - \exp(-\alpha x e^{-\lambda/x}) ; x > 0, (\alpha, \lambda) > 0 \tag{2.2}$$

Chaudhary and Kumar (2020) have introduced the half logistic exponential extension distribution using exponential extension distribution as a base distribution [1]. Chaudhary et al. (2020) have introduced an extension of exponential distribution named truncated Cauchy power exponential distribution [2]. Following the similar approach as used by (Gupta & Kundu, 1999) we have introduced the new distribution named new exponentiated exponential extension (NEEE) distribution [4]. The key objective of this study is to generate a more flexible model by considering just one extra parameter to the EEN distribution to attain a better fit for the lifetime datasets. The rest of the article is arranged as follows. In Section 2 we have introduced the new exponentiated exponential extension (NEEE) distribution and discussed its important mathematical and statistical properties. In Section 3 we present methods of parameter estimation. We have made use of some well-known estimation methods, namely the maximum likelihood estimation (MLE), least-square estimation (LSE) and Cramer-Von-Mises estimation (CVME) methods. For the maximum likelihood (ML) estimate, we have created the asymptotic confidence intervals using the observed information matrix. In Section 4, a real data set has been analyzed to explore the applications and capability of the NEEE distribution. In this section, we present the estimated value of the parameters and log-likelihood, AIC, BIC and AICC criterion for ML, LSE, and CVME methods also the goodness of fit of the proposed distribution is also assessed by fitting it in comparison with some other existing distributions using a real data set. Finally, in Section 5 we present some concluding remarks.

II. THE NEW MODEL

In this section, the three-parameter new exponentiated exponential extension (NEEE) distribution is introduced. This distribution is generated by using new extended exponential distribution defined by (Joshi, 2015) [5]. The CDF of NEEE distribution can be expressed as

$$F(x) = \left\{ 1 - \exp(-\alpha x e^{-\lambda/x}) \right\}^\beta ; \alpha > 0, \beta > 0, \lambda > 0, x > 0 \tag{2.1}$$

The corresponding probability density function (PDF) is given by

$$f(x) = \alpha \beta \left(1 + \frac{\lambda}{x} \right) e^{-\lambda/x} \exp(-\alpha x e^{-\lambda/x}) \left\{ 1 - \exp(-\alpha x e^{-\lambda/x}) \right\}^{\beta-1} \tag{2.2}$$

The survival function of NEEE distribution can be expressed as

$$S(x) = 1 - \left\{ 1 - \exp(-\alpha x e^{-\lambda/x}) \right\}^\beta ; x > 0 \tag{2.3}$$

Similarly, the hazard rate function (HRF) is

$$h(x) = \frac{\alpha \beta \left(1 + \frac{\lambda}{x} \right) e^{-\lambda/x} \exp(-\alpha x e^{-\lambda/x}) \left\{ 1 - \exp(-\alpha x e^{-\lambda/x}) \right\}^{\beta-1}}{1 - \left\{ 1 - \exp(-\alpha x e^{-\lambda/x}) \right\}^\beta} ; x > 0 \tag{2.4}$$

2.1 Quantile and Generating Functions

Quantile Function

The quantile function, related with the probability distribution of a random variable, defines the value of the random variable in probability theory and statistics in such a way that the probability of the variable being less than or equal to that value equals the probability given. The definition of the v^{th} quantile is the real solution of the following equation

$$Q(v) = F^{-1}(v)$$

the quantile function is obtained by inverting CDF (2.1) as

$$\frac{\lambda}{x} - \ln x + \ln \left\{ -\frac{1}{\alpha} \ln(1 - v^{1/\beta}) \right\} = 0 ; 0 < v < 1 \tag{2.5}$$

Generation of the random numbers:

For the random numbers generation of the NEEE distribution, we consider simulating values of random variable X with the CDF (2.1). Let W denote a uniform random variable in (0, 1), then the simulated values of X are obtained by



$$\frac{\lambda}{x} - \ln x + \ln \left\{ -\frac{1}{\alpha} \ln(1 - w^{1/\beta}) \right\} = 0 ; 0 < w < 1 \tag{2.6}$$

Skewness and Kurtosis:

The measures of Skewness based on quantiles is Bowley's coefficient of skewness and it can be written as

$$S_k (Bowley) = \frac{Q(3/4) + Q(1/4) - 2Q(1/2)}{Q(3/4) - Q(1/4)}, \text{ and} \tag{2.7}$$

The coefficient of kurtosis based on octiles was defined by (Moors, 1988)[12] which can be expressed as

$$K_u (M) = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(3/4) - Q(1/4)}, \tag{2.8}$$

Plots of probability density function and hazard rate function of NEEE(α, β, λ) with different values of parameters are shown in Figure 1.

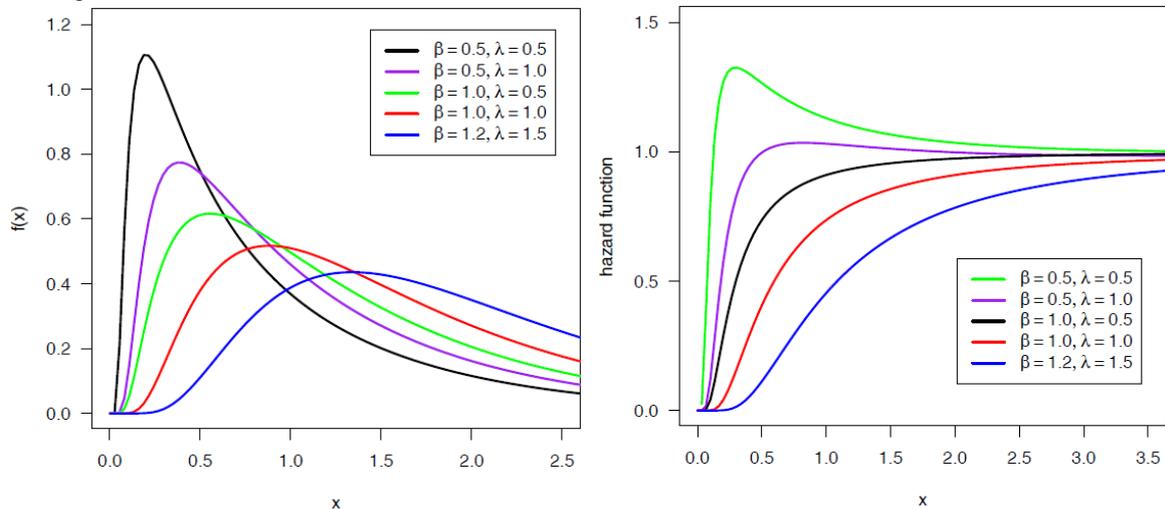


Figure 1. Plots of density function (left panel) and hazard function (right panel) for different values of $\alpha, \beta,$ and λ .

III. METHODS OF PARAMETER ESTIMATION

3.1 Maximum Likelihood Estimation (MLE)

The theory of estimation is a statistics discipline that deals with the determination of parameter values premised on calculated empirical data having a random element. An underlying physical structure is portrayed by the parameters such that their value affects the distribution of the measured data. With the help of measurements, an estimator tries to calculate the unknown parameters. In this study, we have discussed the well-known method maximum likelihood method of estimation.

In this section, we have discussed the maximum likelihood estimators (MLE's) of the NEEE distribution. Let $\underline{x} = (x_1, \dots, x_n)$ be a random sample of size 'n' from NEEE(α, β, λ) then the log likelihood function $l(\alpha, \beta, \lambda / \underline{x})$ can be written as,

$$\begin{aligned} l(\alpha, \beta, \lambda | \underline{x}) = & n \ln \alpha + n \ln \beta + \sum_{i=1}^n \ln \left(1 + \frac{\lambda}{x_i} \right) - \lambda \sum_{i=1}^n (1/x_i) \\ & - \alpha \sum_{i=1}^n x_i e^{-\lambda/x_i} + (\beta - 1) \sum_{i=1}^n \ln \left[1 - \exp(-\alpha x_i e^{-\lambda/x_i}) \right] \end{aligned} \tag{3.1}$$

By differentiating (3.1) with respect to unknown parameters α, β and λ , we get



$$\begin{aligned} \frac{\partial \ell}{\partial \alpha} &= \frac{n}{\alpha} - \sum_{i=1}^n x_i e^{-\lambda/x_i} + (\beta - 1) \sum_{i=1}^n \frac{x_i e^{-\lambda/x_i} \exp(-\alpha x_i e^{-\lambda/x_i})}{1 - \exp(-\alpha x_i e^{-\lambda/x_i})} \\ \frac{\partial \ell}{\partial \beta} &= \frac{n}{\beta} + \sum_{i=1}^n \ln [1 - \exp(-\alpha x_i e^{-\lambda/x_i})] \\ \frac{\partial \ell}{\partial \lambda} &= \sum_{i=1}^n \frac{1}{(x_i + \lambda)} - \sum_{i=1}^n (1/x_i) + \alpha \sum_{i=1}^n e^{-\lambda/x_i} - \alpha (\beta - 1) \sum_{i=1}^n \frac{e^{-\lambda/x_i} \exp(-\alpha x_i e^{-\lambda/x_i})}{1 - \exp(-\alpha x_i e^{-\lambda/x_i})} \end{aligned} \tag{3.2}$$

After equating these non-linear equations to zero and solving for the unknown parameters (α, β, λ) we will obtain the ML estimators of the NEEE distribution. Manually, it is difficult to solve (3.2) hence using appropriate computer software one can solve these equations. Let us denote the parameter vector by $\underline{\Theta} = (\alpha, \beta, \lambda)$ and the corresponding MLE of $\underline{\Theta}$ as $\underline{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then the asymptotic normality results in, $(\underline{\Theta} - \underline{\Theta}) \rightarrow N_3 [0, (I(\underline{\Theta}))^{-1}]$ where

$I(\underline{\Theta})$ is the Fisher's information matrix given by,

$$I(\underline{\Theta}) = - \begin{pmatrix} E\left(\frac{\partial^2 l}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \lambda}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \theta}\right) \\ E\left(\frac{\partial^2 l}{\partial \lambda \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \lambda^2}\right) & E\left(\frac{\partial^2 l}{\partial \lambda \partial \theta}\right) \\ E\left(\frac{\partial^2 l}{\partial \alpha \partial \theta}\right) & E\left(\frac{\partial^2 l}{\partial \lambda \partial \theta}\right) & E\left(\frac{\partial^2 l}{\partial \theta^2}\right) \end{pmatrix}$$

In practice, we don't know $\underline{\Theta}$ hence it is useless that the MLE has an asymptotic variance $(I(\underline{\Theta}))^{-1}$. Hence we approximate the asymptotic variance by plugging in the estimated value of the parameters. The observed fisher information matrix $O(\underline{\Theta})$ is used as an estimate of the information matrix $I(\underline{\Theta})$ given by

$$O(\underline{\Theta}) = - \begin{pmatrix} \frac{\partial^2 l}{\partial \alpha^2} & \frac{\partial^2 l}{\partial \alpha \partial \hat{\beta}} & \frac{\partial^2 l}{\partial \alpha \partial \hat{\lambda}} \\ \frac{\partial^2 l}{\partial \alpha \partial \hat{\beta}} & \frac{\partial^2 l}{\partial \hat{\beta}^2} & \frac{\partial^2 l}{\partial \hat{\beta} \partial \hat{\lambda}} \\ \frac{\partial^2 l}{\partial \alpha \partial \hat{\lambda}} & \frac{\partial^2 l}{\partial \hat{\beta} \partial \hat{\lambda}} & \frac{\partial^2 l}{\partial \hat{\lambda}^2} \end{pmatrix}_{(\hat{\alpha}, \hat{\beta}, \hat{\lambda})} = -H(\underline{\Theta})_{(\underline{\Theta}=\underline{\Theta})}$$

where H is the Hessian matrix.

The Newton-Raphson algorithm to maximize the likelihood produces the observed information matrix. Therefore, the variance-covariance matrix is given by,

$$\left[-H(\underline{\Theta})_{(\underline{\Theta}=\underline{\Theta})} \right]^{-1} = \begin{pmatrix} \text{var}(\hat{\alpha}) & \text{cov}(\hat{\alpha}, \hat{\beta}) & \text{cov}(\hat{\alpha}, \hat{\lambda}) \\ \text{cov}(\hat{\alpha}, \hat{\beta}) & \text{var}(\hat{\beta}) & \text{cov}(\hat{\lambda}, \hat{\beta}) \\ \text{cov}(\hat{\alpha}, \hat{\lambda}) & \text{cov}(\hat{\lambda}, \hat{\beta}) & \text{var}(\hat{\lambda}) \end{pmatrix} \tag{3.3}$$

Hence from the asymptotic normality of MLEs, approximate $100(1-\alpha)\%$ confidence intervals for α, β, λ can be constructed as,

$$\hat{\alpha} \pm Z_{\alpha/2} \sqrt{\text{var}(\hat{\alpha})}, \hat{\beta} \pm Z_{\alpha/2} \sqrt{\text{var}(\hat{\beta})} \text{ and } \hat{\lambda} \pm Z_{\alpha/2} \sqrt{\text{var}(\hat{\lambda})},$$

where $Z_{\alpha/2}$ is the upper percentile of standard normal variate.

3.2. Method of Least-Square Estimation (LSE)

Swain et al. (1988) have introduced the ordinary least square estimators and weighted least square estimators to estimate the parameters of Beta distributions[18]. Here we apply the same technique for the estimation of the parameters of the NEEE distribution. The least-square estimators of the unknown parameters α , β and λ of NEEE distribution can be calculated by minimizing

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n \left[F(X_i) - \frac{i}{n+1} \right]^2 \tag{3.2.1}$$

with respect to unknown parameters α , β and λ .

Consider $F(X_i)$ represent the distribution function of the ordered random variables $X_{(1)} < X_{(2)} < \dots < X_{(n)}$ where $\{X_1, X_2, \dots, X_n\}$ is a random sample of size n from a distribution function $F(\cdot)$. The least-square estimators of α , β and λ say $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\lambda}$ respectively, can be obtained by minimizing

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n \left[\left\{ 1 - \exp(-\alpha x e^{-\lambda/x}) \right\}^\beta - \frac{i}{n+1} \right]^2; x \geq 0, (\alpha, \beta, \lambda) > 0. \tag{3.2.2}$$

with respect to α , β and λ .

Differentiating (3.2.2) with respect to α , β and λ we get,

$$\begin{aligned} \frac{\partial M}{\partial \alpha} &= -2\beta \sum_{i=1}^n \left[\left\{ A_i(x) \right\}^\beta - \frac{i}{n+1} \right] x_i e^{-\lambda/x_i} \left\{ A_i(x) \right\}^{\beta-1} \exp(-\alpha x_i e^{-\lambda/x_i}) \\ \frac{\partial M}{\partial \beta} &= -2 \sum_{i=1}^n \left[\left\{ A_i(x) \right\}^\beta - \frac{i}{n+1} \right] \left\{ A_i(x) \right\}^\beta \ln A_i(x) \\ \frac{\partial M}{\partial \lambda} &= 2\alpha \beta \sum_{i=1}^n x_i^\beta \left[\left\{ A_i(x) \right\}^\beta - \frac{i}{n+1} \right] e^{-\lambda/x_i} \left\{ A_i(x) \right\}^{\beta-1} \exp(-\alpha x_i e^{-\lambda/x_i}) \end{aligned}$$

Where $A_i(x) = 1 - \exp(-\alpha x_i e^{-\lambda/x_i})$

Similarly the weighted least square estimators can be obtained by minimizing

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n w_i \left[F(X_{(i)}) - \frac{i}{n+1} \right]^2$$

with respect to α , β and λ . The weights w_i are $w_i = \frac{1}{Var(X_{(i)})} = \frac{(n+1)^2 (n+2)}{i(n-i+1)}$

Hence, the weighted least square estimators of α , β and λ respectively can be obtained by minimizing,

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n \frac{(n+1)^2 (n+2)}{i(n-i+1)} \left[\left\{ 1 - \exp(-\alpha x_i e^{-\lambda/x_i}) \right\}^\beta - \frac{i}{n+1} \right]^2 \tag{3.2.3}$$

with respect to α , β and λ .

3.4. Method of Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimators of α , β and λ are obtained by minimizing the function



$$\begin{aligned}
 C(X; \alpha, \beta, \lambda) &= \frac{1}{12n} + \sum_{i=1}^n \left[F(x_{i:n} | \alpha, \beta, \lambda) - \frac{2i-1}{2n} \right]^2 \\
 &= \frac{1}{12n} + \sum_{i=1}^n \left[\left\{ 1 - \exp(-\alpha x_i e^{-\lambda/x_i}) \right\}^\beta - \frac{2i-1}{2n} \right]^2
 \end{aligned}
 \tag{3.4.1}$$

Differentiating (3.4.1) with respect to α , β and λ we get,

$$\begin{aligned}
 \frac{\partial C}{\partial \alpha} &= -2\beta \sum_{i=1}^n \left[\left\{ A_i(x) \right\}^\beta - \frac{2i-1}{2n} \right] x_i e^{-\lambda/x_i} \left\{ A_i(x) \right\}^{\beta-1} \exp(-\alpha x_i e^{-\lambda/x_i}) \\
 \frac{\partial C}{\partial \beta} &= -2 \sum_{i=1}^n \left[\left\{ A_i(x) \right\}^\beta - \frac{2i-1}{2n} \right] \left\{ A_i(x) \right\}^\beta \ln A_i(x) \\
 \frac{\partial C}{\partial \lambda} &= 2\alpha\beta \sum_{i=1}^n x_i^\beta \left[\left\{ A_i(x) \right\}^\beta - \frac{2i-1}{2n} \right] e^{-\lambda/x_i} \left\{ A_i(x) \right\}^{\beta-1} \exp(-\alpha x_i e^{-\lambda/x_i})
 \end{aligned}$$

Where $A_i(x) = 1 - \exp(-\alpha x_i e^{-\lambda/x_i})$

By solving $\frac{\partial C}{\partial \alpha} = 0, \frac{\partial C}{\partial \beta} = 0$ and $\frac{\partial C}{\partial \lambda} = 0$ simultaneously we will get the CVM estimators.

IV. APPLICATION TO REAL DATASET

In this section, we demonstrate the applicability of the NEEE distribution by using a real dataset used by earlier researchers. The data represent the waiting times (in minute) of 100 bank customers (Ghitany et al., 2008)[3]. The data are presented below,

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5

The MLEs are calculated by utilizing the optim() function in R software (R Core Team, 2020) and (Ming, 2019) by maximizing the likelihood function (3.1)[11][15]. We have obtained $\hat{\alpha} = 0.1529, \hat{\beta} = 1.8552, \hat{\lambda} = 0.4699$ and corresponding Log-Likelihood value is $l = -316.9109$. We have presented the MLE's with their standard errors (SE) and 95% confidence interval for α, β , and λ in Table 1.

Table 1
MLE and SE and 95% confidence interval for α, β and λ

Parameter	MLE	SE	95% ACI
alpha	0.1529	0.0206	(0.1125, 0.1933)
beta	1.8552	0.6278	(0.6247, 3.0857)
lambda	0.4699	0.0488	(0.3743, 0.5655)

In Figure 2 we have plotted the Q-Q plot and P-P plot and it is seen that the proposed distribution fits the data very well.

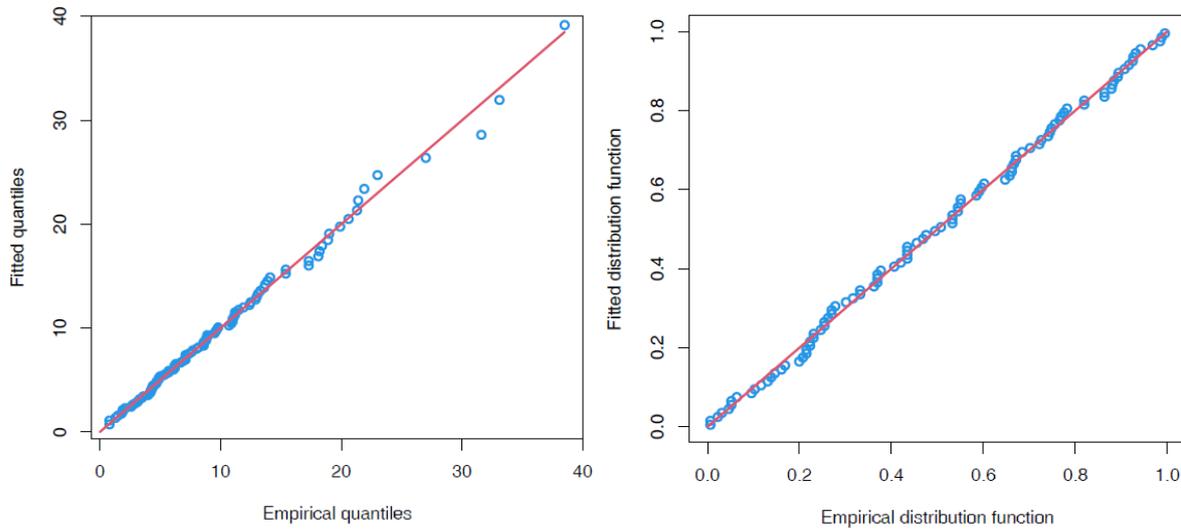


Figure 2. The Q-Q plot (left panel) and P-P plot (right panel) of the NEEE distribution.

We have displayed the graph of the profile log-likelihood function of α , β , and λ in Fig. 3 (Kumar & Ligges, 2011) and observed that the MLEs are unique [6].

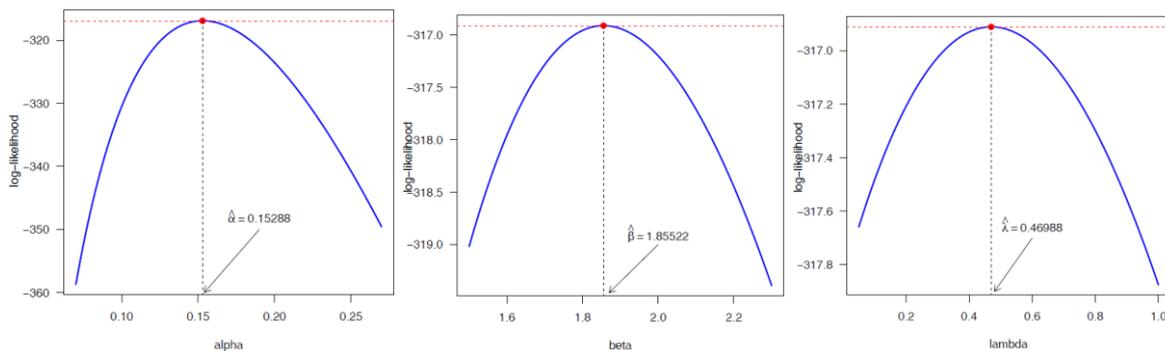


Figure 3. Graph of profile log-likelihood function of α , β , and λ .

In Table 2 we have presented the estimated value of the parameters of NEEE distribution using MLE, LSE and CVE method and their corresponding negative log-likelihood, and AIC criterion.

Table 2
Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC
MLE	0.15288	1.85522	0.46988	316.9109	639.8217
LSE	0.14640	1.52362	1.25103	317.1302	640.2604
CVE	0.14880	1.51414	1.41219	317.2664	640.5328

In Table 3 we have presented The KS, W and A^2 statistics with their corresponding p-value of MLE, LSE and CVE estimates.

Table 3
The KS, W and A^2 statistics with a p-value

Method of Estimation	KS(p-value)	W(p-value)	A^2 (p-value)
MLE	0.0395(0.9977)	0.0178(0.9987)	0.1347(0.9994)
LSE	0.0406(0.9966)	0.0176(0.9988)	0.1550(0.9982)
CVE	0.0368(0.9993)	0.0169(0.9991)	0.1654(0.9971)

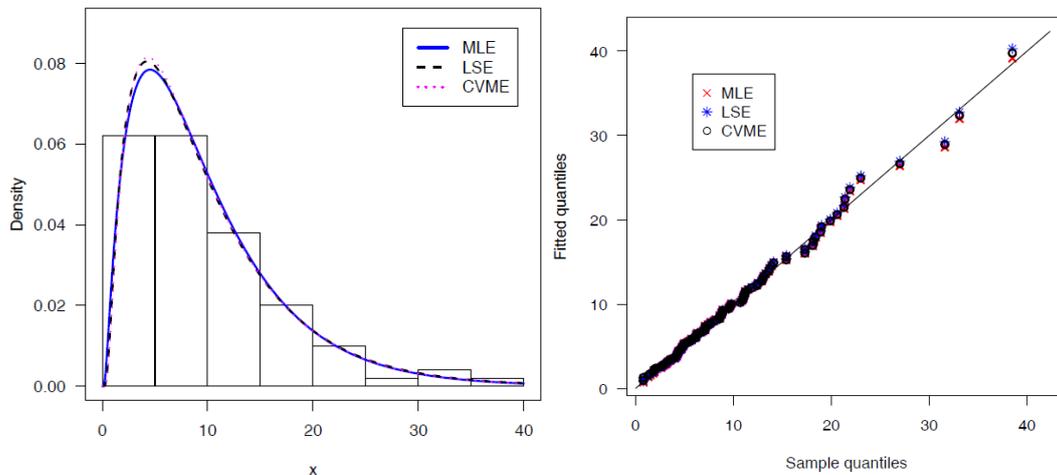


Figure 4. The Histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM (left panel) and fitted quantiles (right panel).

To illustrate the goodness of fit of the NEEE distribution, we have selected some well known distribution for comparison purpose which are listed below,

A. Generalized Exponential Extension (GEE) distribution:

The probability density function of GEE introduced by (Lemonte, 2013)[9] having upside down bathtub-shaped hazard function distribution with parameters α, β and λ is

$$f_{GEE}(x; \alpha, \beta, \lambda) = \alpha\beta\lambda (1 + \lambda x)^{\alpha-1} \exp\{1 - (1 + \lambda x)^\alpha\} \left[1 - \exp\{1 - (1 + \lambda x)^\alpha\}\right]^{\beta-1}; x \geq 0.$$

B. Generalized Rayleigh distribution

The probability density function of Generalized Rayleigh (GR) distribution (Kundu & Raqab, 2005)[7] is

$$f_{GR}(x; \alpha, \lambda) = 2\alpha\lambda^2 x e^{-(\lambda x)^2} \left\{1 - e^{-(\lambda x)^2}\right\}^{\alpha-1}; (\alpha, \lambda) > 0, x > 0$$

Here α and λ are the shape and scale parameters respectively.

C. Exponential Extension (EE) distribution

The density of exponential extension (EE) distribution (Nadarajah & Haghghi, 2011)[13] with parameters α and λ is

$$f_{EE}(x) = \alpha\lambda (1 + \lambda x)^{\alpha-1} \exp\{1 - (1 + \lambda x)^\alpha\}; x \geq 0, \alpha > 0, \lambda > 0.$$

D. Extended Exponential New distribution

The density of EEN distribution (Joshi, 2015)[5] with parameters α and λ is

$$f_{EEN}(x) = \alpha \left(1 + \frac{\lambda}{x}\right) e^{-\lambda/x} \exp(-\alpha x e^{-\lambda/x}); x > 0$$

For the assessment of potentiality of the proposed model we have calculated the Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC) and Hannan-Quinn information criterion (HQIC) which are presented in Table 4.



Table 4
Log-likelihood (LL), AIC, BIC, CAIC and HQIC

Distribution	-LL	AIC	BIC	CAIC	HQIC
EEE	316.9109	639.8217	647.6372	640.0717	642.9848
GEE	317.0364	640.0728	647.8883	640.3228	643.2358
EEN	317.8478	639.6955	644.9058	639.8192	641.8042
GR	321.5182	647.0364	652.2467	647.1601	649.1451
EE	323.4487	650.8973	656.1077	651.0185	653.0060

The Histogram and the density function of fitted distributions and Empirical distribution function with the estimated distribution function of NEEE and some selected distributions are presented in Figure 5.

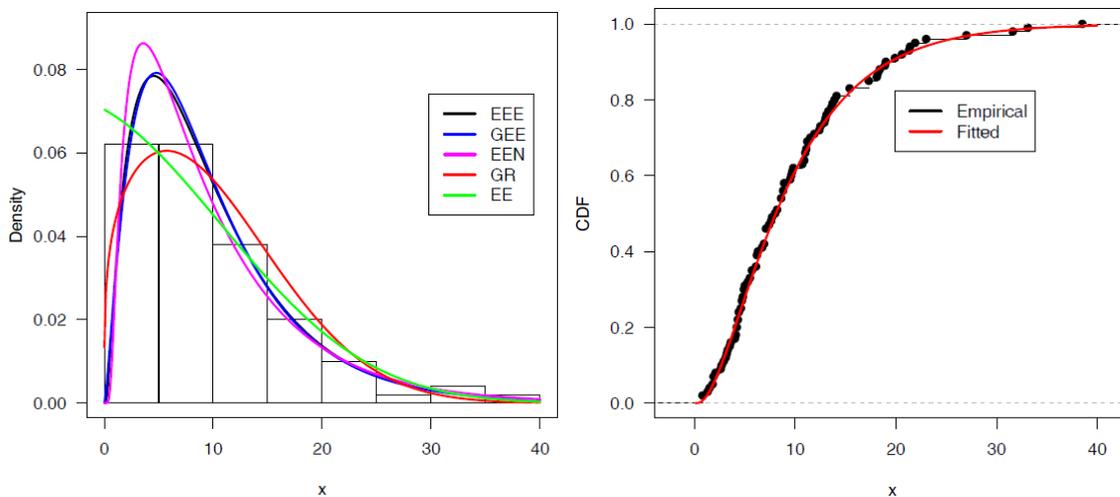


Figure 5. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

To compare the goodness-of-fit of the NEEE distribution with other competing distributions we have presented the value of Kolmogorov-Simnorov (KS), the Anderson-Darling (W) and the Cramer-Von Mises (A^2) statistics in Table 5. It is observed that the NEEE distribution has the minimum value of the test statistic and higher p -value thus we conclude that the NEEE distribution gets quite better fit and more consistent and reliable results from others taken for comparison.

Table 5
The goodness-of-fit statistics and their corresponding p-value

Distribution	KS(p-value)	W(p-value)	A^2 (p-value)
EEE	0.0395(0.9977)	0.0178(0.9987)	0.1347(0.9994)
GEE	0.0366(0.9994)	0.0177(0.9988)	0.1279(0.9996)
EEN	0.0639(0.8093)	0.0520(0.8650)	0.3465(0.8993)
GR	0.0945(0.3337)	0.2043(0.2595)	1.0911(0.3126)
EE	0.1069(0.2028)	0.2096(0.2499)	1.5539(0.1642)

V. CONCLUSION

This article introduced a three-parameter univariate distribution named new exponentiated exponential extension (NEEE) distribution. Some important statistical properties of the NEEE distribution are illustrated such as the shapes of the probability density, cumulative density and hazard rate functions, survival function, reverse hazard rate function. Further quantile function, the skewness, and kurtosis measures are derived and established. We have employed three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods to estimate the model parameters and we concluded that the MLEs

are quite better than LSE, and CVM estimators. A real dataset is considered to explore the applicability and suitability of the proposed distribution and found that the NEEE distribution is quite better than other lifetime model taken into consideration. We hope this distribution may be an alternative in the field of survival analysis, probability theory and applied statistics.

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