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Acceptance Sampling Plan Based on Marshall Olkin Exponential Power Distribution

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ABSTRACT: A reliability test plan is developed by assuming that the life times of the test units follow Marshall Olkin Exponential Power distribution when life test is terminated with a prefixed time .The minimum sample size necessary to ensure the specified average life are obtained and the operating characteristic values of the sampling plans and the associated producer’s risks are given. An example is given to illustrate the methodology.

KEYWORDS: Acceptance sampling, operating characteristic function, Producers risk, Marshall Olkin Exponential Power distribution

1. INTRODUCTION:

Acceptance sampling plans are used to determine the acceptability of a product unit, where the consumer can accept or reject a lot based on a random sample selected from the lot. The process starts by obtaining the minimum sample size necessary to ascertain a certain average life when the life test is terminated at a predetermined time. Such tests are called truncated life tests. Nowadays acceptance sampling plans are an important tool in quality control because they can help manufacturers to minimize variability and safeguard the outgoing quality of their products.

The concept of an acceptance sampling plan based on truncated life tests has been studied by several authors. For example, Sobel and Tischendr of [1959] considered a life test based on an exponential distribution, Gompertz distribution was investigated by Gui and Zhang [2014], Al-Omari [2014] studied a three-parameter kappa distribution, Al-Nasser and Al-Omari [2013] proposed an acceptance sampling plan based on truncated life tests for the Exponentiated Fréchet distribution, Kantam, et al. [2001] considered truncated life tests for a log-logistic distribution; Al-Omari [2016] proposed transmuted inverse Rayleigh distribution and Aslam and Shabaz [2007] considered reliability test plans for a generalized exponential distribution.

The main objective of this paper is to present a new acceptance sampling plan based on truncated life tests following Marshall Olkin Exponential Power distribution. This paper is organized as follows. Section II delivers the basic statistical properties of MOEP distribution. Section III is devoted to the proposed acceptance sampling plan based on MOEP distribution and its properties such as the minimum sample size, the operating characteristic function, and the producer’s risk. Some useful tables and examples are presented in Section IV. Application of a real data set is given in Section V. The main conclusions are reported in Section VI.

The present study deals with the development of acceptance sampling plan, assuming the life time distribution of the product follows MOEP distribution.

II. MARSHALL OLKIN EXPONENTIAL POWER DISTRIBUTION

Jose and Paul [2018] suggested a new family of distribution namely Marshall Olkin Exponential Power (MOEP) distribution with parameters (β, η, θ) with survival function and probability density function as follows,

$$\bar{G}(x) = \frac{\theta \left(\exp \left(1 - \exp \left(\frac{x}{\eta} \right)^\beta \right) \right)}{\left(1 - \bar{\theta} \exp \left(1 - \exp \left(\frac{x}{\eta} \right)^\beta \right) \right)} ; x > 0, \beta, \eta, \theta > 0, \bar{\theta} = 1 - \theta \quad (1)$$

$$g(x) = \frac{\theta \left(\frac{\beta}{\eta}\right) \left(\frac{x}{\eta}\right)^{\beta-1} \exp\left(\frac{x}{\eta}\right)^\beta \exp\left(1 - \exp\left(\frac{x}{\eta}\right)\right)}{\left(1 - (1 - \theta) \exp\left(1 - \exp\left(\frac{x}{\eta}\right)^\beta\right)\right)^2} \quad x > 0, \beta, \eta, \theta > 0, \bar{\theta} = 1 - \theta \quad (2)$$

The hazard rate function of MOEP distribution is given as follows

$$h(x) = \frac{\left(\frac{\beta}{\eta}\right) \left(\frac{x}{\eta}\right)^{\beta-1} \exp\left(\frac{x}{\eta}\right)^\beta}{\left(1 - (1 - \theta) \exp\left(1 - \exp\left(\frac{x}{\eta}\right)^\beta\right)\right)} \quad (3)$$

The MOEP distribution can be consider as generalization of Exponential Power distribution (Smith and Bain [1975]) and is obtained by using the general method given by Marshall and Olkin [1997].

III.RELIABILITY TEST PLAN

We want to test whether the producer’s mean life time is longer than our expectations. Let η_0 be the mean life of each product in a sample of n items. According to this plan the lot will be accepted if the number of failures does not exceed c (the acceptance number) at the end of the time t .

The proposed acceptance sampling plan can be characterized by

1. n : The number of units of test.
2. c : The acceptance number c such that if c or fewer failures occur during the test time t ,the lot is accepted.
3. t : The maximum test duration.
4. The ratio $\frac{t}{\eta_0}$ where η_0 is the specified average life.

Assuming that probability of rejecting a lot with $\eta < \eta_0$ is P^* . For testing the hypothesis $\eta > \eta_0$, we have to fix consumer’s risk $(1 - P^*)$, i.e. the probability of accepting a bad lot not to exceed $1 - P^*$. Assume that lot is large to be considered infinitely. We can apply the theory of binomial distribution. The required sample size n will be the smallest positive integer that satisfies the inequality,

$$\sum_{i=0}^c nC_i p^i (1 - p)^{n-i} \leq 1 - P^* \quad (4)$$

Where $p = F(t, \theta, \beta, \eta, \eta_0)$. The minimum values of n satisfying the above equation are obtained from Table1.For varying values of P^* and $\frac{t}{\eta_0}$.

The operating characteristic function of sampling plan $(n, c, \frac{t}{\eta_0})$ gives the probability of accepting the lot. The probability is given by

$$L(P) = \sum_{i=0}^c nC_i p^i (1 - p)^{n-i}, \text{ where } P = F(t, \theta, \beta, \eta, \eta_0)$$

Considered as a function of η the operating characteristic values for different combinations of the values of confidence level, mean ratio and experiment time ratio have been computed and listed in Table2 .Further, for a given value of producer’s risk 0.5, one may be interested to know, what value of $\eta > \eta_0$ will ensure the producers risk less than or equal to 0.05.The values of η or η_0 is the smallest number $\eta > \eta_0$ for which P satisfies the inequality

$$\sum_{i=0}^c nC_i p^i (1 - p)^{n-i} \geq 0.95 \quad (5)$$

For a given sampling plan $(n, c, \frac{t}{\eta_0})$ at a specified confidence level P^* , the minimum value of

$\eta > \eta_0$ satisfies (5) are computed and presented in Table 3.



IV. TABLES

Table 1. Minimum values of n necessary to ensure an average life exceeding η_0 with Probability p^* and an acceptance number c.

p*	c	$\frac{t}{\eta_0}$						
		1	1.25	1.5	1.75	2	2.25	2.5
0.75	0	24	19	16	13	12	10	9
	1	48	38	31	26	23	20	18
	2	70	55	46	39	33	29	26
	3	92	72	60	51	44	39	34
	4	113	89	74	62	54	47	42
	5	133	106	87	74	64	56	50
	6	154	122	101	85	74	65	58
	7	175	138	114	96	84	74	66
0.9	0	40	32	26	22	19	16	14
	1	69	54	44	37	32	28	25
	2	95	75	61	52	45	39	35
	3	119	94	77	65	56	49	44
	4	143	113	93	79	68	59	53
	5	166	131	108	91	79	69	62
	6	189	149	123	104	90	79	70
	7	211	167	138	116	101	88	79
0.95	0	53	41	34	28	24	21	19
	1	84	66	54	46	39	34	30
	2	112	88	72	61	53	46	41
	3	138	109	90	76	65	57	51
	4	164	129	106	90	77	68	60
	5	188	149	122	103	89	78	69
	6	212	168	138	117	101	88	78
	7	236	186	153	130	112	98	87
0.99	0	81	64	52	44	38	33	29
	1	118	93	76	64	55	48	43
	2	149	118	97	81	70	61	54
	3	179	141	116	98	84	74	65
	4	207	163	134	113	98	86	76
	5	234	185	152	128	111	97	86
	6	261	206	169	143	123	108	96
	7	270	226	186	157	136	119	106
8	285	246	202	171	148	130	115	



Table 2. Operating characteristic values of the sampling plan $(n, c, \frac{t}{\eta_0})$

For a given p^* when $c=2$

p^*	n	t/η_0	η/η_0						
			1	1.25	1.5	1.75	2	2.25	2.5
0.75	70	1	0.2538	0.4063	0.5332	0.6321	0.707	0.765	0.80913
	55	1.25	0.2572	0.4124	0.5404	0.6395	0.7146	0.7715	0.81493
	46	1.5	0.2474	0.4036	0.5335	0.6342	0.7106	0.7684	0.812571
	39	1.75	0.2464	0.4049	0.5362	0.6375	0.7140	0.7718	0.81572
	33	2	0.2595	0.4218	0.5538	0.6541	0.7290	0.7849	0.82714
	29	2.25	0.2590	0.4236	0.5569	0.6577	0.7327	0.7884	0.83032
	26	2.5	0.9999	0.4201	0.5550	0.6570	0.7326	0.7888	0.8309
0.90	95	1	0.1010	0.2114	0.3278	0.4345	0.5262	0.6025	0.6653
	75	1.25	0.1009	0.2131	0.3313	0.4392	0.5314	0.6079	0.67063
	61	1.5	0.1050	0.2212	0.3421	0.4514	0.5439	0.6200	0.68200
	52	1.75	0.1020	0.2185	0.3405	0.4508	0.5441	0.6208	0.6830
	45	2	0.1010	0.2191	0.3426	0.4540	0.5479	0.6248	0.68714
	39	2.25	0.1056	0.2280	0.3544	0.4671	0.561	0.6375	0.69889
	35	2.5	0.9998	0.2238	0.3511	0.4648	0.5597	0.6367	0.6985
0.95	112	1	0.0510	0.1290	0.2254	0.3240	0.4157	0.4968	0.5667
	88	1.25	0.0520	0.1322	0.2309	0.3313	0.4240	0.5056	0.5755
	72	1.5	0.0530	0.1357	0.2368	0.3389	0.4326	0.5146	0.58455
	61	1.75	0.0525	0.1361	0.2388	0.3421	0.4368	0.5192	0.58933
	53	2	0.0508	0.1346	0.2380	0.3423	0.4377	0.5208	0.59125
	46	2.25	0.0532	0.1406	0.2474	0.3539	0.4503	0.5335	0.60364
	41	2.5	0.9987	0.1402	0.2482	0.3559	0.4532	0.5369	0.60724
0.99	149	1	0.0103	0.0400	0.0914	0.1580	0.2316	0.3058	0.37674
	118	1.25	0.0101	0.0397	0.0918	0.1593	0.2338	0.3088	0.38040
	97	1.5	0.0100	0.0402	0.0934	0.1623	0.2381	0.3142	0.38647
	81	1.75	0.0108	0.0433	0.0996	0.1717	0.2500	0.3278	0.40103
	70	2	0.0106	0.0435	0.1009	0.1742	0.2538	0.3325	0.40635
	61	2.25	0.0110	0.0453	0.1050	0.1807	0.2621	0.342	0.41677
	54	2.5	0.9810	0.0465	0.1080	0.1855	0.2686	0.3497	0.42498



Table 3. Minimum ratio of true mean life to specified mean life for the acceptability of a lot with producer's risk of .05.

p*	c	$\frac{t}{\eta_0}$						
		1	1.25	1.5	1.75	2	2.25	2.5
0.75	0	60.66	75.44	89.98	104.85	119.70	133.34	146.51
	1	57.72	71.06	83.15	95.07	106.49	116.24	127.87
	2	55.43	67.23	78.28	88.62	97.30	105.19	114.57
	3	53.78	64.58	74.45	83.46	91.66	98.12	103.77
	4	52.18	62.35	71.53	79.41	87.22	92.47	95.91
	5	51.15	60.87	69.43	76.32	83.01	88.03	92.23
	6	50.13	59.02	67.39	74.09	79.02	84.65	88.69
	7	49.63	58.40	65.41	71.22	76.73	81.00	85.31
0.90	0	61.17	76.38	91.41	106.39	120.83	135.24	150.27
	1	58.90	72.88	86.14	99.09	111.99	123.47	134.46
	2	57.14	69.36	81.17	93.18	104.37	114.05	122.85
	3	55.32	67.23	78.28	88.62	97.30	106.24	114.57
	4	54.11	65.23	75.20	84.29	92.57	101.09	107.95
	5	53.24	63.30	72.98	81.81	89.85	96.20	101.74
	6	52.18	62.04	71.53	78.62	86.36	92.47	96.86
	7	51.15	61.43	69.43	77.08	83.84	88.90	94.05
0.95	0	61.40	76.68	91.55	106.63	121.81	136.62	151.34
	1	59.50	73.62	87.46	101.01	113.13	125.99	138.58
	2	57.54	70.70	83.15	95.55	107.03	118.01	128.51
	3	56.28	68.59	79.87	90.41	101.28	109.47	119.23
	4	54.87	66.69	77.73	87.73	96.34	104.15	111.21
	5	53.83	65.29	75.20	84.29	92.57	100.09	106.89
	6	53.24	63.93	72.98	81.81	89.85	96.20	102.75
	7	52.45	62.66	71.53	80.20	87.22	93.39	98.78
0.99	0	61.59	76.83	92.11	107.35	122.50	137.60	152.58
	1	60.11	74.75	88.80	103.08	116.61	129.87	142.85
	2	58.60	72.51	85.71	97.99	110.87	122.24	134.46
	3	57.43	70.35	82.98	94.98	105.32	116.13	126.60
	4	56.56	68.59	80.67	91.32	101.28	111.68	120.42
	5	55.43	67.23	78.28	88.62	98.28	107.30	114.57
	6	54.71	65.89	76.72	89.42	94.44	103.11	110.11
	7	53.51	63.93	73.71	82.63	90.75	98.12	103.77
	8	52.71	63.30	72.25	81.00	88.09	95.25	99.76



V.ILLUSTRATION OF THE TABLES AND EXAMPLES

Assume that the life distribution is MOEP distribution and the experimenter is interested in showing that the true unknown average life is at least 1000 hours. Let the consumers risk be set to $1-p^*=0.25$. It is desired to stop the experiment at $t=1000$ hours. Then for an acceptance number $c=2$, the required n is the entry in Table 1 corresponding to the values of $1-p^*=0.25, \frac{t}{\eta_0}=1.0$ and $c=2$ is 70. Thus $n=70$ items have to be put on test. If during 1000 hours, no more than 2 failures out of 70 units are observed, then the experimenter can assert with a confidence level of $p^*=0.75$ that the average life is at least 1000 hours.

For the sampling plan $n=70, c=2, \frac{t}{\eta_0}=1.0$ under MOEP model, the operating characteristic values from Table 2 are

η/η_0	1	1.25	1.5	1.75	2	2.25	2.5
L(p)	0.2538	0.4063	0.5332	0.6321	0.707	0.765	0.80913

This shows that if the true mean life is twice as the specified mean life $\eta/\eta_0=2$ producer’s risk is approximately 0.001.

From Table 3 we can get the value of η/η_0 for various choices of $c, \frac{t}{\eta_0}$ in order that the producer’s risk may not exceed 0.05. Thus in this case we get $\eta/\eta_0 = 55.43$.

Example

Here we consider an application of the model to a data set. The data represent lifetimes of 50 devises (Aarset [1987]) and are used here for a comparison of the acceptance sampling plan. Data set is given below

0.1 0.2 1 1 1 1 1 2 3 6 7 11 12 18 18 18
 18 18 21 32 36 40 45 46 47 50 55 60
 63 63 67 67 67 67 72 75 79 82 82 83
 84 84 84 85 85 85 85 85 86 86

Let the specified average life be 10hours and the testing time be 25hours, this leads to the ratio 2.5 with corresponding n and c as 50 and 5 from Table 1 for $p^*=0.75$. Therefore, the sampling plan for the above sample data is ($n=50, c=5, \frac{t}{\eta_0}=2.5$). Based on the observations we have to decide whether to accept the product or reject it. We accept the product only, if the number of failure after 25 hours is less than or equal to 5. From the given data we can see that the earliest failure for 18 devises are less than 25 hours in the given sample of $n=50$ observations. Since the number of failures is greater than c , our decision is to reject the lot.

VI.CONCLUSION

In this paper an acceptance sampling plan was derived on the basis that the life distribution of the test item follows the MOEP distribution. Also we provide some useful tables corresponding to the sampling plan and is applied to a real data set.

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