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Numerical Solution of Black-Scholes Equation using Variational Iteration Method

Pradeep Kashyap¹, Ajay Kumar Sharma^{2,*}

Department of Mathematics, Shri Sadguru Saibaba Science & Commerce College, Gadchiroli, India¹

Department of Mathematics, Mahatma Gandhi College of Science, Gadchandur, India²

ABSTRACT: This paper deals with the numerical solution of Black-Scholes equation, which is solved by variational iteration method (VIM). One of the important applications of Black-Scholes equation is that it is widely used for option pricing. VIM is a very powerful, efficient and effective mathematical tool which provides approximate solutions for a wide range of real-world problems arising in engineering and natural sciences, modeled in terms of differential equations. Convergence analysis and numerical examples are presented to show the efficacy of the proposed numerical method.

KEYWORDS: Variational iteration method; Non-linear phenomenon; Partial differential equation; Black-Scholes equation

I. INTRODUCTION

Fractional order derivative is a non-local property while integer order derivative is local in nature. It shows that the upcoming state of physical system is also dependent on all of its historical states in addition to its present state. Hence, the fractional models are more realistic and fractional derivatives are often used in mathematical modeling of acoustics, fluid mechanics, anomalous diffusion, electrochemistry, signal processing, biology, etc., [1,8,13,20,21]. Most of the nonlinear FDEs do not possess exact solutions, therefore some numerical techniques are necessary to be used. In the past two decades, the problem of handling numerical solutions of FDEs has attracted the attention of many researchers. They are taking keen interest in developing numerical techniques for FPDEs. Recently, various techniques have been developed to solve nonlinear FPDEs such as, Adomian decomposition method [28], differential transform method [2], homotopy perturbation method [9], homotopy analysis method [7], homotopy analysis transform method [25], homotopy perturbation transform method [18,4], etc.

Differential equations which govern the systems with memory are fractional differential equations (FDEs). The arbitrariness in their order introduces more degrees of freedom in design and analysis, resulting in more accurate modelling, better robustness in control and greater flexibility in signal processing [22]. By this time, it is established that the electrochemical phenomena like double layer charge distribution or the diffusion process can be better explained with fractional order system. As a result, the modelling of lithium in battery, fuel cells, super capacitors are carried out with FDEs. The characterization of ceramic bodies, fractal structures, viscoelastic materials, the decay rate of fruits and meats, study of corrosion in metal surface are also promising areas of its applications. Fractional order system is also a popular choice to study real time events such as earthquake propagation, volcanic phenomenon, design of thermo-kinetics, modelling of human lungs and skin. Even the characteristics of economic market fluctuation adopts fractional calculus based system modelling. So, F.O. analysis has now reached from inert physical network to living network of biology, ecology, physiology and sociology reminding us Leibnitz's predication in his letter to L'Hopital in 1695 that fractional differential operator is "an apparent paradox from which one day useful consequences will be drawn".

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In 1969, Fisher Black and Myron Scholes [5] got an idea that have changed the world of finance forever. The main idea of their study centered around the discovery that it is not required to estimate the expected return of a stock in order to price an option written on that stock. The Black-Scholes model for pricing stock options has been applied to many different commodities and payoff structures. Further, nonlinear Black-Scholes equations have attracted many researchers, as they have provided more accurate results by taking into account realistic assumptions. These assumptions include transaction costs, risks from an unprotected portfolio, large investor's preferences or illiquid markets (which may have an impact on the stock price), the volatility, the drift and the option price itself [10]. The main idea of Black and Scholes lies in the construction of a riskless portfolio taking positions in cash, option and the underlying stock. Derivation of a closed form solution to the Black-Scholes equation particularly depends on the fundamental solution of the heat equation. Therefore, it is important to transform the Black-Scholes equation into heat equation by using change of variables. Due to availability of closed form solution of the heat equation, it is possible to transform it back to find the corresponding solution of the Black-Scholes PDE.

The Black-Scholes model for the value of an option is described by the equation

$$\frac{\partial v}{\partial t} + \frac{\sigma^2 x^2}{2} \frac{\partial^2 v}{\partial x^2} + r(t) x \frac{\partial v}{\partial x} - r(t) v = 0, \quad (x, t) \in R^+ \times (0, T) \quad (1)$$

where $v(x, t)$ is the European call option price at asset price x and at time t , K is the exercise price, T is the maturity, $r(t)$ is the risk free interest rate, and $\sigma(x, t)$ represents the volatility function of underlying asset. Many researchers studied the existence of solutions of the Black-Scholes model using many methods in [14,23,24,29,6,19,27].

It is used due to its advantage that with it, initial conditions for FDEs undertake similar form as for integer order differential equations [8]. Kumar et al. [26] Presented a numerical algorithm for Black-Scholes equation with boundary condition for a European option problem by using homotopy perturbation method [HPM] and homotopy analysis method [HAM]. Ozdemir and Yavuz [17] used multivariate Pade approximation method for solving European vanilla call option pricing problem.

Motivated by the above discussions, in this paper, we propose the variational iteration method (VIM) [11,12,3] to get the approximate analytic solution of nonlinear Black-Scholes equation and results are compared with recently developed techniques [14,23,24,29,6,19,27,26,17]. VIM directly attacks the nonlinear FDEs without a need to find certain polynomials for nonlinear terms and gives result in an infinite series, that rapidly converges to analytical solution. This method does not require linearization, discretization, little perturbations or any restrictive assumptions.

This paper is organized in the following manner. Section 1 is introductory. In section 2, we represents the brief review of preliminary descriptions of Caputo fractional derivative. In section 3, the working of numerical method VIM. Section 4, illustrates two numerical examples on which, VIM is applied to find the approximate solutions. In last Section 5, we have concluded the paper.

II. BASIC PLAN OF FVIM

To illustrate the process of solution by VIM, we consider coupled nonlinear partial differential equation

$$Dv(x, t) + S(x, t) + Q(x, t) = g(x, t), \quad (2)$$

By FVIM, correction functional is formed as

$$v_{n+1}(x, t) = v_n(x, t) + \int_0^t \lambda \left[\frac{\partial v_n(x, \tau)}{\partial \tau} + S(x, \tau) + Q(x, \tau) - g(x, \tau) \right] (d\tau) \quad (3)$$

where λ is a Lagrangian multiplier.

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Now by variational theory, Lagrangian multiplier λ must satisfy

$$\frac{\partial \lambda}{\partial t} \Big|_{\tau=t} \quad \text{and} \quad 1 + \lambda \Big|_{\tau=t} = 0.$$

Consequently, we obtain $\lambda = -1$,
and hence, from Eq. (3), We have

$$v_{n+1}(x, t) = v_n(x, t) - \int_0^t \left[\frac{\partial v_n(x, \tau)}{\partial \tau} + S(x, t) + Q(x, t) - g(x, t) \right] (d\tau) \quad (4)$$

Now from Eq. (4), we can obtain successive approximations $v_n(x, t)$, $n \geq 0$. The functions v_n are restricted variations which means $\delta v_n = 0$. In this way, we get sequences $v_{n+1}(x, t)$, $n \geq 0$. Finally, the exact solution is obtained as

$$v(x, t) = \lim_{n \rightarrow \infty} v_n(x, t).$$

Convergence Analysis of VIM. Now, our emphasis is on convergence of the proposed VIM applied to Eq. (2). Sufficient conditions for convergence of VIM & its error estimate [15] are provided.

We define the operator S as:

$$S = \int_0^x (-1) \left(D v_n(x, t) + \frac{\sigma^2 x^2}{2} D_x^2 v_n(x, t) + r(t) x \frac{\partial v}{\partial x} - r(t) v \right) (d\tau), \quad (5)$$

also, we define the components v_k , $k = 0, 1, 2, \dots$, as

$$v(x, t) = \lim_{n \rightarrow \infty} v_n(x, t) = \sum_{k=0}^{\infty} v_k. \quad (6)$$

Theorem 1 [30]. Let S , defined in Eq. (5), be an operator from Banach space BS to BS. The series solution as defined in Eq. (6), converge if $0 < q < 1$ exists such that

$$\|S[v_0 + v_1 + v_2 + \dots + v_{k+1}]\| \leq q \|S[v_0 + v_1 + v_2 + \dots + v_k]\|,$$

(i.e. $\|v_{k+1}\| \leq q \|v_k\|$), $\forall k \in \mathbb{N} \cup \{0\}$.

Theorem 1 is an exceptional case of Banach fixed point theorem used in [16] as sufficient condition to discuss convergence of VIM for various differential equations.

Theorem 2 [30]. If the solution defined in Eq. (6) converges, it is exact solution of problem (2).

Theorem 3 [30]. Suppose series solution defined in Eq. (6) converges to solution $v(x, t)$ of problem (2). If the truncated series $\sum_{k=0}^j v_k$ is used as an approximation to the solution $v(x, t)$ of problem (2), then the maximum error, $E_j(x, t)$, is assessed as: $E_j(x, t) \leq \frac{1}{1-q} q^{j+1} \|v_0\|$.

If $\forall i \in \mathbb{N} \cup \{0\}$, we define parameters, $\chi_i = \begin{cases} \frac{\|v_{i+1}\|}{\|v_i\|}, & \|v_i\| \neq 0 \\ 0, & \|v_i\| = 0 \end{cases}$ then the series solution $\sum_{k=0}^{\infty} v_k$ of problem (2)

converges to exact solution $v(x, t)$, when $0 \leq \chi_i < 1$, $\forall i \in \mathbb{N} \cup \{0\}$. Moreover, as stated in theorem 3, the maximum absolute truncation error is estimated as

$$\|u(x, y) - \sum_{k=0}^{\infty} v_k\| \leq \frac{1}{1-\chi} \chi^{j+1} \|v_0\|, \text{ where } \chi = \max \{\chi_i, i = 0, 1, 2, \dots, j\}.$$

Remark 1 [30]. If the first finite χ_i 's, $i = 1, 2, \dots, j$ are not less than one and $\chi_i \leq 1$ for $i > j$, then, of course, the series solution $\sum_{k=0}^{\infty} v_k$ of problem (2) converges to exact solution. In other words, the first finite terms don't affect convergence of the series solution. In this case, the convergence of VIM approach depends on χ_i , for $i > j$.

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III. NUMERICAL EXPERIMENTS

In this section, we apply the proposed technique FVIM to some test examples.

Example 1. Consider the fractional Black-Scholes equation as [26]

$$Dv = v_{xx} + (k - 1)v_x - kv, \tag{7}$$

with initial condition $v(x, 0) = \max(e^x - 1, 0)$ (8)

$$v_1(x, t) = v_0 - \int_0^t \left[\frac{\partial v_0}{\partial t} - \frac{\partial^2 v_0}{\partial x^2} + (k - 1) \frac{\partial v_0}{\partial x} - kv_0 \right] (d\tau)^\alpha, \tag{9}$$

$$= \max(e^x - 1, 0) - \max(e^x, 0) \frac{(-kt^\alpha)}{\Gamma(\alpha+1)} + \max(e^x - 1, 0) \frac{(-kt^\alpha)}{\Gamma(\alpha+1)},$$

$$v_2(x, t) = v_1 - \int_0^t \left[\frac{\partial v_1}{\partial t} - \frac{\partial^2 v_1}{\partial x^2} + (k - 1) \frac{\partial v_1}{\partial x} - kv_1 \right] (d\tau)^\alpha, \tag{10}$$

=

$$\max(e^x - 1, 0) - \max(e^x, 0) \frac{(-kt^\alpha)}{\Gamma(\alpha+1)} + \max(e^x - 1, 0) \frac{(-kt^\alpha)}{\Gamma(\alpha+1)} - \max(e^x, 0) \frac{(-kt^\alpha)^2}{\Gamma(2\alpha+1)} + \max(e^x - 1, 0) \frac{(-kt^\alpha)^2}{\Gamma(2\alpha+1)}.$$

Proceeding in this way, rest components may be obtained using Maple package. At last, we can get solution as

$$v(x, t) = \lim_{n \rightarrow \infty} v_n(x, t) = \max(e^x, 0)(1 - E_\alpha(-kt^\alpha)) + \max(e^x - 1, 0)E_\alpha(-kt^\alpha). \tag{10}$$

Here, the expansion of the series in Eq. (10) for $\alpha = 1$ is

$$v(x, t) = kt + kxt - \frac{k^2t^2}{2} + \frac{k^3t^3}{6} - \frac{k^2xt^2}{2} + \frac{kx^2t}{2} + \frac{k^3xt^3}{6} - \frac{k^2x^2t^2}{4} + \dots, \tag{11}$$

The expansion of the series in Eq. (14) for $\alpha = 0.5$ is

$$v(x, t) = 1.12837 kt^{0.5} - k^2t + 0.75225 k^3t^{1.5} + 1.12837 kxt^{0.5} - k^2xt + 0.75225 k^3xt^{1.5} + 0.56418 kx^2t^{0.5} - 0.5k^2x^2t + 0.37616 k^3x^2t^{1.5} + \dots, \tag{12}$$

In view of Eqs. (4) and (5), the iteration formula for problem (2) can be constructed as,

$$w_0 = \max(e^x - 1, 0),$$

$$w_1 = -\max(e^x, 0) \frac{(-kt^\alpha)}{\Gamma(\alpha+1)} + \max(e^x - 1, 0) \frac{(-kt^\alpha)}{\Gamma(\alpha+1)},$$

$$w_2 = -\max(e^x, 0) \frac{(-kt^\alpha)^2}{\Gamma(2\alpha+1)} + \max(e^x - 1, 0) \frac{(-kt^\alpha)^2}{\Gamma(2\alpha+1)},$$

$$\dots$$

$$w_n = -\max(e^x, 0) \frac{(-kt^\alpha)^n}{\Gamma(n\alpha+1)} + \max(e^x - 1, 0) \frac{(-kt^\alpha)^n}{\Gamma(n\alpha+1)}.$$

Clearly, we can conclude that the obtained solution, $\sum_{n=0}^{\infty} w_n$ converges to the exact solution $w(x, t) = \max(e^x, 0)E_\alpha(kt^\alpha) - \max(e^x - 1, 0)E_\alpha(kt^\alpha)$, where $E_\alpha(z)$ is the one parameter Mittag-Leffler function.

In addition, by computing χ_i 's for this problem, we have,

$$\chi_i = \frac{\|w_{i+1}\|}{\|w_i\|} = \left\| t^\alpha \frac{\Gamma(1 + i\alpha)}{\Gamma(1 + (i + 1)\alpha)} \right\| < 1,$$

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when for example, $i > 1$ and $0 < \alpha \leq 1$. This confirms that the variational approach for problem (2) gives the positive and bounded solution, which converges to the exact solution.

Remark 2 [30]. The above test problem is considered when $0 < t \leq 1$ in order to discuss the condition of convergence. Of course, we can length the interval and examine the condition of convergence after neglecting the first few terms of series solution. For example, if we consider the time-fractional Black-Scholes equation (2) when $0 < x \leq a$ and $\alpha = 1$, where $a > 0$, then

$$\chi_i = \left\| t \frac{\Gamma(1+i)}{\Gamma(1+(i+1))} \right\| = \left\| t \frac{i!}{(i+1)!} \right\| \leq \frac{a}{i+1} < 1, \text{ for } i > a.$$

It is the same solution as obtained by homotopy perturbation method HPM [27,26], homotopy analysis method HAM [26] and multivariate pade approximation MPA [17]. The numerical results obtained by using FVIM and exact solution are depicted through figs. 1 and 2. It is observed from figs. 1-2 that the approximate solution obtained by FVIM is almost identical with exact solution at $\alpha = 1$ and $\alpha = 0.5$ for different values of x and t . The numerical results indicate that FVIM work very well, even if lower order approximate solution $v_2(x, t)$ is used. However, the accuracy can be improved by using higher order approximations in solution.

Example 2. Consider the fractional Black-Scholes equation as [29]

$$Dv + 0.08(2 + \sin x)^2 x^2 v_{xx} + 0.06 x v_x - 0.06 v = 0, \tag{13}$$

$$\text{with initial condition } v(x, 0) = \max(x - 25 e^{-0.06}, 0) \tag{14}$$

$$v_1(x, t) = v_0 - \int_0^t \left[\frac{\partial v_0}{\partial t} + 0.08(2 + \sin x)^2 x^2 \frac{\partial^2 v_0}{\partial x^2} + 0.06 x \frac{\partial v_0}{\partial x} - 0.06 v_0 \right] d\tau, \tag{15}$$

$$= \max(x - 25 e^{-0.06}, 0) - \frac{x(0.06 t^\alpha)}{\Gamma(\alpha+1)} + \max(x - 25 e^{-0.06}, 0) \frac{(0.06 t^\alpha)}{\Gamma(\alpha+1)},$$

$$v_2(x, t) = v_1 - \int_0^t \left[\frac{\partial^\alpha v_1}{\partial t^\alpha} + 0.08(2 + \sin x)^2 x^2 \frac{\partial^2 v_1}{\partial x^2} + 0.06 x \frac{\partial v_1}{\partial x} - 0.06 v_1 \right] (d\tau)^\alpha, \tag{16}$$

$$= \max(x - 25 e^{-0.06}, 0) - \frac{x(0.06 t^\alpha)}{\Gamma(\alpha+1)} + \max(x - 25 e^{-0.06}, 0) \frac{(0.06 t^\alpha)}{\Gamma(\alpha+1)} - \frac{x(0.06 t^\alpha)^2}{\Gamma(2\alpha+1)} + \max(x - 25 e^{-0.06}, 0) \frac{(0.06 t^\alpha)^2}{\Gamma(2\alpha+1)},$$

Proceeding in this way, rest components may be obtained using Maple package. At last, we can get solution as,

$$v(x, t) = \lim_{n \rightarrow \infty} v_n(x, t) = \max(x - 25 e^{-0.06}, 0) E_\alpha(0.06 t^\alpha) \tag{17}$$

In view of Eqs. (4) and (5), the iteration formula for problem (2) can be constructed as,

$$\begin{aligned} w_0 &= \max(x - 25 e^{-0.06}, 0), \\ w_1 &= -\frac{x(0.06 t^\alpha)}{\Gamma(\alpha+1)} + \max(x - 25 e^{-0.06}, 0) \frac{(0.06 t^\alpha)}{\Gamma(\alpha+1)}, \\ w_2 &= -\frac{x(0.06 t^\alpha)^2}{\Gamma(2\alpha+1)} + \max(x - 25 e^{-0.06}, 0) \frac{(0.06 t^\alpha)^2}{\Gamma(2\alpha+1)}, \\ &\vdots \\ w_n &= -\frac{x(0.06 t^\alpha)^n}{\Gamma(2\alpha+1)} + \max(x - 25 e^{-0.06}, 0) \frac{(0.06 t^\alpha)^n}{\Gamma(2\alpha+1)}. \end{aligned}$$

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Clearly, we can conclude that the obtained solution, $\sum_{n=0}^{\infty} w_n$ converges to the exact solution $w(x, t) = x E_{\alpha}(-0.06t^{\alpha}) + \max(x - 25 e^{-0.06}, 0) E_{\alpha}(0.06 t^{\alpha})$, where $E_{\alpha}(z)$ is the one parameter Mittag-Leffler function.

In addition, by computing χ_i 's for this problem, we have,

$$\chi_i = \frac{\|w_{i+1}\|}{\|w_i\|} = \left\| t^{\alpha} \frac{\Gamma(1 + i\alpha)}{\Gamma(1 + (i + 1)\alpha)} \right\| < 1,$$

when for example, $i > 1$ and $0 < \alpha \leq 1$. This confirms that the variational approach for problem (2) gives the positive and bounded solution, which converges to the exact solution.

Remark 2 [30]. The above test problem is considered when $0 < t \leq 1$ in order to discuss the condition of convergence. Of course, we can length the interval and examine the condition of convergence after neglecting the first few terms of series solution. For example, if we consider the time-fractional Black-Scholes equation (2) when $0 < x \leq a$ and $\alpha = 1$, where $a > 0$, then

$$\chi_i = \left\| t \frac{\Gamma(1+i)}{\Gamma(1+(i+1))} \right\| = \left\| t \frac{i!}{(i+1)!} \right\| \leq \frac{a}{i+1} < 1, \text{ for } i > a.$$

It is the same solution as obtained by homotopy perturbation method HPM [29, 26] and homotopy analysis method HAM [26]. The numerical results indicate that FVIM work very well, even if lower order approximate solution $v_2(x, t)$ is used. However, the accuracy can be improved by using higher order approximations in solution.

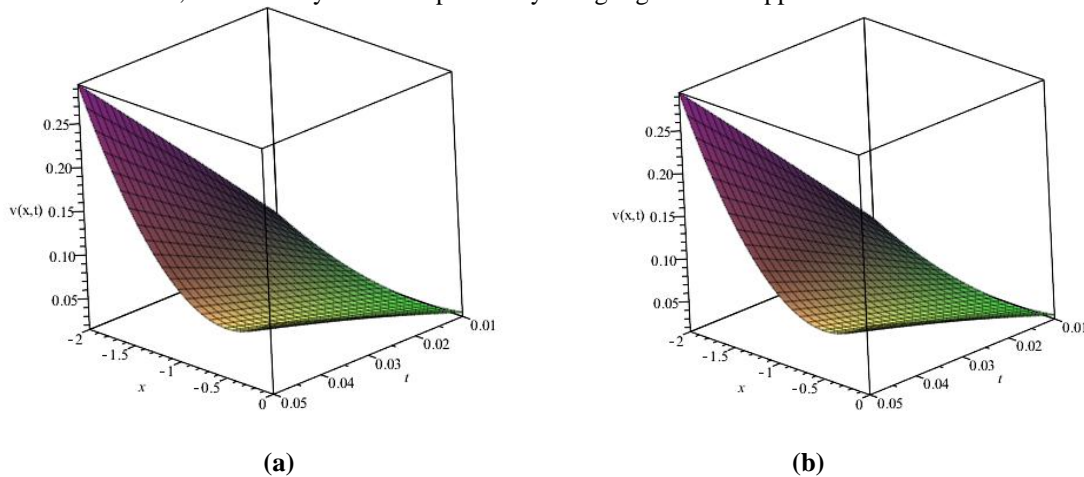


Fig. 1. The surface shows the (a) exact solution $v(x, t)$ and (b) approximate solution for (9) with respect to x and t at $\alpha = 1$ and $k = 2$.

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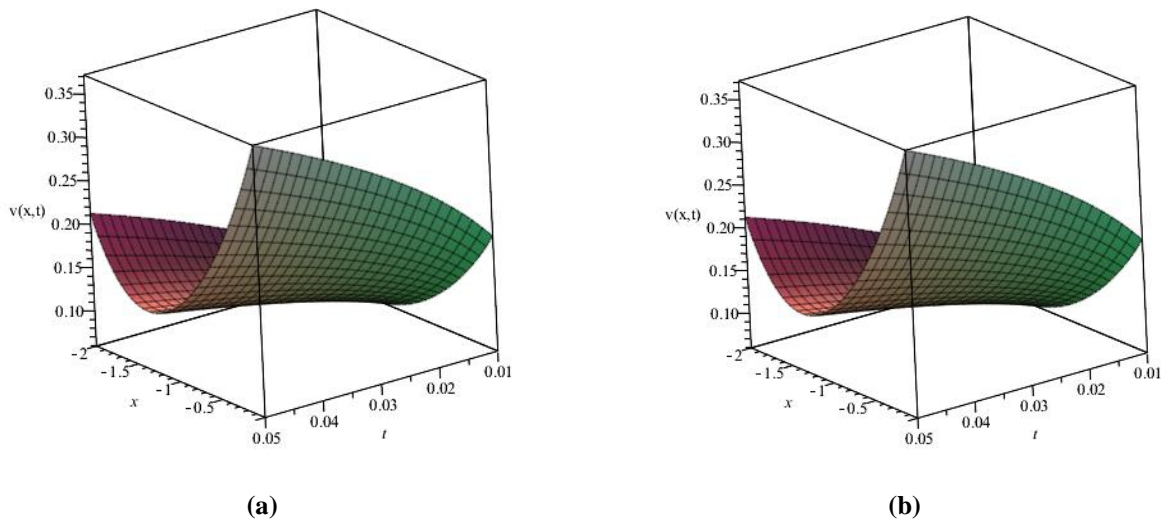


Fig. 2. The surface shows the (a) exact solution $v(x, t)$ and (b) approximate solution for (9) with respect to x and t at $\alpha = 0.5$ and $k = 2$

IV. CONCLUSION

In this work, VIM is successfully applied to solve the Black-Scholes equation for a European option problem. Outcomes reveal that the results derived using VIM are more general and contains results of HPM, HAM and MPA as particular cases. VIM is extremely simple and easy to handle the nonlinear terms. It is observed that VIM is used directly without using linearization, perturbation, Adomian polynomial or any restrictive assumptions. Hence, VIM is more convenient and also easier than other existing methods. Maple 13 package is used to calculate series obtained from iteration. Further, the method needs much less computational work, which shows the fast convergent for solving nonlinear partial differential equations.

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