

# International Journal of Innovative Research in Science, Engineering and Technology

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## Hydro magnetic Convection Flow in a Porous Medium Bounded between Vertical Wavy Wall & Parallel Flat Wall: Analysis Using Darcy-Brinkman-Forchheimer Model

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**ABSTRACT:** The present paper deals with two-dimensional convection flow and heat transfer of a viscous, incompressible, conducting fluid, through porous medium confined in a vertical wavy wall and parallel flat wall under the influence of transverse magnetic field. The non-linear equations governing the flow are solved by linearization technique under the assumption that the flow consists of two parts; a mean part and a perturbed part. Darcy-Brinkman-Forchheimer model is used to visualize the problem. Exact solutions are obtained for the mean part and the perturbed part is solved using long wave approximation method. The effects of various parameters, zeroth order and total axial velocity fields, transverse velocity, temperature distribution, shear stress and heat transfer rate are presented graphically and discussed. The problem finds possible applications controlling the heat transfer efficiency in industrial processes and nuclear energy system.

**KEYWORDS:** heat transfer, Brinkman-Forchheimer model, Vertical Wavy Wall, Parallel Flat Wall

### I. INTRODUCTION

The study of heat transfer from irregular surfaces is of immense importance, because irregular surfaces are often present in engineering applications. Therefore, the study of convective flows in wavy channels is essential to observe the flow characteristics and heat transfer processes in several heat devices with variations in material parameters. The channel consisting of a corrugated wall and flat wall is one of the several devices employed for enhancing the heat transfer efficiency of industrial processes. Surface non-uniformities of such channels are encountered in insulating systems and grain storage containers. The presence of roughness on the surface disturbs the flow past surfaces and alerts the heat transfer rate. As such, the natural convection in vertical channels consisting of a wavy surface and a flat surface is a model problem for the investigation of momentum and heat transfer from roughened and flat surfaces of the wall in order to understand effect of material parameters on flow characteristics.

The references pertaining to flow through porous media are based on Darcy law. However its importance and ramifications in process design and operation have been recognized only during the last three/four decades. However, heat transfer in fluid flow by natural convection as a result of difference in temperature gradient of two vertical walls is also affected by temperature ratio of the channel walls which is of immense importance particularly in heat transfer devices in industrial processes and aerospace technology. Therefore, the present study addresses the free convective flow of viscous incompressible fluid through porous medium confined in vertical channel consisting of a wavy wall and a parallel flat wall. A generalized porous media model, namely, Brinkman-Forchheimer extended Darcy model is considered in order to simulate momentum transfer in porous medium. Since new emerging technological models are applicable over a wide range of porosity, the present paper deals with flow through porous medium using Brinkman-Forchheimer extended Darcy model in the presence of transversely applied uniform magnetic field. Besides, the

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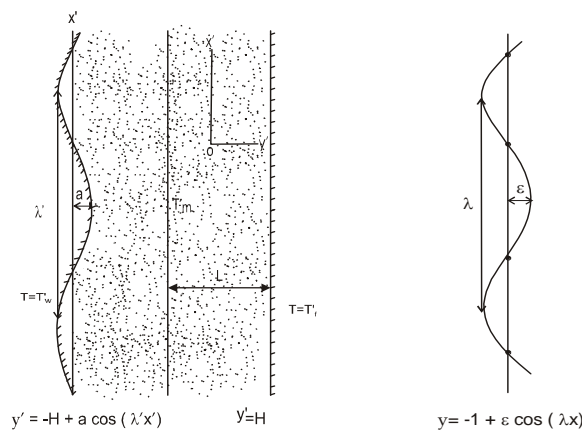
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convection phenomenon between the walls is set by changing the temperature of the walls to that of the fluid temperature and a non-dimensional parameter (buoyancy force parameter) is used to characterize the temperature of vertical walls with respect to the fluid temperature. The present model enhances the applicability of the models suggested by Paul et al (2006) and Singh et al. (2011). The model can have possible application in heat transfer devices of industrial processes, nuclear energy technology and geothermal systems.

## II. FORMULATION OF THE PROBLEM

We consider steady two-dimensional laminar free convection flow of an incompressible, electrically conducting, viscous fluid through a homogeneous porous medium confined in an open-ended vertical channel consisting of a wavy wall and a flat wall as shown in Fig.-1. A uniform magnetic field is applied perpendicular to the walls, i.e. normal to the flow region. It is assumed that the magnetic Reynolds number is small, so that the induced magnetic field is neglected in comparison to the applied magnetic field (Ferraro & Plumpton, 1961).

The wavy and flat walls are respectively maintained at different constant temperatures  $T'_w$  and  $T'_f$  such that the fluid temperature  $T'_m \neq T'_w$ . Besides, the present analysis is based on the following assumptions:



**Fig. 1:** Physical model and coordinate system.

- (i) All the fluid properties are constant except influence of density variation with temperature in buoyancy term in the momentum equation.
- (ii) The density is a linear function of temperature so that usual Boussinesq approximation is taken into account.
- (iii) The wavelength  $\lambda'$  of the wavy wall is large compared with width of the channel.
- (iv) In the energy equation the dissipative effect is negligible.
- (v) The inter-particle and inertia-particle heat transfer effects are negligible.
- (vi) Darcy-Brinkman-Forchheimer model is considered to simulate momentum transfer in porous media, which is applicable over a wide range of porosity (Nield & Bejan, 2006).
- (vii) Transport properties of the fluid are constant and the fluid rises in the channel driven by buoyancy forces.

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(viii) Both the walls are in stationary position, as such the boundary conditions on velocity components  $u'$  and  $v'$  are the no slip conditions.

Under the above stated configuration and assumptions, the equations governing the flows are:

Continuity equation:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (1)$$

Momentum equations:

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \frac{\mu_{eff}}{\rho} \left( \frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} \right) + g\beta(T' - T'_m) - \frac{\nu}{k} u' - \frac{\sigma B_0^2}{\rho} u' - \frac{F}{\sqrt{k}} u'^2 \quad (2)$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial y'} + \frac{\mu_{eff}}{\rho} \left( \frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2} \right) - \frac{\nu}{k} v' - \frac{F}{\sqrt{k}} v'^2 \quad (3)$$

Energy equation:

$$u' \frac{\partial T'}{\partial x'} + v' \frac{\partial T'}{\partial y'} = \frac{K_T}{\rho C_p} \left( \frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2} \right) \quad (4)$$

The relevant boundary conditions on velocity and temperature are:

$$\begin{aligned} u' = 0, \quad v' = 0, \quad T' = T'_w & \quad \text{at} \quad y' = -H + a \cos(\lambda' x'), \\ u' = 0, \quad v' = 0, \quad T' = T'_f & \quad \text{at} \quad y' = H. \end{aligned} \quad (5)$$

We introduce following non-dimensional variables and parameters:

$$\begin{aligned} x = \frac{x'}{H}, \quad y = \frac{y'}{H}, \quad u = \frac{Hu'}{\nu}, \quad v = \frac{Hv'}{\nu}, \quad \gamma = \frac{\mu_{eff}}{\mu}, \\ p = \frac{H^2 \rho p'}{\mu^2}, \quad T = \frac{T' - T'_m}{T'_w - T'_m}, \quad \lambda = \lambda' H \quad \text{and} \quad \varepsilon = \frac{a}{H}, \quad Da = \frac{k}{H^2}. \end{aligned}$$

$$M = \sqrt{\frac{\sigma}{\rho \nu}} B_0 H \quad (\text{magnetic parameter}), \quad Gr = \frac{g\beta H^3 (T'_w - T'_m)}{\nu^2} \quad (\text{buoyancy parameter}), \quad Pr = \frac{\mu C_p}{K_T}$$

(Prandtl number).

Introducing above mentioned non-dimensional variables and parameters, the Equations. (1)-(4) become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \gamma \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + GrT - \frac{1}{Da} u - M^2 u - \frac{F}{\sqrt{Da}} u^2 \quad (7)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \gamma \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{1}{Da} v - \frac{F}{\sqrt{Da}} v^2 \quad (8)$$

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$$Pr \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}. \quad (9)$$

The boundary conditions (5) on velocity and temperature in non-dimensional form are:

$$\begin{aligned} u = 0, \quad v = 0, \quad T = 1, \quad y = -1 + \varepsilon \cos \lambda x, \\ u = 0, \quad v = 0, \quad T = T_R, \quad y = 1. \end{aligned} \quad (10)$$

where  $T_R = \frac{T'_f - T'_m}{T'_w - T'_m}$  (temperature ratio parameter).

Now we proceed for the solution of Equations. (6)-(9) under boundary conditions (10).

### III. SOLUTION OF THE PROBLEM

In order to obtain the solution of the Equations. (7)-(9) satisfying the boundary conditions (10), we assume that the solution consists of a mean part and a perturbed part so that the velocity, temperature and pressure distribution can be expressed as:

$$\left. \begin{aligned} u(x, y) &= u_0(y) + \varepsilon u_1(x, y), \\ v(x, y) &= \varepsilon v_1(x, y), \\ T(x, y) &= T_0(y) + \varepsilon T_1(x, y), \\ p(x, y) &= p_0(x, y) + \varepsilon p_1(x, y), \end{aligned} \right\} \quad (11)$$

where  $\varepsilon$  is the amplitude, used as perturbation parameter. The perturbed quantities  $u_1$ ,  $v_1$ ,  $T_1$  and  $p_1$  are small compared to mean quantities.

Introducing (11) in Equations. (6)-(9) and boundary conditions (10), we obtain:

*Zeroth order equation and boundary conditions:*

$$\frac{d^2 T_0}{dy^2} = 0. \quad (12)$$

$$\gamma \frac{d^2 u_0}{dy^2} - \left( \frac{1}{Da} + M^2 \right) u_0 + Gr T_0 + C - \frac{F}{\sqrt{Da}} u_0^2 = 0. \quad (13)$$

where  $-\frac{\partial p_0}{\partial x} = C$ .

$$\begin{aligned} u_0 = 0, T_0 = 1 \quad \text{at} \quad y = -1, \\ u_0 = 0, T_0 = T_R \quad \text{at} \quad y = 1. \end{aligned} \quad (14)$$

*First order equation and boundary conditions:*

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0. \quad (15)$$

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$$u_0 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_0}{\partial y} = -\frac{\partial p_1}{\partial x} + \gamma \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right) + GrT_1 - \left( \frac{1}{Da} + M^2 \right) u_1 - \frac{2F}{\sqrt{Da}} u_0 u_1. \quad (16)$$

$$u_0 \frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} + \gamma \left( \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} \right) - \frac{1}{Da} v_1. \quad (17)$$

$$Pr \left( u_0 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_0}{\partial y} \right) = \frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}. \quad (18)$$

$$u_1 = -\cos \lambda x \frac{du_0}{dy}, \quad v_1 = 0, \quad T_1 = -\cos \lambda x \frac{dT_0}{dy} \quad \text{at } y = -1,$$

$$u_1 = 0, \quad v_1 = 0, \quad T_1 = 0 \quad \text{at } y = 1. \quad (19)$$

In arriving at the boundary conditions at the wavy wall  $y = -1$ , the actual boundary conditions are transformed to mean position. This is justified when  $\varepsilon \ll 1$ , i.e., when the amplitude ( $\varepsilon$ ) of the disturbance is small compared with the disturbance of the wavelength (Lekoudis, 1976; Tuck et al., 1995).

The solution of Equation (12) under the boundary conditions (14) is:

$$T_0(y) = \frac{(T_R - 1)}{2} y + \frac{T_R + 1}{2}. \quad (20)$$

To solve Eq. (13), we assume:

$$u_0(y) = u_{00}(y) + Fu_{01}(y). \quad (21)$$

Introducing (21) into Equation (13), we obtain:

$$\frac{d^2 u_{00}}{dy^2} - A_1 u_{00} = -A_2 y - A_3. \quad (22)$$

$$\frac{d^2 u_{01}}{dy^2} - A_1 u_{01} = \frac{1}{\gamma \sqrt{Da}} u_{00}^2. \quad (23)$$

where  $A_1 = \frac{1}{\gamma} \left( \frac{1}{Da} + M^2 \right)$ ,  $A_2 = \frac{Gr(T_R - 1)}{2\gamma}$ ,  $A_3 = \frac{Gr(T_R + 1) + 2C}{2\gamma}$ .

The boundary conditions (14), become:

$$u_{00} = 0, \quad u_{01} = 0 \quad \text{at } y = -1; \quad u_{00} = 0, \quad u_{01} = 0 \quad \text{at } y = 1, \quad (24)$$

The solutions of Equations. (22) and (23) satisfying boundary conditions (24) are:

$$u_{00}(y) = C_1 e^{\sqrt{A_1} y} + C_2 e^{-\sqrt{A_1} y} + \frac{A_2}{A_1} y + \frac{A_3}{A_1}. \quad (25)$$

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$$u_{01}(y) = C_3 e^{\sqrt{A_1}y} + C_4 e^{-\sqrt{A_1}y} + K_3 e^{2\sqrt{A_1}y} + K_4 e^{-2\sqrt{A_1}y} + K_5 y e^{\sqrt{A_1}y} + K_6 y e^{-\sqrt{A_1}y} + K_7 \left( y^2 - \frac{y}{\sqrt{A_1}} \right) e^{\sqrt{A_1}y} + K_8 \left( y^2 + \frac{y}{\sqrt{A_1}} \right) e^{-\sqrt{A_1}y} + K_9 \left( y^2 + \frac{2}{A_1} \right) + K_{10} y + K_{11} \quad (26)$$

In order to solve the first order Equations. (16)-(18), we introduce the stream function  $\psi(x, y)$  such that:

$$u_1 = -\frac{\partial \psi}{\partial y}, \quad v_1 = \frac{\partial \psi}{\partial x} \quad (27)$$

Introducing (27) into Equations. (16)-(18), in view of the continuity equation (14), the governing equation for the perturbed flow and heat transfer after eliminating  $p_1$  are as follows:

$$u_0 \left( \frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2} \right) - \frac{\partial^2 u_0}{\partial y^2} \frac{\partial \psi}{\partial x} - \gamma \left( 2 \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial y^4} \right) + \left( \frac{1}{Da} + M^2 \right) \frac{\partial^2 \psi}{\partial y^2} + Gr \frac{\partial T_1}{\partial y} + \frac{1}{Da} \frac{\partial^2 \psi}{\partial x^2} + \frac{2F}{\sqrt{Da}} \left( u_0 \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial u_0}{\partial y} \frac{\partial \psi}{\partial y} \right) = 0 \quad (28)$$

$$Pr \left( u_0 \frac{\partial T_1}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T_0}{\partial y} \right) = \frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} \quad (29)$$

In view of boundary conditions (19), we assume general solution for  $\psi$  and  $T_1$  in the following form:

$$\psi(x, y) = \text{Real} \left( \sum_{r=0}^{\infty} \psi_r \lambda^r e^{i\lambda x} \right) = \text{Real} \left[ \psi_0 e^{i\lambda x} + \psi_1 \lambda e^{i\lambda x} \right] \quad (30)$$

$$T_1(x, y) = \text{Real} \left( \sum_{r=0}^{\infty} \theta_r \lambda^r e^{i\lambda x} \right) = \text{Real} \left[ \theta_0 e^{i\lambda x} + \theta_1 \lambda e^{i\lambda x} \right] \quad (31)$$

Introducing (30) and (31) into the Equations. (28) and (29), we get the following set of ordinary differential equations for ( $r = 0, 1$ ):

$$\frac{d^2 \theta_0}{dy^2} = 0 \quad (32)$$

$$\gamma \frac{d^4 \psi_0}{dy^4} - \left( \frac{1}{Da} + M^2 \right) \frac{d^2 \psi_0}{dy^2} = Gr \frac{d\theta_0}{dy} + \frac{2F}{\sqrt{Da}} \left( u_0 \frac{d^2 \psi_0}{dy^2} + \frac{du_0}{dy} \frac{d\psi_0}{dy} \right) \quad (33)$$

$$\gamma \frac{d^4 \psi_1}{dy^4} - \left( \frac{1}{Da} + M^2 \right) \frac{d^2 \psi_1}{dy^2} = Gr \frac{d\theta_1}{dy} + i \left( u_0 \frac{d^2 \psi_0}{dy^2} - \psi_0 \frac{d^2 u_0}{dy^2} \right) + \frac{2F}{\sqrt{Da}} \left( u_0 \frac{d^2 \psi_1}{dy^2} + \frac{du_0}{dy} \frac{d\psi_1}{dy} \right) \quad (34)$$

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$$\frac{d^2\theta_1}{dy^2} = iPr \left( u_0\theta_0 + \psi_0 \frac{dT_0}{dy} \right). \quad (35)$$

In the light of (30), (31) and (27), the boundary conditions (19) corresponding to  $\psi_0$ ,  $\theta_0$ ,  $\psi_1$  and  $\theta_1$  become:

$$\begin{aligned} \psi_0 = 0, \quad \frac{d\psi_0}{dy} = \frac{du_0}{dy}, \quad \theta_0 = -\frac{dT_0}{dy} \quad \text{at } y = -1. \\ \psi_0 = 0, \quad \frac{d\psi_0}{dy} = 0, \quad \theta_0 = 1 \quad \text{at } y = 1. \end{aligned} \quad (36.1)$$

$$\psi_1 = 0, \quad \frac{d\psi_1}{dy} = 0, \quad \theta_1 = 0 \quad \text{at } y = -1. \quad \psi_1 = 0, \quad \frac{d\psi_1}{dy} = 0, \quad \theta_1 = 0 \quad \text{at } y = 1.$$

$$(\theta_1)_{y=-1} = 0, \quad \left( \frac{d\theta_1}{dy} \right)_{y=1} = 0, \quad (\psi_1)_{y=\pm 1} = 0, \quad \left( \frac{d\psi_1}{dy} \right)_{y=\pm 1} = 0. \quad (36.2)$$

The solution of Eq. (32) satisfying corresponding boundary condition (36.1) is:

$$\theta_0 = C_5 y + C_6. \quad (37)$$

The Equations. (33) and (34) are coupled equations and cannot be solved directly. Hence we assume:

$$\psi_0 = \psi_{00} + F\psi_{01} \quad \text{and} \quad \psi_1 = \psi_{10} + F\psi_{11}. \quad (38)$$

Introducing (38) into Equations. (33) and (34) we obtain:

$$\frac{d^4\psi_{00}}{dy^4} - A_1 \frac{d^2\psi_{00}}{dy^2} = \frac{C_5}{\gamma} Gr. \quad (39)$$

$$\frac{d^4\psi_{01}}{dy^4} - A_1 \frac{d^2\psi_{01}}{dy^2} = \frac{2}{\gamma\sqrt{Da}} \left( u_{00} \frac{d^2\psi_{00}}{dy^2} + \frac{du_{00}}{dy} \frac{d\psi_{00}}{dy} \right). \quad (40)$$

$$\frac{d^4\psi_{10}}{dy^4} - A_1 \frac{d^2\psi_{10}}{dy^2} = \frac{Gr}{\gamma} \frac{d\theta_1}{dy} + \frac{i}{\gamma} \left( u_{00} \frac{d^2\psi_{00}}{dy^2} - \psi_{00} \frac{d^2u_{00}}{dy^2} \right). \quad (41)$$

$$\begin{aligned} \frac{d^4\psi_{11}}{dy^4} - A_1 \frac{d^2\psi_{11}}{dy^2} = \frac{i}{\gamma} \left( u_{00} \frac{d^2\psi_{01}}{dy^2} + u_{01} \frac{d^2\psi_{00}}{dy^2} - \psi_{01} \frac{d^2u_{00}}{dy^2} \right. \\ \left. - \psi_{00} \frac{d^2u_{01}}{dy^2} \right) + \frac{2}{\gamma\sqrt{Da}} \left( u_{00} \frac{d^2\psi_{10}}{dy^2} + \frac{du_{00}}{dy} \frac{d\psi_{10}}{dy} \right). \end{aligned} \quad (42)$$

The boundary conditions corresponding to  $\psi_{00}$ ,  $\psi_{01}$ ,  $\psi_{10}$  and  $\psi_{11}$  are:

$$\psi_{00} = 0, \quad \psi_{01} = 0, \quad \frac{d\psi_{00}}{dy} = \frac{du_{00}}{dy}, \quad \frac{d\psi_{01}}{dy} = \frac{du_{01}}{dy} \quad \text{at } y = -1.$$

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$$\begin{aligned} \psi_{10} = 0, \quad \psi_{11} = 0, \quad \frac{d\psi_{10}}{dy} = 0, \quad \frac{d\psi_{11}}{dy} = 0 \quad \text{at } y = -1. \\ \psi_{00} = 0, \quad \psi_{01} = 0, \quad \frac{d\psi_{00}}{dy} = 0, \quad \frac{d\psi_{01}}{dy} = 0 \quad \text{at } y = 1. \\ \psi_{10} = 0, \quad \psi_{11} = 0, \quad \frac{d\psi_{10}}{dy} = 0, \quad \frac{d\psi_{11}}{dy} = 0 \quad \text{at } y = 1. \end{aligned} \quad (43)$$

The solutions of Equations. (39)-(42) satisfying (43) under corresponding boundary conditions (36):

$$\psi_{00}(y) = C_7 y + C_8 + C_9 e^{\sqrt{A_1} y} + C_{10} e^{-\sqrt{A_1} y} - K_{14} y^2. \quad (44)$$

$$\begin{aligned} \psi_{01}(y) = C_{11} y + C_{12} + C_{13} e^{\sqrt{A_1} y} + C_{14} e^{-\sqrt{A_1} y} + K_{16} e^{2\sqrt{A_1} y} + K_{17} e^{-2\sqrt{A_1} y} + K_{18} y e^{\sqrt{A_1} y} - K_{19} y e^{-\sqrt{A_1} y} \\ + K_{20} y^2 e^{\sqrt{A_1} y} - K_{21} y^2 e^{-\sqrt{A_1} y} - K_{22} y^3 + K_{23} y^2. \end{aligned} \quad (45)$$

The solution of Eq. (35) introducing (21) and (38) satisfying conditions (43) is:

$$\begin{aligned} \theta_1 = C_{15} y + C_{16} + iPr \left[ (K_{31} y + K_{32}) e^{2\sqrt{A_1} y} + (K_{33} y + K_{34}) e^{-2\sqrt{A_1} y} \right. \\ \left. + (K_{35} y^3 + K_{36} y^3 + K_{37} y + K_{38}) e^{\sqrt{A_1} y} \right. \\ \left. + (K_{39} y^3 + K_{40} y^3 + K_{41} y + K_{42}) e^{-\sqrt{A_1} y} \right. \\ \left. + K_{43} y^5 + K_{44} y^4 + K_{45} y^3 + K_{46} y^2 \right]. \end{aligned} \quad (46)$$

$$\begin{aligned} \psi_{10} = C_{17} y + C_{18} + C_{19} e^{\sqrt{A_1} y} + C_{20} e^{-\sqrt{A_1} y} + i \left\{ (K_{51} y + K_{52}) e^{2\sqrt{A_1} y} + (K_{53} y + K_{54}) e^{-2\sqrt{A_1} y} + (K_{55} y^4 + K_{56} y^3 + K_{57} y^2 + K_{58} y) e^{\sqrt{A_1} y} \right. \\ \left. + (K_{59} y^4 + K_{60} y^3 + K_{61} y^2 + K_{62} y) e^{-\sqrt{A_1} y} \right. \\ \left. + K_{63} y^6 + K_{64} y^5 + K_{65} y^4 - K_{66} y^3 - K_{67} y^2 \right\}. \end{aligned} \quad (47)$$

$$\begin{aligned} \psi_{11} = C_{21} y + C_{22} + C_{23} e^{\sqrt{A_1} y} + C_{24} e^{-\sqrt{A_1} y} + i \left\{ K_{78} e^{3\sqrt{A_1} y} + K_{79} e^{-3\sqrt{A_1} y} + K_{80} e^{2\sqrt{A_1} y} + K_{81} e^{-2\sqrt{A_1} y} + K_{82} y e^{3\sqrt{A_1} y} + K_{83} y e^{-3\sqrt{A_1} y} \right. \\ \left. + K_{84} y e^{2\sqrt{A_1} y} + K_{85} y e^{-2\sqrt{A_1} y} + K_{86} y e^{\sqrt{A_1} y} + K_{87} y e^{-\sqrt{A_1} y} \right. \\ \left. + K_{88} y^2 e^{2\sqrt{A_1} y} + K_{89} y^2 e^{-2\sqrt{A_1} y} + K_{90} y^2 e^{\sqrt{A_1} y} + K_{91} y^2 e^{-\sqrt{A_1} y} \right. \\ \left. + K_{92} y^3 e^{2\sqrt{A_1} y} + K_{93} y^3 e^{-2\sqrt{A_1} y} + K_{94} y^3 e^{\sqrt{A_1} y} + K_{95} y^3 e^{-\sqrt{A_1} y} \right. \\ \left. + K_{96} y^4 e^{2\sqrt{A_1} y} + K_{97} y^4 e^{-2\sqrt{A_1} y} + K_{98} y^4 e^{\sqrt{A_1} y} + K_{99} y^4 e^{-\sqrt{A_1} y} \right. \\ \left. + K_{100} y^5 e^{\sqrt{A_1} y} + K_{101} y^5 e^{-\sqrt{A_1} y} + K_{102} y^6 e^{\sqrt{A_1} y} + K_{103} y^6 e^{-\sqrt{A_1} y} \right. \\ \left. - K_{104} y^7 - K_{105} y^6 - K_{106} y^5 + K_{107} y^4 + K_{108} y^3 + K_{109} y^2 \right\}. \end{aligned} \quad (48)$$

From these solutions the first order quantities  $\psi_1$  and  $\theta_1$  can be expressed in the following form:

$$\psi_1 = (\psi_r + i\psi_i) = \psi_{10} + \lambda \psi_{11}. \quad (49)$$



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$$\theta_1 = (\theta_r + i\theta_i) = \theta_{10} + \lambda\theta_{11}, \quad (50)$$

where the suffix  $r$  denotes the real part and  $i$  denotes the imaginary part.

Considering only the real part, the expressions for first order velocity and first order temperature become:

$$u_1 = \varepsilon \left( \frac{d\psi_i}{dx} \lambda \sin(\lambda x) - \frac{d\psi_r}{dy} \cos(\lambda x) \right).$$

$$(51) v_1 = \varepsilon \left( -\lambda\psi_r \sin(\lambda x) - \lambda^2\psi_i \cos(\lambda x) \right). \quad (52)$$

$$T_1 = \varepsilon (\theta_r \cos(\lambda x) - \lambda \sin(\lambda x) \theta_i). \quad (53)$$

The entire solution for the velocity components and temperature distribution is the summation of the mean part and the perturbed part.

### IV. SHEAR STRESS

The shearing stress ( $\tau_{xy}$ ) at any point in the fluid in non-dimensional form is given by:

$$\tau_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{du_0}{dy} + \varepsilon e^{i\lambda x} \frac{du_1}{dy} + \varepsilon i \lambda e^{i\lambda x} v_1(y). \quad (54)$$

Therefore, the shear stress ( $\tau_w$ ) at the wavy wall  $y = -1 + \varepsilon \cos(\lambda x)$  and the shear stress ( $\tau_f$ ) at the parallel flat wall  $y = 1$  in non-dimensional form can be expressed as:

$$\tau_w = \left( \frac{du_0}{dy} \right)_{y=-1} + \varepsilon Re \left[ e^{i\lambda x} \left( \frac{d^2 u_0}{dy^2} (-1) + \frac{d^2 \psi_1}{dy^2} \right) \right]. \quad (55)$$

and

$$\tau_f = \left( \frac{du_0}{dy} \right)_{y=1} + \varepsilon Re \left( e^{i\lambda x} \frac{d^2 \psi}{dy^2} (1) \right). \quad (56)$$

### V. HEAT TRANSFER RATE

The heat transfer rate in terms of Nusselt number ( $Nu$ ) in non-dimensional form is given by:

$$Nu = \frac{\partial T}{\partial y} = \frac{dT_0}{dy} + \varepsilon Re \left( e^{i\lambda x} \frac{dT_1}{dy} \right). \quad (57)$$

Hence, the Nusselt number ( $Nu_w$ ) at the wavy wall and the Nusselt number ( $Nu_f$ ) at the flat wall in non-dimensional form take the form:

$$Nu_w = \left( \frac{d\theta_0}{dy} \right)_{y=-1} + \varepsilon Re \left( e^{i\lambda x} \frac{d\theta}{dy} (-1) \right). \quad (58)$$

$$Nu_f = \left( \frac{d\theta_0}{dy} \right)_{y=1} + \varepsilon Re \left( e^{i\lambda x} \frac{d\theta}{dy} (1) \right). \quad (59)$$

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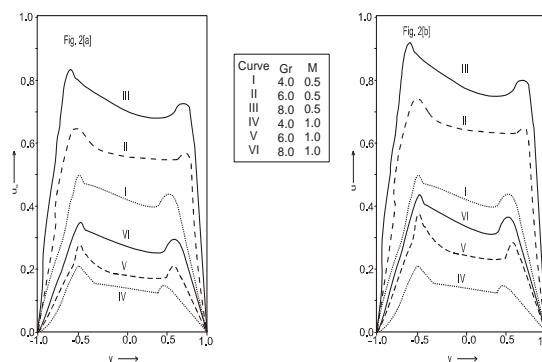
## VI. RESULTS AND DISCUSSION

Magneto hydrodynamic (MHD) mixed convection flow through porous medium bounded in a vertical channel consisting of a wavy wall and a parallel flat wall is investigated analytically. In order to simulate the momentum transfer within the porous medium, the Brinkman-Forchheimer extended Darcy model commonly known as Darcy-Brinkman-Forchheimer model is considered. Consideration of the Darcy-Brinkman-Forchheimer model gives rise to an additional empirical parameter ( $F$ ), which is known as Forchheimer parameter (inertia constant). The numerical value of the Forchheimer parameter ( $F$ ) depends upon the porosity of the porous medium ( $\epsilon$ ). If the porous medium consists of concrete (ordinary mixes) or concrete (bituminous) then  $\epsilon$  lies in the range  $0.02 \leq \epsilon \leq 0.07$  [Neild & Bejan, 2006]. When  $\epsilon = 0.02$ , the value of  $F$  is found as 0.01 approximately. As such  $F \ll 1$ , which is used as perturbation parameter in (21). Non-linear equations governing the motion have been solved by linearization technique, wherein the flow is assumed to be consisting of two parts; a mean part and a perturbed part. Exact solutions are obtained for the mean part and are expressed in (20), (25) and (26). The perturbed part is solved using long wave approximation, which are expressed in (51)-(53). The solutions for axial velocity ( $u$ ), transverse velocity ( $v$ ) and temperature distribution ( $T$ ) are numerically evaluated for several sets of values of parameters.

The Forchheimer parameter (inertia constant)  $F$ , appearing in Eq. (1) is evaluated from the following empirical formula suggested by Ergun (1952).

$$F = \frac{1.75}{\sqrt{150 \epsilon^3}}$$

The effects of material parameters on the axial velocity ( $u$ ), zeroth order velocity ( $u_0$ ), transverse velocity ( $v$ ), temperature distribution ( $T$ ), shear-stress at the wavy wall ( $\tau_w$ ); shear stress at the flat wall ( $\tau_f$ ); rate of heat transfer (Nusselt number) at the wavy wall ( $Nu_w$ ) and the parallel flat wall ( $Nu_f$ ) are presented graphically and discussed.



**Fig. 2 :** Effect of Grashof number ( $Gr$ ) and magnetic field ( $M$ ) on [a] zeroth order axial velocity ( $u_0$ ) and [b] total axial velocity ( $u$ ) at  $Da = 10^{-5}$ ,  $\lambda = 0.05$ ,  $F = 0.01$ ,  $Pr = 1.0$ ,  $\epsilon = 0.02$ ,  $\gamma = 6.0$ ,  $T_R = 0.5$  and  $\lambda_x = 0.78$ .

Fig.-2a is intended to illustrate the effect of Grashof number ( $Gr$ ) and the magnetic parameter ( $M$ ) on the zeroth order axial velocity ( $u_0$ ). It is observed that the effect of increasing Grashof number ( $Gr$ ) is to increase the zeroth order axial velocity ( $u_0$ ), whereas the effect of the magnetic parameter ( $M$ ) is to decrease it. Also, the zeroth order axial velocity in the vicinity of the wavy wall is more enhanced as compared to the velocity in the vicinity of the flat plate. The effect of Grashof number ( $Gr$ ) and the magnetic parameter ( $M$ ) on the total axial velocity ( $u$ ) is shown in Fig.-2b. It is noted that the effect of Grashof number ( $Gr$ ) and the magnetic parameter ( $M$ ) on the axial velocity ( $u$ ) is similar to that on the

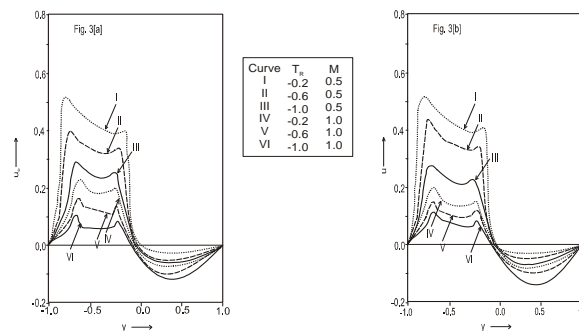
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zeroth order axial velocity ( $u_0$ ). Also, maximum magnitudes of the velocities  $u_0$  and  $u$  are recorded for maximum value of the Grashof number ( $Gr$ ) and minimum magnitude of the magnetic parameter ( $M$ ).



**Fig. 3:**Effect of temperature ratio ( $T_R < 0$ ) on [a] zeroth order axial velocity ( $u_0$ ) and [b] total axial velocity ( $u$ ) at  $Gr = 4.0$ ,  $Da = 10^{-5}$ ,  $\lambda = 0.05$ ,  $F = 0.01$ ,  $Pr = 1.0$ ,  $\epsilon = 0.02$ ,  $\gamma = 6.0$  and  $\lambda x = 0.78$ .

Fig.-3a illustrates the effect of the temperature ratio parameter ( $T_R$ ) and the magnetic field parameter ( $M$ ) on the zeroth order axial velocity ( $u_0$ ) for the case  $T_R < 0$ , in the range  $0 < T_R \leq -1$ . It is observed that an increase in the numerical value of the temperature ratio ( $T_R < 0$ ) decreases the zeroth order axial velocity ( $u_0$ ) and the presence of the magnetic field ( $M$ ) also decreases it. The velocity ( $u_0$ ) is positive upto half width of the channel, approximately, from the wavy wall ( $y = -1$ ), whereas reverse flow is observed in the remaining flow region. The effect of the temperature ratio parameter ( $T_R$ ) and the magnetic field ( $M$ ) on the axial velocity ( $u$ ) is shown in Fig.-3b.

## VII. CONCLUSION

Comparison of the effect of temperature ratio parameter of the present paper with that of steady case of Paul et al. (2006) show that findings of the present paper are in agreement under the same physical situation of the channel walls and the boundary conditions. From the obtained graphs, the following conclusions are drawn:

The effect of increasing Grashof number ( $Gr$ ) is to increase the zeroth order axial velocity ( $u_0$ ). The effect of the magnetic parameter ( $M$ ) is to decrease the zeroth order axial velocity and total axial velocity.

An increase in the numerical value of the temperature ratio  $T_R < 0$  in the range  $0 \leq T_R \leq -1$  decreases the zeroth order axial velocity ( $u_0$ ) and the total axial velocity ( $u$ ), whereas an increase in  $T_R$  increases these velocities and the reverse flow exists. For the case  $T_R > 0$  an increase in  $T_R$  increases  $u_0$  as well as  $u$  in the range  $1 \geq T_R \geq 0$ . When  $T_R = 0$ , the enhanced velocity is noted near the wavy wall and when  $T_R = 1$ , the velocity increases symmetrically. The magnitude of velocity is optimum for  $T_R = 1$  and minimum  $T_R = -1$ . When  $T_R = -1$ , the reverse flow is noted at the right flat wall. The effect Darcy number ( $Da$ ) is to increase in the zeroth order velocity ( $u_0$ ) as well as in total axial velocity ( $u$ ). For  $T_R = 1$ , the temperature field is constant and is independent of the distance parallel to the  $y$ -axis; whereas for  $T_R = -1$ , the temperature decreases at constant rate towards the  $y$ -axis. The temperature increases with increase in the magnitude of Prandtl number.

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The shear stress ( $\tau_f$ ) at the flat wall decreases whereas the shear stress ( $\tau_w$ ) at the wavy wall increases as Grashof number ( $Gr$ ) increases. The effect of Grashof number ( $Gr$ ) is to decrease the Nusselt number ( $Nu_w$ ) at the wavy wall and to increase the Nusselt number ( $Nu_f$ ) at the flat wall.

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