

Alternating Current Best Power Flow Formulation in a deregulated Orbitary Energy Market

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ABSTRACT: The electricity deregulation of Power System with its attendant legislation poses problem for the Power System Operation in Nigeria. The aspect of retail Orbitary whereby the customers reserved the right of choice of Provider of the product has not been given much needed attention since the industry has only the major three segments of Generation, Transmission, and Distribution. In this paper attempt is made to formulate the concept of generation and load sets in modeling the behavior of Power System Supply and distribution as it affects retail wheeling or Orbitary market. The formulation of Best Power Flow(BPF) or Optimal Power Flow (OPF) in which the public interest are factored into the equation is demonstrated.

KEYWORD: Best Power Flow Formulation, deregulated energy market, Orbitary,

I. INTRODUCTION

Some years back, the Nigerian electricity market was deregulated and this had given rise to competition in her energy market. The quality of energy supply to the end-users was moribund during the time the Nigerian Energy market was vertically integrated. With the implementation of deregulation policy of 2009, various Generating Companies (GENCOs), and Distribution Companies (DISCOs), penetrated the energy market utilizing the services of Transmission Company of Nigeria (TCN or TRANSCOP) to deliver quality and on-demand electricity to end-users. In a largely populated nation like Nigeria, power demand fluctuation is expected and this poses problems for the energy players in the market. In order to tackle such scenario effectively, various energy/power optimization techniques are resorted. Among the prominent solution in this order include Alternating Current Optimal Power Flow (AC-OPF) solutions, Decoupled Optimal Power Flow (DC-OPF) solutions, Security Constrained Optimal Power Flow (SC-OPF) solutions, mixed-integer programming (MIP)etc.,problems. These problems are complex economically, electrically and computationally [1]. A multi-part nonlinear pricing model is required to establish efficient market equilibrium economically. The alternating current (AC) power flow in the system, introduces additional nonlinearities electrically and computationally, the optimization problem has nonconvexities, including both binary variables and continuous functions. The power system must be able to withstand contingencies, and the system operator must make binary decisions to start up and shut down generation and transmission assets in response to system events optimally.

An economically efficient and reliable Independent System Operator (ISO) power market depends on credible formulation and operation of AC-OPF problem. Over the years engineers use approximations, decompositions, and engineering judgment to obtain reasonably acceptable solutions to this problem.

The optimal power flow algorithms or software, finds the optimal solution to an objective function subject to the power flow constraints and other operational constraints, such as generator minimum output constraints, transmission stability, voltage constraints, and limits on switching mechanical equipment. Optimal power flow is sometimes referred to as security-constrained economic dispatch (SCED) when rotor angle synchronization or fall-out and other time variant factors are considered. Most implementations of SCED include only thermal limits, and proxies for voltage limits.

In the 1960's, Newton-Raphson method became commonly used for ACOPF solution [2] alongside Gauss Seidel method, after [3] developed sparsity techniques to take advantage of the structure of the admittance matrix in the AC-OPF problem. The admittance matrix is sparse, meaning it has many zero elements; this is because power system networks are not densely connected. The techniques of sparsity can be used to reduce data storage and increase computational speed [4]. Chaotic Whale Optimization Algorithm [5] is among the recent solution software applied to ACOPF and SCOPF problems.

II. CONCEPT OF ORBITARY ENERGY MARKET

This concept is used to describe demand-supply driven energy market. The structure of the system is presented in figure 1. It features a system in which the end-users of electricity enjoy the liberty of cross-selectivity.

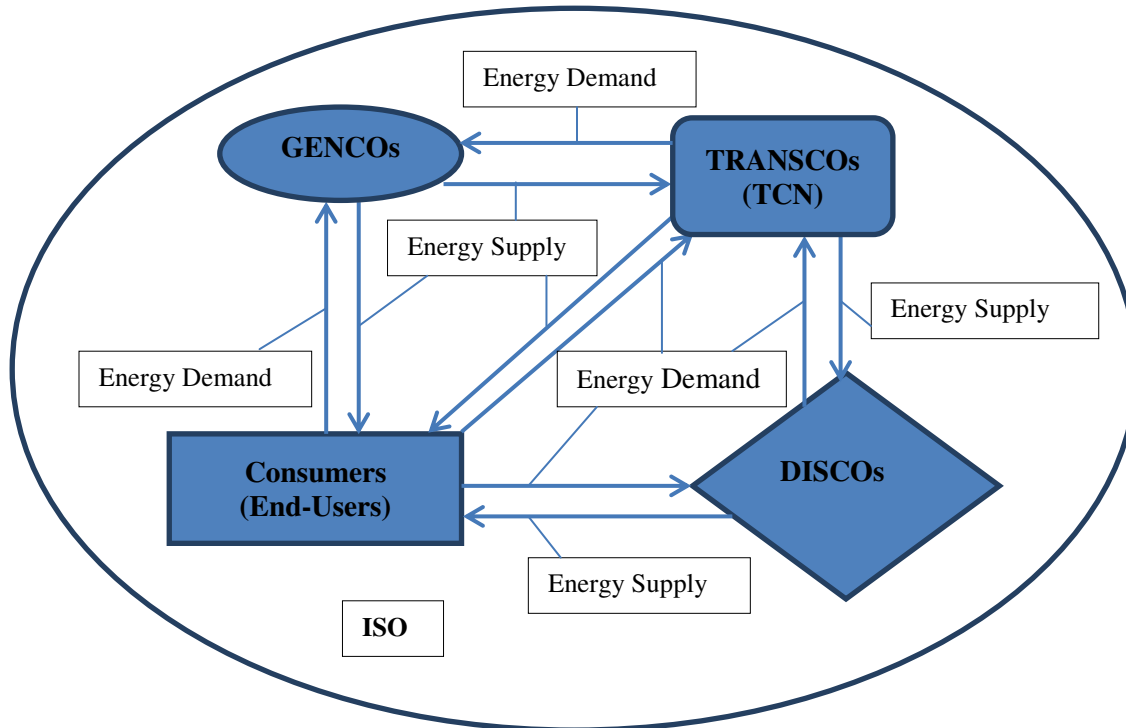


Figure 1: Structure of an Orbital energy market.

They can order or demand power and absorb power from the DISCOs at consumer level or from nearby GENCOs. At Retail or semi-wholesale level they can demand and absorb power directly from the TCN (TRANSCOP) in the form of large companies or independent bulk power users. The system allows free-wheeling [7] of electric power in the energy market. The ISO ensures proper transactions are carried out to the benefit of all parties involved in the energy market.

III. TECHNICAL CONCEPT

AC-OPF or Alternating Current Best Power Flow (AC-BPF) is an optimization algorithm that considers the full AC power flow equations. With the assumption that the model parameters are correct, this is the most accurate representation of the power flows in a system [6]. The solutions determined by this optimization correspond almost to reality. The AC power flow equations are quadratic equations (due to the power to voltage square dependency), and including them in an optimization problem as equality constraints, result to a non-linear non-convex problem. Thus the usual procedure to solving this equation is to relax the non-convex problem to a convex one. That is, defining a convex function around the non-convex function and solving them to obtain the global optimum. The convex relaxations of the AC-BPF focusing on the Semi-definite Programming formulation shall form the basis of our AC-BPF problem formulation. One of the central elements for the implementation of the AC-BPF is the modeling of the transmission lines.

Transmission line Modeling: The π -model: The most usual representation of a Transmission line in power systems is usually by the so-called π -model or pi-Network. For lines between 25 km and 250 km, the π -model is the most preferred modeling approach [6].

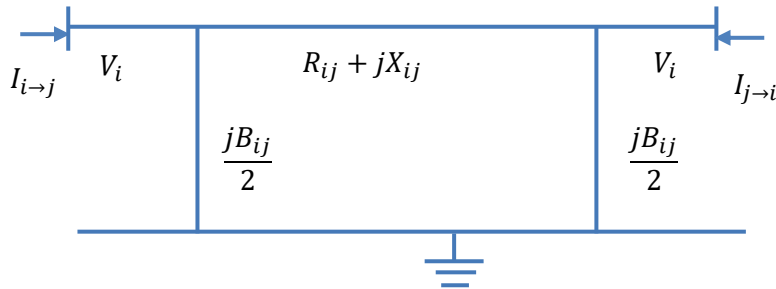


Figure 2: Sample transmission line

Assume a transmission line connected between the nodes i and j in a network, as shown in Figure: 2. The π -model of the transmission line consists of a series impedance $R_{ij} + jX_{ij}$ connected at the nodes i and j , and two equal shunt susceptances, one connected at node i and the other at node j . The π -model is usually represented by the series admittance:

$$y_{ij} = \frac{1}{R_{ij} + jX_{ij}} \tag{1}$$

and by the shunt susceptances at both ends of the line:

$$y_{sh,i} = y_{sh,j} = \frac{jB_{ij}}{2} \tag{2}$$

The total current entering node i

$$I_{i \rightarrow j} = I_{sh,i} + I_{ij} = y_{sh,i}V_i + y_{ij}(V_i - V_j) \tag{3}$$

In Matrix form equation (3) can be written as follows:

$$I_{i \rightarrow j} = [y_{sh,i} + y_{ij} \quad -y_{ij}] \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{4}$$

Similarly, $I_{j \rightarrow i} = [-y_{sh,j} - y_{ij} \quad y_{ij}] \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{5}$

The line admittance can be written as: $I_{line} = Y_{line}V \tag{6}$

Taken equations (4) and (5) into consideration this can be further expanded into;

$$\begin{bmatrix} I_{1 \rightarrow 2} \\ \vdots \\ I_{i \rightarrow j} \\ \vdots \\ I_{m \rightarrow n} \end{bmatrix} = \begin{bmatrix} y_{sh,1} + y_{12} & -y_{12} & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & y_{sh,i} + y_{ij} & \cdots & -y_{ij} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & y_{sh,m} + y_{mn} & \cdots & -y_{mn} & \cdots \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_j \\ \vdots \\ V_n \end{bmatrix} \tag{7}$$



and,

$$\begin{bmatrix} I_{2 \rightarrow 1} \\ \vdots \\ I_{j \rightarrow i} \\ \vdots \\ I_{n \rightarrow m} \end{bmatrix} = \begin{bmatrix} -y_{12}y_{sh,1} + y_{12} & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -y_{ij} & \cdots & y_{sh,i} + y_{ij} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & -y_{mn} & \cdots & y_{sh,m} + y_{mn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_j \\ \vdots \\ V_n \end{bmatrix} \tag{8}$$

The AC net apparent power at bus i is equal to the product of the bus voltage (complex number) and the conjugate of the bus current (complex number), equation (9).

$$S_i = V_i I_i^* \tag{9}$$

The bus current;

$$I_i = \sum_k I_{ik} \tag{10}$$

where k are all the buses connected to bus i . In terms of the line or network admittances;

$$I_i = V^* \sum_k Y_{ik} = V^* \sum_k \frac{1}{Z_k} \tag{11}$$

Therefore the apparent power can be given by;

$$S_i = V_i I_i^* = V^* V_i \sum_k Y_{ik} \tag{12}$$

The net apparent power injection at every bus is equal to the total generation minus the total load connected at the bus. In vector form:

$$S = S_{gen} - S_{load} = V^* V_i \sum_k Y_{ik} \tag{13}$$

$$\text{Or } P_i - Q_i = V^* V_i \sum_k Y_{ik} \tag{14}$$

IV. AC-BPF FORMULATION

The AC-BPF can be formulated as minimize the total cost of generation, taken line current, apparent power, bus voltage magnitudes, and angle as constraints.

$$AC - BPF = \underset{P_{Gi}}{Min} c^T P_G \tag{15}$$

Subject to:

$$AC \text{ flow } S_{gen} - S_{load} = V^* V_i \sum_k Y_{ik} \tag{a}$$

$$\text{Line Current } \mathbf{V} \sum_k \mathbf{Y}_{ik} \leq I_{i,max} \quad (\text{b})$$

$$\text{Line Apparent flow } \mathbf{V}^* \mathbf{V}_i \sum_k \mathbf{Y}_{ik} \leq S_{i,max} \quad (\text{c})$$

$$\text{Generator Active power } 0 \leq \mathbf{P}_G \leq \mathbf{P}_{G,max} \quad (\text{d})$$

$$\text{Generator Reactive Power } -Q_{G,max} \leq \mathbf{Q}_G \leq Q_{G,max} \quad (\text{e})$$

$$\text{Voltage Magnitude } V_{min} \leq V \leq V_{max} \quad (\text{f})$$

$$\text{Voltage Angle } \delta_{min} \leq \delta \leq \delta_{max} \text{ or } \delta_{min} - \pi \leq \delta \leq \pi = \delta_{max} \quad (\text{g})$$

Where **P**-subscript G refers to the active power generation injected at bus *i* and **c**-superscript T is the total cost of generation. Note that all variables are vectors or matrices. The asterisk sign or operator (*) denotes complex conjugate values.

Constraints (a) are the only equality constraints in the standard AC-BPF formulation, and they denote the AC power flow equations. Any operating point that will be determined through the optimization must satisfy the AC power flow equations in order to be a true operating point of the power system [6]. Constraints (b) represent the line current inequality constraints. If our transmission line limits are represented by the current thermal limit, these inequality constraints must be satisfied. If on the other hand, the line thermal limits are sets by the apparent power, then (c) must be used; which refer to the apparent power flow. *In any case, only one constraint must be used:* either the line current limit (b) or the apparent power line flow limit (c). Constraints (d) refer to the active power bounds of the generators, while constraints (e) refer to the generators' reactive power bounds. Constraints (f) refer the maximum and minimum allowable limits for the bus voltage magnitudes, and constraints (g) refer to maximum and minimum allowable limits for the voltage angles.

V. AC-BPF APPLICATION TO NIGERIAN EASTERN GRID

The formulated AC-BPF problem is tested on eastern regional grid (E-G) network shown in figure 3 with parameters obtained from Nigerian Transmission Company (TCN) Osogbo and Abuja. The network contains 6 buses and 9 branches similar to Matpower case 6 example obtained from pp. 104, 112, 119, 123-124, 549 of [10]. The simulation was done with Matpower Version 7.0; a power system simulation toolbox developed for use in Matlab and Octave environment by Power Systems Engineering Research Center (PSERC).

MATPOWER Interior Point Solver (MIPS), Version 1.3.1 of Matpower 7.0 with primal/dual interior point method computed the AC-Best Power Flow of AC-BPF formulation of equation (15) subject to voltages and power balance constraints of (a, d and f).

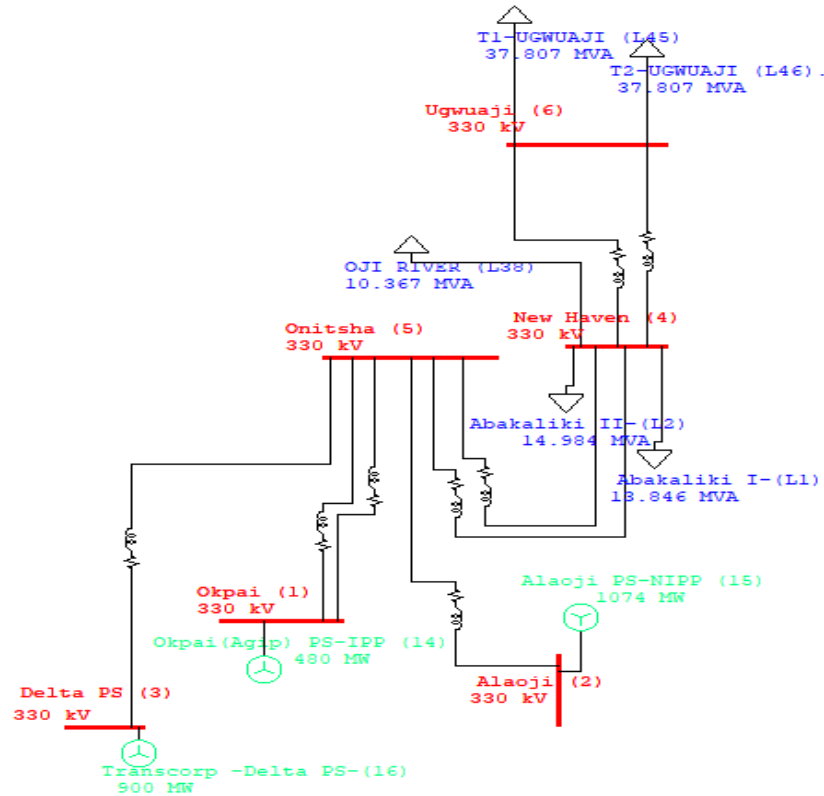


Figure 3: Eastern Grid Nigerian 6 buses Network.

VI. AC-BPF RESULTS AND ANALYSIS

The simulation Converged in **0.08seconds** and the Objective Function Value = **2515.95 \$/hr.**

Table 1: System Summary

How many?	How much?	P (MW)	Q (MVar)
Buses 6	Total Gen Capacity	530.0	-300.0 to 300.0
Generators 3	On-line Capacity	530.0	-300.0 to 300.0
Committed Gens 3	Generation (actual)	165.0	89.2
Loads 3	Load	162.0	162.0
Fixed 3	Fixed	162.0	162.0
Dispatchable 0	Dispatchable	-0.0 of -0.0	-0.0
Shunts 0	Shunt (inj)	-0.0	0.0
Branches 9	Losses (I ² * Z)	3.01	24.14
Inter-ties 0	Total Inter-tie Flow	0.0	0.0
Areas 1			
	Minimum	Maximum	
Voltage Magnitude	0.954 p.u. @ bus 4	1.070 p.u. @ bus 3	
Voltage Angle	-4.45 deg @ bus 6	0.00 deg @ bus 1	
P Losses (I ² *R)	-	0.64 MW @ line 1-4	
Q Losses (I ² *X)	-	5.00 MVar @ line 3-6	
Lambda P	11.52 \$/MWh @ bus 1	11.81 \$/MWh @ bus 6	
Lambda Q	0.00 \$/MWh @ bus 1	0.30 \$/MWh @ bus 4	

The system performance summary is shown in Table 1 whereas table 2 shows the bus performance



Table: 2 Bus Data

Bus #	Voltage		Generation		Load		Lambda(\$/MVA-hr)	
	Mag(pu)	Ang. (deg)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)	P	Q
1	1.050	0.000*	50.00	17.79	-	-	11.517	-
2	1.050	-0.222	67.63	21.19	-	-	11.535	-
3	1.070	-0.176	47.38	50.20	-	-	11.535	-
4	0.954	-4.442	-	-	54.00	54.00	11.806	0.302
5	0.998	-2.834	-	-	54.00	54.00	11.696	0.167
6	0.965	-4.452	-	-	54.00	54.00	11.809	0.292
Total			165.01	89.18	162	162		

Table 3 shows the branch data generated by the simulation. In other words the branch/line flows and losses are presented in table 3. Table 4 and 5 presents voltage and generation constraints respectively.

Table 3: Branch Data

Branch #	Bus		From Bus Injection		To Bus Injection		Loss (I ² * Z)	
	From	To	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)
1	1	2	1.28	-27.16	-1.27	-26.86	0.001	0.00
2	1	4	29.56	29.75	-28.92	-26.01	0.643	4.94
3	1	5	19.16	15.20	-18.94	-14.75	0.221	1.70
4	2	3	-1.01	-11.19	1.02	1.19	0.014	0.12
5	2	4	25.60	23.29	-25.08	-27.99	0.514	4.36
6	2	5	16.18	9.87	-16.01	-17.87	0.170	1.45
7	2	6	28.14	26.08	-27.60	-23.16	0.538	4.14
8	3	5	19.37	22.50	-19.05	-21.37	0.314	2.42
9	3	6	26.99	26.51	-26.40	-30.84	0.590	5.00
Total							3.005	24.14

Table 4: Voltage Constraints

Bus #	Vmin mu	Vmin	V	Vmax	Vmax mu
1	0.000	1.050	1.050	1.050	28.150
2	0.000	1.050	1.050	1.050	72.036
3	13.343	1.070	1.070	1.070	0.000

Table 5: Generation Constraints

Gen #	Bus #	Active Power Limits				
		Pmin mu	Pmin	Pg	Pmax	Pmax mu
1	1	0.685	50.00	50.00	200.00	-

VII. CONCLUSION

Alternating Current Best Power Flow cost model in an Orbital energy market was formulated in this article, taking all its attendant constraints into consideration. The formulated model was applied to Nigerian Eastern Grid Power Network and the results obtained presented. The optimized cost of total generation in the network was found to be **2515.95 \$/hr**. The Lambda (λ) or cost of generation required to meet the load demand at bus 4 and 6 are presented in table 1 among others.



Worthy of note is the total demand (162 MW) as against total actual generation (165 MW) in table 1. The network was well optimized such that excess power was not generated since the extra 3 MW was required to feed the line losses as shown in table 3.

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