

Modular Product of Fuzzy Graphs with Totally Regular Property

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ABSTRACT: In this paper, the total degree of vertices in the Modular Product of two fuzzy graphs in terms of those in G_1 and G_2 are determined for some particular cases and using them the totally regular property of Modular Product of two fuzzy graphs is studied. In general, the Modular Product of two totally fuzzy graphs need not be a totally regular fuzzy graph. The necessary and sufficient conditions for the Modular Product of two totally regular fuzzy graphs to be totally regular under some restrictions are obtained.

KEYWORDS: Total degree of a vertex, Totally Regular fuzzy graph, Cartesian, Alpha, Beta, Gamma, Tensor, Normal and Modular Product.

I. INTRODUCTION

A fuzzy subset of a set V is a mapping σ from V to $[0,1]$. A fuzzy graph G is a pair of functions $G:(\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ , (i.e.) $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The underlying crisp graph of $G : (\sigma, \mu)$ is denoted by $G^*:(V, E)$ where $E \subseteq V \times V$. Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness in fuzzy graphs. Mordeson. J. N and Peng. C. S introduced the concept of operations on fuzzy graphs.

The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Mordeson.J.N. and Peng.C.S[2]. Sunitha. M. S and Vijayakumar. A discussed about the complement of the operations of union, join, Cartesian product and composition on two fuzzy graphs. The degree of a vertex in some fuzzy graphs and the Regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operations Union, Join, Cartesian product, Composition and Conjunction was discussed by Nagoorgani.A and Radha. K. The degree of a vertex in alpha, beta, gamma product of two fuzzy graphs was discussed by Nagoorgani .A and Fathima kani.B. The total degree of a vertex in some fuzzy graphs and the totally regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operation union, join, Cartesian product, composition, disjunction, alpha, beta and gamma product was discussed by Radha.K. and Vijaya.M. Different types of product of fuzzy graphs by Shovan Dogra were studied. Regular property in Modular product of fuzzy graphs was discussed. In this paper we study about the total degree of a vertex in Modular Product of two fuzzy graphs and the totally regular property of Modular Product of two fuzzy graphs.

First we go through some preliminaries which can be found in [1]-[15].

II. PRELIMINARIES

Throughout this paper we assume that μ is reflexive and need not consider loops. Also, the underlying set V is assumed to be finite and σ can be chosen in any manner so as to satisfy the definition of a fuzzy graph in all the examples and all these properties are satisfied for all fuzzy graphs except null graphs.

Definition 2.1[3] : Let $G^*:(V, E)$ be a graph. The degree $d_{G^*}(v)$ of a vertex v in G^* is the number of edges incident with v . If all the vertices of G^* have the same degree r , then G^* is called a regular graph of degree r . Here r is an integer.

Definition 2.2[6] : Let $G:(\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u in G is defined by

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$$

Definition 2.3[6] : Let $G:(\sigma, \mu)$ be a fuzzy graph on G^* . The total degree of a vertex $u \in V$ is defined by



$$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u)$$

If each vertex of G has the same total degree k , then G is said to be a totally regular fuzzy graph of total degree k or a k -totally regular fuzzy graph.

Definition 2.4 [6]: If the underlying graph G^* is regular, then G is said to be a partially regular fuzzy graph. If G is both regular and partially regular, then G is said to be a full regular fuzzy graph.

Lemma 2.5[6]: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^*:(V, E)$. Then σ is a constant function if and only if the following are equivalent :

- (1) G is a regular fuzzy graph.
- (2) G is a totally regular fuzzy graph.

Definition 2.7[2]: The Cartesian product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph

$$G = G_1 \times G_2: (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2) \text{ on } G^* : (V, E) \text{ where } V = V_1 \times V_2 \text{ and } E = \{((u_1, v_1)(u_2, v_2)) / u_1 = v_1, u_2 v_2 \in E_2 \text{ or } u_2 = v_2, u_1 v_1 \in E_1\} \text{ with } (\sigma_1 \times \sigma_2)(u_1, v_1) = \sigma_1(u_1) \wedge \sigma_2(v_1), \text{ for all } (u_1, v_1) \in V_1 \times V_2 \text{ and } (\mu_1 \times \mu_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(v_1 v_2), & \text{if } u_1 = u_2, v_1 v_2 \in E_2 \\ \sigma_2(v_1) \wedge \mu_1(u_1 u_2), & \text{if } v_1 = v_2, u_1 u_2 \in E_1 \end{cases}$$

Definition 2.8 [21]: Let $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ be two fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively.. Then tensor product of G_1 and G_2 is a pair of functions $(\sigma_1 \otimes \sigma_2, \mu_1 \otimes \mu_2)$ with underlying vertex set $V_1 \otimes V_2 = \{(u_1, v_1) : u_1 \in V_1, v_1 \in V_2\}$ and underlying edge set

$$E_1 \otimes E_2 = \{((u_1, v_1)(u_2, v_2)) : u_1 u_2 \in E_1, v_1 v_2 \in E_2 \text{ or } u_1 u_2 \notin E_1, v_1 v_2 \notin E_2\} \text{ with } (\sigma_1 \otimes \sigma_2) = \sigma_1(u_1) \wedge \sigma_2(v_1), \text{ where } u_1 \in V_1, v_1 \in V_2$$

$$(\mu_1 \otimes \mu_2)((u_1, u_2)(v_1, v_2)) = \mu_1(u_1, v_1) \wedge \mu_2(u_2, v_2), \text{ if } u_1 v_1 \in E_1, u_2 v_2 \in E_2$$

Definition 2.9[21]: Let $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ be two fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then normal product of G_1 and G_2 is defined as a fuzzy graph

$$G_1 \circ G_2 = ((\sigma_1 \circ \sigma_2), (\mu_1 \circ \mu_2)) \text{ on } G^* : (V, E) \text{ where } V = V_1 \circ V_2 \text{ and } E = \{((u_1, v_1)(u_2, v_2)) : u_1 = u_2, v_1 v_2 \in E_2 \text{ (or) } v_1 = v_2, u_1 u_2 \in E_1 \text{ (or) } u_1 u_2 \in E_1, v_1 v_2 \in E_2\} \text{ with } \sigma_1 \circ \sigma_2 = \sigma_1(u_1) \wedge \sigma_2(v_1) \quad \forall (u_1, v_1) \in V_1 \times_\alpha V_2$$

$$(\mu_1 \circ \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2), & \text{if } u_1 = u_2, v_1 v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1), & \text{if } v_1 = v_2, u_1 u_2 \in E_1 \\ \mu_1(u_1, v_1) \wedge \mu_2(u_2, v_2), & \text{if } u_1 u_2 \in E_1, v_1 v_2 \in E_2 \end{cases}$$

Definition 2.10[11]: The α – product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph

$$G_1 \times_\alpha G_2 = ((\sigma_1 \times_\alpha \sigma_2), (\mu_1 \times_\alpha \mu_2)) \text{ on } G^* : (V, E) \text{ where } V = V_1 \times_\alpha V_2 \text{ and } E = \{((u_1, v_1)(u_2, v_2)) : u_1 = u_2, v_1 v_2 \in E_2 \text{ (or) } v_1 = v_2, u_1 u_2 \in E_1 \text{ (or) } u_1 u_2 \in E_1, v_1 v_2 \notin E_2 \text{ (or) } u_1 u_2 \notin E_1, v_1 v_2 \in E_2\} \text{ with } \sigma_1 \times_\alpha \sigma_2 = \sigma_1(u_1) \wedge \sigma_2(v_1) \quad \forall (u_1, v_1) \in V_1 \times_\alpha V_2$$

$$(\mu_1 \times_\alpha \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(v_1 v_2), & \text{if } u_1 = u_2, v_1 v_2 \in E_2 \\ \sigma_2(v_1) \wedge \mu_1(u_1 u_2), & \text{if } v_1 = v_2, u_1 u_2 \in E_1 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1), & \text{if } u_1 u_2 \in E_1, v_1 v_2 \notin E_2 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2 v_2), & \text{if } u_1 u_2 \notin E_1, v_1 v_2 \in E_2 \end{cases}$$

Definition 2.11[11]: The β – product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph

$$G_1 \times_\beta G_2 = ((\sigma_1 \times_\beta \sigma_2), (\mu_1 \times_\beta \mu_2)) \text{ on } G^* : (V, E) \text{ where } V = V_1 \times V_2 \text{ and } E = \{((u_1, v_1)(u_2, v_2)) : u_1 u_2 \in E_1, v_1 v_2 \notin E_2 \text{ (or) } u_1 u_2 \notin E_1, v_1 v_2 \in E_2 \text{ (or) } u_1 u_2 \in E_1, v_1 v_2 \in E_2\} \text{ with } (\sigma_1 \times_\beta \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(v_1) \quad \forall (u_1, v_1) \in V_1 \times_\beta V_2$$



$$(\mu_1 \times_{\beta} \mu_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \mu_1(u_1v_1) \wedge \mu_2(u_2v_2), & \text{if } u_1u_2 \in E_1, v_1v_2 \in E_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), & \text{if } u_1u_2 \in E_1, v_1v_2 \notin E_2 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2), & \text{if } u_1u_2 \notin E_1, v_1v_2 \in E_2 \end{cases}$$

Definition 2.12[11]: The γ – product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G_1 \times_{\gamma} G_2 = ((\sigma_1 \times_{\gamma} \sigma_2), (\mu_1 \times_{\gamma} \mu_2))$ on $G^*: (V, E)$ where $V = V_1 \times_{\gamma} V_2$ and $E = \{((u_1, v_1)(u_2, v_2)): u_1 = u_2, v_1v_2 \in E_2 \text{ (or) } v_1 = v_2, u_1u_2 \in E_1 \text{ (or) } u_1u_2 \in E_1, v_1v_2 \notin E_2 \text{ (or) } u_1u_2 \notin E_1, v_1v_2 \in E_2 \text{ (or) } u_1u_2 \in E_1, v_1v_2 \in E_2\}$

with $\sigma_1 \times_{\gamma} \sigma_2 = \sigma_1(u_1) \wedge \sigma_2(v_1) \quad \forall (u_1, v_1) \in V_1 \times_{\gamma} V_2$

$$(\mu_1 \times_{\gamma} \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2v_2), & \text{if } u_1 = u_2, v_1v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1v_1), & \text{if } v_1 = v_2, u_1u_2 \in E_1 \\ \mu_1(u_1v_1) \wedge \mu_2(u_2v_2), & \text{if } u_1u_2 \in E_1, v_1v_2 \in E_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), & \text{if } u_1u_2 \in E_1, v_1v_2 \notin E_2 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2), & \text{if } u_1u_2 \notin E_1, v_1v_2 \in E_2 \end{cases}$$

Notations :

The relation $\mu_1 \leq \mu_2$ means that $\mu_1(e) \leq \mu_2(e), \forall e \in E_1$ and $\forall e \in E_2$ where μ_1 is a fuzzy subset of E_1 and μ_2 is a fuzzy subset of E_2 .

Lemma 2.13 [6]:

- (1) If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$, then $\sigma_2 \geq \mu_1$.
- (2) The relation $\sigma_1 \geq \sigma_2$ means that $\sigma_1(u) \geq \sigma_2(v)$, for every $u \in V_1$ and for every $v \in V_2$, where σ_i is a fuzzy subset of $V_i, i = 1, 2$.

Note : Throughout this paper $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying crisps $G_1^*:(V_1, E_1)$ and $G_2^*:(V_2, E_2)$ with $|V_i| = p_i, i = 1, 2$. Also $d_{G_i^*}(u_i)$ denotes the degree of u_i in G_i^* .

Definition 2.14[21] : Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then modular product of G_1 and G_2 is a pair of functions

$(\sigma_1 \odot \sigma_2, \mu_1 \odot \mu_2)$ with underlying vertex set $V_1 \odot V_2 = \{ (u_1, v_1) : u_1 \in V_1, v_1 \in V_2 \}$ and underlying edge set $E_1 \odot E_2 = \{ ((u_1, u_2)(v_1, v_2)) : u_1u_2 \in E_1, v_1v_2 \in E_2 \text{ or } u_1u_2 \notin E_1, v_1v_2 \notin E_2 \}$ with $(\sigma_1 \odot \sigma_2) = \sigma_1(u_1) \wedge \sigma_2(v_1)$, where $u_1 \in V_1, v_1 \in V_2$
 $(\mu_1 \odot \mu_2)((u_1, u_2)(v_1, v_2)) = \mu_1(u_1, v_1) \wedge \mu_2(u_2, v_2)$, if $u_1v_1 \in E_1, u_2v_2 \in E_2$
 $= \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)$, if $u_1u_2 \notin E_1, v_1v_2 \notin E_2$

Definition 2.15[21] : Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then degree of a vertex (u_1, v_1) in their modular fuzzy graph product is

$$d_{G_1 \odot G_2}(u_1, u_2) = \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_1(u_1v_1) \wedge \mu_2(u_2, v_2) + \sum_{u_1u_2 \notin E_1, v_1v_2 \notin E_2} \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2) \text{ --- (3.1)}$$

$$= \sum_{u_1v_1 \in E_1} \mu_1(u_1v_1), \text{ if } \mu_1 \leq \mu_2$$

$$= d_{G_1}(u_1), \text{ if both fuzzy graphs are complete}$$

$$d_{G_1 \odot G_2}(u_1, u_2) = \sum_{u_2v_2 \in E_2} \mu_2(u_2, v_2), \text{ if } \mu_2 \leq \mu_1$$

$$= d_{G_2}(v_1), \text{ if both fuzzy graphs are complete}$$



III. TOTAL DEGREE OF A VERTEX IN MODULAR PRODUCT OF TWO FUZZY GRAPHS

Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then total degree of a vertex (u_1, v_1) in their modular fuzzy graph product is

$$\begin{aligned} \text{td}_{G_1 \odot G_2}(u_1, u_2) &= \sum_{u_1 v_1 \in E_1, u_2 v_2 \in E_2} \mu_1(u_1 v_1) \wedge \mu_2(u_2, v_2) \\ &\quad + \sum_{u_1 u_2 \notin E_1, v_1 v_2 \notin E_2} \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2) + \sigma_1(u_1) \wedge \sigma_2(v_1) \dots \dots \dots \rightarrow (3.1) \\ &= \sum_{u_1 v_1 \in E_1, u_2 v_2 \in E_2} \mu_1(u_1 v_1) \wedge \mu_2(u_2, v_2) + \sigma_1(u_1) \wedge \sigma_2(v_1), \text{ if both fuzzy graphs are complete.} \\ &= d_{G_1}(u_1) + \sigma_1(u_1) = \text{td}_{G_1}(u_1), \mu_1 \leq \mu_2. \end{aligned}$$

In the following theorems, we find the total degree of vertices in $G_1 \odot G_2$ in terms of those in G_1 and G_2 in some particular cases.

Theorem 3.1: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\mu_1 \leq \mu_2$ and $\sigma_1 \leq \sigma_2$, then $\text{td}_{G_1 \odot G_2}(u_1, u_2) = \text{td}_{G_1}(u_1) + [p_2 - 1 - d_{G_2^*}(v_1)]\sigma_1(u_1)$.

Proof :

Since $\mu_1 \leq \mu_2$ and $\sigma_1 \leq \sigma_2$.

$$\begin{aligned} \text{From (3.1), } \text{td}_{G_1 \odot G_2}(u_1, u_2) &= d_{G_1}(u_1) + d_{G_2^*}(v_1)\sigma_1(u_1) + \sigma_1(u_1) \\ &= \text{td}_{G_1}(u_1) + d_{G_2^*}(v_1)\sigma_1(u_1) [\because d_{G_2^*}(v_1) = p_2 - 1 - d_{G_2^*}(v_1)] \\ &= \text{td}_{G_1}(u_1) + [p_2 - 1 - d_{G_2^*}(v_1)]\sigma_1(u_1) \dots \dots \dots \rightarrow (3.2) \end{aligned}$$

Corollary 3.2: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\mu_1 \leq \mu_2$ and $\sigma_1 \leq \sigma_2$, then $\text{td}_{G_1 \odot G_2}(u_1, u_2) = \text{td}_{G_1}(u_1) + [p_2 - d_{G_2^*}(v_1)]\sigma_1(u_1) \dots \dots \dots \rightarrow (3.3)$

Proof : Proof follows from theorem 3.1.

Theorem 3.3: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\mu_2 \leq \mu_1$ and $\sigma_2 \leq \sigma_1$, then $\text{td}_{G_1 \odot G_2}(u_1, u_2) = \text{td}_{G_2}(v_1) + [p_1 - 1 - d_{G_1^*}(u_1)]\sigma_2(v_1)$.

Proof :

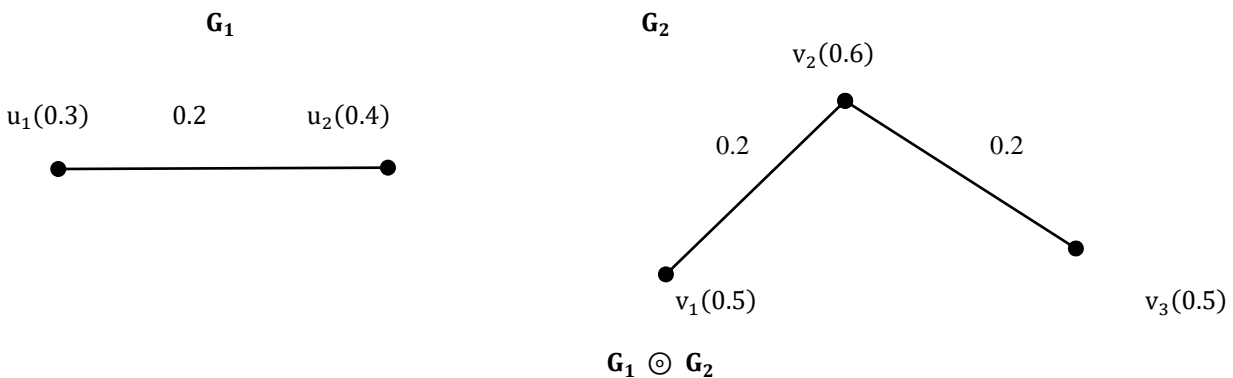
Since $\mu_2 \leq \mu_1$ and $\sigma_2 \leq \sigma_1$.

$$\begin{aligned} \text{From (3.1), } \text{td}_{G_1 \odot G_2}(u_1, u_2) &= d_{G_2}(v_1) + d_{G_1^*}(u_1)\sigma_2(v_1) + \sigma_2(v_1) \\ &= d_{G_2}(v_1) + [p_1 - 1 - d_{G_1^*}(u_1)]\sigma_2(v_1) + \sigma_2(v_1) \\ &= \text{td}_{G_2}(v_1) + [p_1 - 1 - d_{G_1^*}(u_1)]\sigma_2(v_1) \dots \dots \dots \rightarrow (3.4) \end{aligned}$$

Corollary 3.4: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\mu_2 \leq \mu_1$ and $\sigma_2 \leq \sigma_1$, then $\text{td}_{G_1 \odot G_2}(u_1, u_2) = \text{td}_{G_2}(v_1) + [p_1 - d_{G_1^*}(u_1)]\sigma_2(v_1) \dots \dots \dots \rightarrow (3.5)$

Proof : Proof follows from theorem 3.3.

Example 3.3: Consider the fuzzy graphs $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ in Fig 3.1.



Here $\mu_1 \leq \mu_2$ and $\sigma_1 \leq \sigma_2$. $td_{G_1 \otimes G_2}(u_1, v_1) = td_{G_1}(u_1) + [p_2 - d_{G_2}(v_1)]\sigma_1(u_1)$
 $= 0.5 + [2 - 1]0.3 = 0.5 + 0.3 = 0.8$

IV. TOTALLY REGULAR PROPERTY IN THE MODULAR PRODUCT OF TWO FUZZY GRAPHS

In general, there does not exist any relationship between the totally regular property of G_1 and G_2 and the totally regular property of $G_1 \otimes G_2$. In this section, we obtain Characterizations for $G_1 \otimes G_2$ to be totally regular in some particular cases.

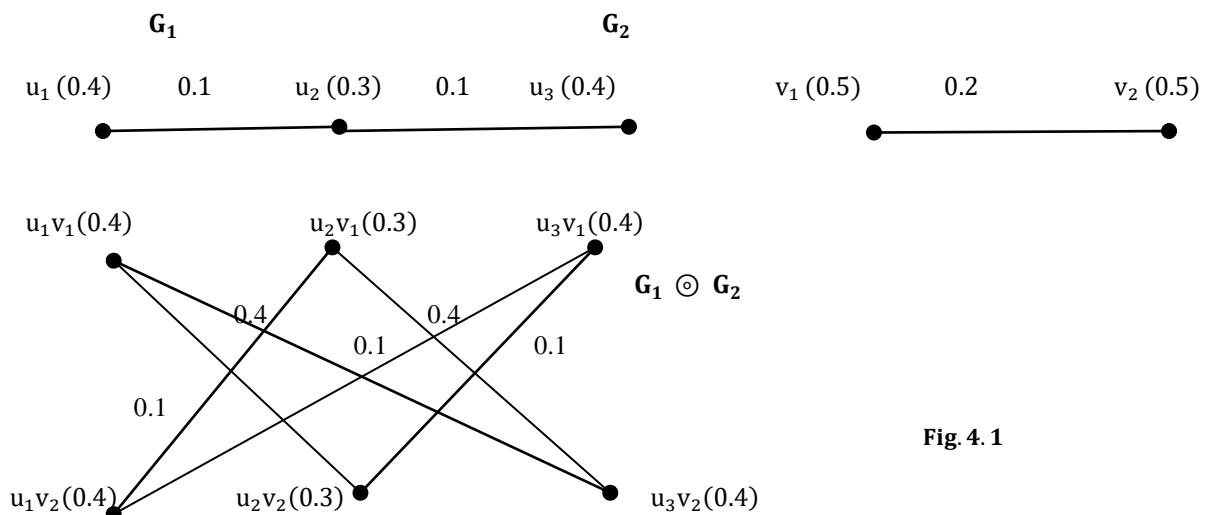


Fig. 4. 1

Here G_1 is 0.5 – totally regular and G_2 is 0.7 – totally regular. But $G_1 \odot G_2$ is not a regular fuzzy graph.

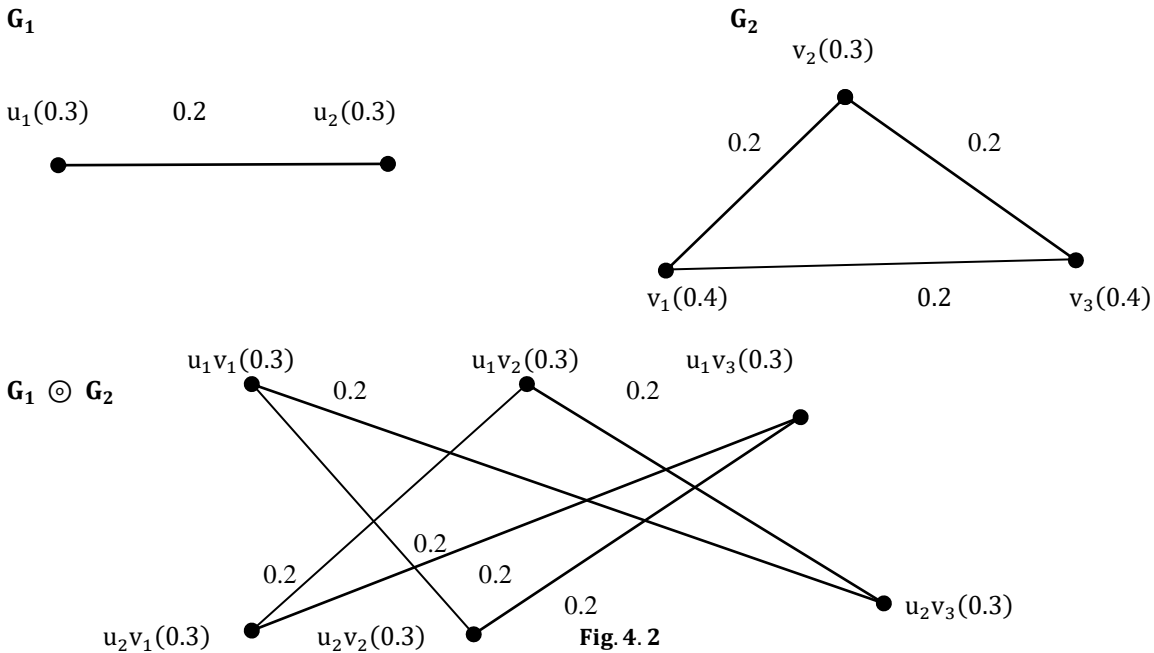


Fig. 4.2

If $G_1 \odot G_2$ is a totally regular fuzzy graph, then G_1 or G_2 need not be a totally regular. In fig. 4.2. $G_1 \odot G_2$ is a totally regular fuzzy graph, but G_2 is not a totally regular fuzzy graph.

Theorem 4.1 : Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_1 is a constant function with $\mu_1 \leq \mu_2$, $\sigma_1 \leq \sigma_2$ and let G_2^* is a regular graph, then $G_1 \odot G_2$ is a totally regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph.

Proof :

Let $\sigma_1(u_i) = c_i$ for all $u \in V_i$, where c_i is a constant, $i = 1, 2$.

Let G_2^* be r_2 – regular graph.

Assume that $G_1 \odot G_2$ is a regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \odot G_2$,

$$td_{G_1 \odot G_2}(u_1, v_1) = td_{G_1 \odot G_2}(u_2, v_2)$$

From (3.2) $td_{G_1}(u_1) + [p_2 - 1 - d_{G_2^*}(v_1)]\sigma_1(u_1) = td_{G_1}(u_2) + [p_2 - 1 - d_{G_2^*}(v_2)]\sigma_1(u_2)$

$$td_{G_1}(u_1) + [p_2 - 1 - r_2]c_1 = td_{G_1}(u_2) + [p_2 - 1 - r_2]c_1$$

$$\Rightarrow td_{G_1}(u_1) = td_{G_1}(u_2) \text{ ----> (4.1)}$$

Thus G_1 is a totally regular fuzzy graph.

Conversely, Let G_1 be a k_1 – totally regular fuzzy graph and G_2^* be a r_2 – regular graph

Then for any vertex (u_1, u_2) of $G_1 \odot G_2$, from (1),

$$td_{G_1 \odot G_2}(u_1, u_2) = k_1 + [p_2 - 1 - r_2]c_1$$

$$= [k_1 - c_1] + [p_2 - r_2]c_1$$

Hence $G_1 \odot G_2$ is a totally regular fuzzy graph.

Theorem 4.2 : Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_1 is a constant function with $\mu_1 \leq \mu_2$, $\sigma_1 \leq \sigma_2$ and let G_2^* is a regular graph, then $G_1 \odot G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph.



Proof : The proof of the theorem is similar to that of theorem 4.1.

Theorem 4.3 : Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_2 is a constant function with $\mu_2 \leq \mu_1$, $\sigma_2 \leq \sigma_1$ and let G_1^* is a regular graph, then $G_1 \odot G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph.

Proof :

Let $\sigma_i(u_i) = c_i$ for all $u \in V_i$, where c_i is a constant, $i = 1, 2$.

Let G_1^* be r_1 – regular graph.

Assume that $G_1 \odot G_2$ is a totally regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \odot G_2$,

$$td_{G_1 \odot G_2}(u_1, v_1) = td_{G_1 \odot G_2}(u_2, v_2)$$

$$\begin{aligned} \text{From (3.4), } \quad td_{G_2}(v_1) + [p_1 - 1 - d_{G_1^*}(u_1)]\sigma_2(v_1) &= td_{G_2}(v_2) + [p_1 - 1 - d_{G_1^*}(u_2)]\sigma_2(v_2) \\ td_{G_2}(v_1) + [p_1 - 1 - r_1]c_2 &= td_{G_2}(v_2) + [p_1 - 1 - r_1]c_2 \\ \implies td_{G_1}(v_1) &= td_{G_1}(v_2) \implies (4.2) \end{aligned}$$

Thus G_1 is a totally regular fuzzy graph.

Conversely, Let G_2 be a k_2 – totally regular fuzzy graph and G_1^* be a r_1 – regular graph

Then for any vertex (u_1, u_2) of $G_1 \odot G_2$, from (1),

$$\begin{aligned} td_{G_1 \odot G_2}(u_1, u_2) &= k_2 + [p_1 - 1 - r_1]c_2 \\ &= [k_2 - c_2] + [p_1 - r_1]c_2 \end{aligned}$$

Hence $G_1 \odot G_2$ is a totally regular fuzzy graph.

Theorem 4.4 : Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_2 is a constant function with $\mu_2 \leq \mu_1$, $\sigma_2 \leq \sigma_1$ and let G_1^* is a regular graph, then $G_1 \odot G_2$ is a totally regular fuzzy graph if and only if G_2 is a regular fuzzy graph.

Proof :

The proof of the theorem is similar to that of theorem 4.3.

V. CONCLUSION

In this paper, we have found the total degree of vertices in modular product of two fuzzy graphs in some particular cases and we have shown that the modular product of two totally regular fuzzy graphs need not be a totally regular fuzzy graph. We have obtained necessary and sufficient condition for the modular product of two fuzzy graphs to be totally regular under some restrictions. The formulae obtained for total degree and the properties of totally regular fuzzy graph will be helpful in studying various properties of operations of two fuzzy graphs.

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