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Nano Generalized Homeomorphisms in Nano Topological Space

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ABSTRACT: The purpose of this paper is to define and study some properties of Nano generalized homeomorphisms in Nano topological spaces.

KEYWORDS: Nano Topology, Ng-closed sets, Ng-closed map, Ng-open map, Ng-homeomorphisms.

I. INTRODUCTION

The notion of homeomorphism plays a very important role in topology. Many researchers have generalized the notions of homeomorphism in topological spaces. Maki *et al.*[5] have introduced and investigated g -homeomorphism and gc -homeomorphism in topological spaces. Lellis Thivagar [4] introduced Nano homeomorphisms in Nano Topological Spaces. Bhuvaneshwari *et al.* [2] introduced and studied some properties of Nano generalized continuous function in Nano topological spaces. In this paper, new class of homeomorphisms called Nano generalized homeomorphism (shortly, Ng-homeomorphism) and Ngc-homeomorphism in Nano topological space are introduced and some of its properties are discussed.

II. RELATED WORK

In general topology, g -homeomorphism and gc -homeomorphism in topological space was introduced by Maki *et al.*[5]. The concept of Nano Topology was introduced by Lellis Thivagar[3]. He studied about the weak forms of nano open sets such as Nano α -open sets, Nano semi open sets and Nano pre open sets. He also introduced Nano homeomorphisms [4] and investigated its properties. Later Bhuvaneshwari *et al.*[1] introduced Nano generalised closed sets and studied some of its properties. In continuation of the study, Nano Generalized homeomorphism is introduced in this article.

III. PRELIMINARIES

Definition 3.1. A bijective function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a generalized homeomorphism (g -homeomorphism) [5] if f is both g -continuous and g -open.

Definition 3.2. A bijective function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a gc -homeomorphism [5] if both f and f^{-1} are gc -irresolute functions.

Definition 3.3. [6]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes equivalence class determined by x .

- (ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$
- (iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Property 3.4. [6]

If (U, R) is an approximation space and $X, Y \subseteq U$, then

- (i) $L_R(X) \subseteq X \subseteq U_R(X)$
- (ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
- (iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- (iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- (v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- (vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- (vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- (viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- (ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
- (x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 3.5. [3] Let U be the Universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 3.4., $\tau_R(X)$ satisfies the following axioms:

- (i) U and $\phi \in \tau_R(X)$.
- (ii) The union of the elements of any sub – collection of $\tau_R(X)$ is in $\tau_R(X)$
- (iii) The intersection of the elements of any finite sub – collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X .

We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets. The elements of the complement of $\tau_R(X)$ are called as Nano-closed sets.

Definition 3.6. [4] A map $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be Nano closed map (resp. Nano open map) if the image of every Nano closed set (resp. Nano open set) in $(U, \tau_R(X))$ is Nano closed (resp. Nano open) in $(V, \tau_{R'}(Y))$

Definition 3.7. [4] A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be Nano homeomorphism if, f is 1-1 and onto, f is Nano continuous and f is Nano open.

IV. PROPERTIES OF NANO GENERALIZED HOMEOMORPHISM IN NANO TOPOLOGICAL SPACE

Definition 4.1. A map $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be Nano generalized closed map (shortly- Ng-closed map) if the image of every Nano closed set in $(U, \tau_R(X))$ is Ng-closed in $(V, \tau_{R'}(Y))$.

Definition 4.2. A bijection $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is called Nano generalized homeomorphism (shortly, Ng-homeomorphism) if f is both Ng-continuous and Ng-open.

Example 4.3. Let $U = \{a, b, c, d\} = V$. Then $\tau_R(X) = \{\Phi, \{a, b, d\}, \{a, b\}, \{d\}, U\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_{R'}(Y) = \{\Phi, \{a, c, d\}, \{a, d\}, \{c\}, V\}$ with $V/R' = \{\{a\}, \{b\}, \{c, d\}\}$ and $Y = \{a, d\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as $f(a) = d, f(b) = c, f(c) = b, f(d) = a$. Then f is bijective, Ng-continuous and Ng-open. Therefore f is an Ng-homeomorphism.

Theorem 4.4. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological space. If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is a Nano homeomorphism, then f is an Ng-homeomorphism.

Proof.

If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is a Nano homeomorphism, by definition, f is bijective, Nano continuous and Nano open. Therefore $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-continuous and Ng-open respectively. Thus the function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-homeomorphism. Therefore, every Nano homeomorphism $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-homeomorphism.

The function f in Example 4.3 is an Ng-homeomorphism but not a Nano homeomorphism because it is not Nano continuous.

Theorem 4.5.

Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological space. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a bijective Ng-continuous function. Then the following statements are equivalent.

- (i) f is an Ng-open function,
- (ii) f is an Ng-homeomorphism,
- (iii) f is an Ng-closed function.

Proof.

(i) \Rightarrow (ii) By the given hypothesis, the function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is bijective, Ng-continuous and Ng-open. Hence $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-homeomorphism.

(ii) \Rightarrow (iii) Let A be the Nano closed set in $(U, \tau_R(X))$. Then A^C is Nano open in $(U, \tau_R(X))$. By assumption, $f(A^C)$ is Ng-open in $(V, \tau_{R'}(Y))$. i.e., $f(A^C) = (f(A))^C$ is Ng-open in $(V, \tau_{R'}(Y))$ and hence $f(A)$ is Ng-closed in $(V, \tau_{R'}(Y))$ for every Nano closed set A in $(U, \tau_R(X))$. Hence the function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-closed function.

(iii) \Rightarrow (i) Let F be a Nano open set in $(U, \tau_R(X))$. Then F^C is Nano closed set in $(U, \tau_R(X))$. By the given hypothesis, $f(F^C)$ is Ng-closed in $(V, \tau_{R'}(Y))$. But, $f(F^C) = (f(F))^C$ is Ng-closed, i.e., $f(F)$ is Ng-open in $(V, \tau_{R'}(Y))$ for every Nano open set F in $(U, \tau_R(X))$. Hence $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-open function.

Theorem 4.6

Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a one to one onto function. Then f is an Ng-homeomorphism if and only if f is Ng-closed and Ng-continuous.

Proof.

Let f be an Ng-homeomorphism. Then f is Ng-continuous. Let B be an arbitrary Nano closed set in $(U, \tau_R(X))$. Then B is Ng-closed set in $(U, \tau_R(X))$. Then $U - B$ is Ng-open. Since f is Ng-open, $f(U - B)$



is Ng-open in $(V, \tau_{R'}(Y))$. That is $V - f(B)$ is Ng-open in $(V, \tau_{R'}(Y))$. Therefore $f(B)$ is Ng-closed in $(V, \tau_{R'}(Y))$. Thus the image of every Nano closed set in $(U, \tau_R(X))$ is Ng-closed in $(V, \tau_{R'}(Y))$

Conversely, let f be Ng-closed and Ng-continuous. Let B be a Nano open set in $(U, \tau_R(X))$. Then $U - B$ is Nano closed in $(U, \tau_R(X))$. Since f is Ng-closed, $f(U - B) = V - f(B)$ is Ng-closed in V . Therefore $f(B)$ is Ng-open in V . Thus f is Ng-open and hence f is Ng-homeomorphism.

The composition of two Ng-homeomorphisms need not be an Ng-homeomorphism as seen from the example 4.7

Example 4.7

Let $U = \{a, b, c\} = V = W$. Then $\tau_R(X) = \{\Phi, \{b\}, \{a, c\}, U\}$ with $U/R = \{\{b\}, \{a, c\}\}$ and $X = \{b, c\}$. Let $\tau_{R'}(Y) = \{\Phi, \{a\}, \{b, c\}, V\}$ with $V/R' = \{\{a\}, \{b, c\}\}$ and $Y = \{a, c\}$. Let $\tau_{R''}(Z) = \{\Phi, \{a\}, W\}$ with $W/R'' = \{\{a\}, \{b, c\}\}$ and $Z = \{a\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as an identity function. Define $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$ as $g(a) = b$, $g(b) = a$ and $g(c) = c$. Then f and g are Ng-homeomorphism, but their composition $(g \circ f) : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$ is not an Ng-homeomorphism because for the Nano open set $\{a, c\}$ in $(U, \tau_R(X))$, $(g \circ f)(\{a, c\}) = \{b, c\}$ which is not Ng-open set in $(W, \tau_{R''}(Z))$. Therefore $(g \circ f)$ is not an Ng-open function and $(g \circ f)$ is not an Ng-homeomorphism.

Next a new class of function called Ngc-homeomorphism which forms a subclass of Ng-homeomorphism is introduced.

Definition 4.8

A bijection $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be Ngc-homeomorphism if both f and f^{-1} are Ngc-irresolute.

Example 4.9

Let $U = \{a, b, c\} = V$. Then $\tau_R(X) = \{\Phi, \{b\}, \{a, c\}, U\}$ with $U/R = \{\{b\}, \{a, c\}\}$ and $X = \{b, c\}$. Let $\tau_{R'}(Y) = \{\Phi, \{a\}, \{b, c\}, V\}$ with $V/R' = \{\{a\}, \{b, c\}\}$ and $Y = \{a, c\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as an identity function. Then this forms an Ngc-homeomorphism.

The family of all Ng-homeomorphism (resp. Ngc-homeomorphism) of a Nano topological space $(U, \tau_R(X))$ onto itself is denoted by $Ng - h(U, \tau_R(X))$ (resp. $Ngc - h(U, \tau_R(X))$)

Theorem 4.10

Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological space. If a bijection $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is an Ngc-homeomorphism, then f is Ng-homeomorphism.

i.e., For a Nano topological space $(U, \tau_R(X))$, $Ngc - h(U, \tau_R(X)) \subseteq Ng - h(U, \tau_R(X))$

Proof.

We know that a function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ngc-irresolute if and only if for every Ng-open set B in $(V, \tau_{R'}(Y))$, $f^{-1}(B)$ is Ng-open in $(U, \tau_R(X))$. Also every Ngc-open function is Ng-open. Thus $Ngc - h(U, \tau_R(X)) \subseteq Ng - h(U, \tau_R(X))$

Remark 4.11

The converse of the above theorem need not be true as shown from the example 4.12

Example 4.12

Consider the function g in Example 4.7. Then g is an Ng-homeomorphism but not Ngc-homeomorphism since for the Ng-closed set $\{b\}$ in $(V, \tau_{R'}(Y))$, $(g^{-1})^{-1}(\{b\}) = g(b) = \{a\}$ is not Ng-closed in $(W, \tau_{R''}(Z))$. Therefore g^{-1} is not Ngc-irresolute and therefore g is not Ngc-homeomorphism.



Remark 4.13

Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be an Ngc-homeomorphism. Then f is not necessarily a Nano homeomorphism.
 Consider the Example 4.9 of Ngc-homeomorphism. Then f is not Nano homeomorphism because it is not Nano continuous.

Remark 4.14

From Theorems and results from 4.4 to 4.13, the following implications are obtained.

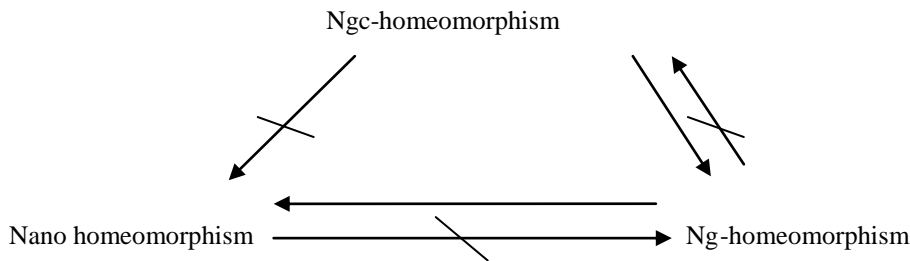


Figure. 4.1

Theorem 4.15

If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$ are Ngc-homeomorphisms, then their composition $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$ is also Ngc-homeomorphism.

Proof.

Let B be an Ng-open set in $(W, \tau_{R''}(Z))$. Now, $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B)) = f^{-1}(D)$, where $D = g^{-1}(B)$. By hypothesis, D is Ng-open in $(V, \tau_{R'}(Y))$ and therefore again by hypothesis, $f^{-1}(D)$ is Ng-open in $(U, \tau_R(X))$. Therefore, $g \circ f$ is Ngc-irresolute.

Also for an Ng-open set G in $(U, \tau_R(X))$, $((g \circ f)^{-1})^{-1}(G) = (g \circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis, $f(G)$ is Ng-open in $(V, \tau_{R'}(Y))$ and again by hypothesis, $g(f(G))$ is Ng-open in $(W, \tau_{R''}(Z))$. i.e., $(g \circ f)(G)$ is Ng-open in $(W, \tau_{R''}(Z))$. Therefore $(g \circ f)^{-1}$ is Ngc-irresolute. Hence $(g \circ f)$ is an Ngc-homeomorphism.

Theorem 4.16

The set $Ngc-h(U, \tau_R(X))$ is a group under the composition of functions.

Proof.

Define $\chi : Ngc-h(U, \tau_R(X)) \times Ngc-h(U, \tau_R(X)) \rightarrow Ngc-h(U, \tau_R(X))$ by $\chi(f, g) = (g \circ f)$ for all $f, g \in Ngc-h(U, \tau_R(X))$ and \circ is the usual operation of composition of functions. Then by theorem 4.15, $(g \circ f) \in Ngc-h(U, \tau_R(X))$. Also the composition of functions is associative and the identity function $I : (U, \tau_R(X)) \rightarrow (U, \tau_R(X))$ belonging to $Ngc-h(U, \tau_R(X))$ serves as the identity element. If $f \in Ngc-h(U, \tau_R(X))$, then $f^{-1} \in Ngc-h(U, \tau_R(X))$, such that $(f \circ f^{-1}) = (f^{-1} \circ f) = I$ and therefore inverse exists for each element of $Ngc-h(U, \tau_R(X))$. Therefore, $(Ngc-h(U, \tau_R(X)), \circ)$ is a group under the operation of composition of functions.

Theorem 4.17.

Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be an Ngc-homeomorphism. Then f induces an isomorphism from the group $Ngc-h(U, \tau_R(X))$ onto the group $Ngc-h(V, \tau_{R'}(Y))$.



Proof.

Using the function f , we define a function $\psi_f : Ngc-h(U, \tau_R(X)) \rightarrow Ngc-h(V, \tau_{R'}(Y))$ by $\psi_f(h) = f \circ h \circ f^{-1}$ for every $h \in Ngc-h(U, \tau_R(X))$. Then ψ_f is a bijection. Further, for all $h_1, h_2 \in Ngc-h(U, \tau_R(X))$, $\psi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \psi_f(h_1) \circ \psi_f(h_2)$. Therefore, ψ_f is a homeomorphism and therefore it is an isomorphism induced by f .

V. CONCLUSION

In this article, some of the properties of Nano Generalized homeomorphism and Ngc-homeomorphism are discussed. This shall be extended in future Research with some applications.

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