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# Vertex Prime Labeling of Umbrella $U(m,n)$ Graphs

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**ABSTRACT:** A Graph  $G = (V, E)$  is said to have a Vertex Prime Labeling if its edges can be labeled with distinct integers from  $\{1, 2, 3, \dots, |E|\}$  such that for each vertex of degree at least two, the Greatest Common Divisor of the labels on its incident edges is 1. A graph that admits a vertex prime labeling is called a Vertex Prime graph. In this paper, we investigate the Umbrella graph  $U(m,n)$   $m > 2 ; n > 1$  is a Vertex Prime Graph and illustrate with some examples.

**KEYWORDS:** Prime Labeling, Vertex Prime Labeling, and Umbrella graph.

## I. INTRODUCTION

In Graph labeling, we consider only a simple, finite, connected and undirected graph  $G(V, E)$  with  $p$  vertices and  $q$  edges. A graph with vertex set  $V$  is said to have a prime labeling if its vertices are labeled with distinct integers  $1, 2, \dots, |V|$  such that for each edge  $xy$  the labels assigned to  $x$  and  $y$  are relatively prime. A dual of Prime labeling (i.e Vertex Prime labeling) has been introduced by T.Deretsky, S.M.Lee, and J.Mitchem[3]. A graph  $G(V,E)$  with edge set  $E$  has a Vertex Prime labeling if its edges can be labeled with distinct integer  $1, 2, 3, \dots, |E|$ , such that for each vertex of degree at least two, the greatest common divisor of the labels on its incident edges is 1. For standard terminology and notation we follow Bondy J.A and Moorthy U.S.R [2]. A current survey of various graph labeling problem can be found in [1] Dynamic Survey of graph labeling. In [5] Dr.V.Ganesan and Dr.K.Balamurugan have proved that Umbrella graph is a prime graph and also they have proved that Sunlet graph is a prime graph. In [4] Mukund V. Bapat have proved some Vertex Prime graph and a new type of graph labeling. Dr.P.Kavitha and Dr.S.Meena have proved vertex prime labeling for some Helm related graphs [6].

We will give brief summary of definitions and notation which are useful for the present investigations.

## II. PRELIMINARIES AND NOTATIONS

### Definition 2.1

Umbrella graph: For any integers  $m > 2; n > 1$  An Umbrella graph  $U(m,n)$  is the graph obtained by identifying the end vertex of Path  $P_n$  with a central vertex of a Fan graph  $F_m$ .



Example:

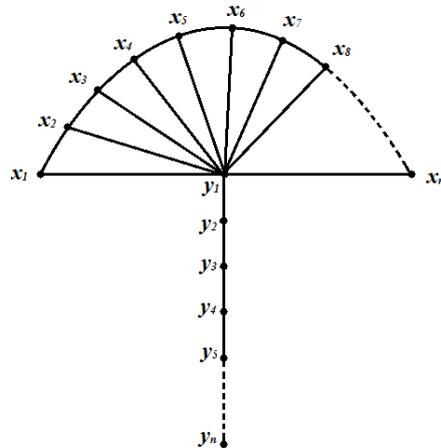


Fig. 1 Umbrella Graph for U (m,n)

Definition 2.2

Prime Labeling: Let  $G = (V, E)$  be a graph with  $p$  vertices. A function  $f: V(G) \rightarrow \{1, 2, \dots, p\}$  is called a prime labeling if for edge  $e = xy$ ,  $\gcd(f(x), f(y)) = 1$ . A graph which admits prime labeling is called Prime graph.

Definition 2.3

Vertex Prime Labeling: Let  $G(V,E)$  be a graph with  $q$  edges, a function  $f : E(G) \rightarrow \{1,2,3,\dots, | E |\}$  is such that for any vertex  $v$  the gcd of all labels on edges incident with  $v$  is 1. This is true for true to all vertices with degree at least two. The graph that admits Vertex Prime labeling is called Vertex Prime graph.

III. MAIN RESULT

Theorem: 3.1

The Umbrella Graph  $U(m,n)$  is a Vertex Prime graph.

Proof

Consider the Umbrella graph  $U(m,n)$  with vertex set,  $V(U(m,n)) = \{x_1, x_2, \dots, x_m ; y_1, y_2, \dots, y_n\}$  and the edge set

$$E(U(m,n)) = \begin{cases} (x_i, x_{i+1}) \text{ for } i = 1, 2, \dots, (m - 1) \\ (y_j, y_{j+1}) \text{ for } j = 1, 2, \dots, (n - 1) \\ (x_i, y_1) \text{ for } i = 1, 2, \dots, m \end{cases}$$

(or)  $E(U(m,n)) = \{x_i x_{i+1} / 1 \leq i \leq m - 1\} \cup \{y_j y_{j+1} / 1 \leq j \leq n\} \cup \{x_i y_1 / 1 \leq i \leq m\}$

Clearly,  $|V(U(m,n))| = m + n$  and

$$|E(U(m,n))| = (2m + n - 2).$$

Define a labeling  $f: E(U(m,n)) \rightarrow \{1, 2, \dots, (2m + n - 2)\}$  as follow,

- (i)  $f(y_j y_{j+1}) = j$  for  $j = 1, 2, \dots, (n - 1)$
- (ii)  $f(y_1 x_i) = (n - 1) + 2i - 1$  for  $i = 1, 2, \dots, m$
- (iii)  $f(x_i x_{i+1}) = (n - 1) + 2i$  for  $i = 1, 2, \dots, (m - 1)$

Let  $f^*(v) = \gcd$  of labels of all edges incident at  $v$  where  $d(v) > 2$

Checking the relative prime of all edges incident at each vertex of  $U(m,n)$  whose degree is at least 2

Now consider

$$\begin{aligned} f^*(y_1) &= \gcd \text{ of labels of all edges incident at } y_1 \\ &= \gcd \{f(y_1 y_2), f(y_1 x_1), f(y_1 x_2), \dots, f(y_1 x_m)\} \\ &= \gcd \{f(y_1 y_2), f(y_1 x_i)\} \text{ for } i = 1, 2, \dots, m \\ &= \gcd \{1, f(y_1 x_i)\} \text{ for } i = 1, 2, \dots, m \\ &= 1. \end{aligned}$$

$$\begin{aligned} f^*(y_j) &= \gcd \text{ of labels of all edges incident at } y_j (j \geq 2) \\ &= \gcd \{f(y_{j-1} y_j), f(y_j y_{j+1})\} \text{ for } j = 2, 3, \dots, (n - 1) \\ &= 1 \text{ since labels of } y_{j-1} y_j \text{ and } y_j y_{j+1} \text{ are consecutive integers} \end{aligned}$$

$$\begin{aligned} f^*(x_1) &= \gcd \{f(x_1 y_1), f(x_1 x_2)\} \\ &= \gcd \{(n - 1) + 2i - 1, (n - 1) + 2i\} \text{ for } i = 1 \\ &= \gcd \{n, n + 1\} \end{aligned}$$



$$\begin{aligned}
 &= 1 \text{ since } n \text{ and } n + 1 \text{ are consecutive integers.} \\
 f^*(x_i) &= \text{gcd of labels of all edges incident at } x_i \text{ ( } i \geq 2 \text{ )} \\
 &= \text{gcd}\{f(x_{i-1}x_i), f(x_ix_{i+1}), f(y_1x_i)\} \text{ for } i = 2, 3, \dots, (m - 1) \\
 &= \text{gcd}\{1, f(y_1x_i)\} \\
 &= 1 \text{ since } f(x_{i-1}x_i) \text{ and } f(x_ix_{i+1}) \text{ are consecutive integers.} \\
 f^*(x_m) &= \text{gcd}\{f(x_mx_{m-1}), f(y_1x_m)\} \\
 &= \text{gcd}\{(n - 1) + 2m - 1, (n - 1) + 2m\} \\
 &= 1
 \end{aligned}$$

Therefore,

$$f^*(y_1) = f^*(y_2) = \dots = f^*(y_n) = 1$$

$$\text{and } f^*(x_1) = f^*(x_2) = \dots = f^*(x_m) = 1$$

Thus  $f$  admits Vertex Prime labeling.

Hence the Umbrella graph  $U(m, n)$  is a Vertex Prime graph.

#### IV. ILLUSTRATION

Case: 4.1

Take the Umbrella graph for the type,  $m = 8$  and  $n = 4$ . Here both  $m, n$  are even numbers. Now consider the Umbrella  $U(8, 4)$ .

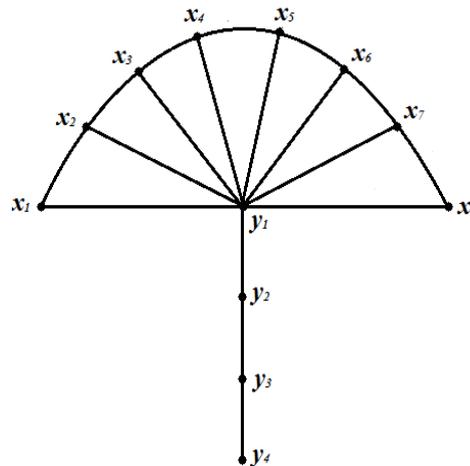


Fig. 2 Umbrella Graph for  $U(8, 4)$

Define a labeling  $f: E(U(m, n)) \rightarrow \{1, 2, \dots, (2m + n - 2)\}$  as follow,

- (i)  $f(y_jy_{j+1}) = j$  for  $j = 1, 2, \dots, (n - 1)$
- (ii)  $f(y_1x_i) = (n - 1) + 2i - 1$  for  $i = 1, 2, \dots, m$
- (iii)  $f(x_ix_{i+1}) = (n - 1) + 2i$  for  $i = 1, 2, \dots, (m - 1)$

Here the edge labels are distinct.

The Vertex Prime labeling of  $U(8, 4)$  is given below

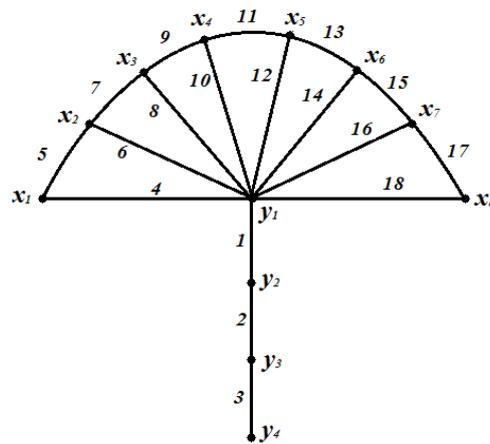


Fig. 3 Vertex Prime Labeling of Umbrella Graph U (8, 4)

Now consider

$$\begin{aligned}
 f^*(y_1) &= \text{gcd of labels of all edges incident at } y_1 \\
 &= \text{gcd}\{f(y_1y_2), f(y_1x_i)\} && \text{for } 1 \leq i \leq m \text{ and } m = 8 \\
 &= \text{gcd}\{1, f(y_1x_i)\} \\
 &= 1. \\
 f^*(y_j) &= \text{gcd}\{f(y_{j-1}y_j), f(y_jy_{j+1})\} && \text{for } 2 \leq j \leq n - 1 \text{ and } n = 4 \\
 &= 1. \\
 f^*(x_1) &= \text{gcd}\{f(x_1y_1), f(x_1x_2)\} \\
 &= \text{gcd}\{(n - 1) + 2j - 1, (n - 1) + 2j\} && \text{for } n = 4, j = 1 \\
 &= 1 \\
 f^*(x_m) &= \text{gcd}\{f(x_my_1), f(x_mx_{m-1})\} && \text{for } m = 8 \\
 &= 1 \\
 f^*(x_j) &= \text{gcd}\{f(x_{i-1}x_i), f(x_ix_{i+1}), f(y_1x_i)\} && \text{for } 2 \leq i \leq m - 1 \text{ and } m = 8 \\
 &= 1
 \end{aligned}$$

(i.e)  $f^*(y_j) = f^*(x_1) = f^*(x_m) = f^*(x_i) = 1$  for all  $i, j$ .

Umbrella graph U (8, 4) is a Vertex Prime Labeling.

Case: 4.2

Take the Umbrella graph for the type,  $m = 9$  and  $n = 5$ . Here both  $m, n$  are odd numbers.

Now consider the Umbrella U (9, 5).

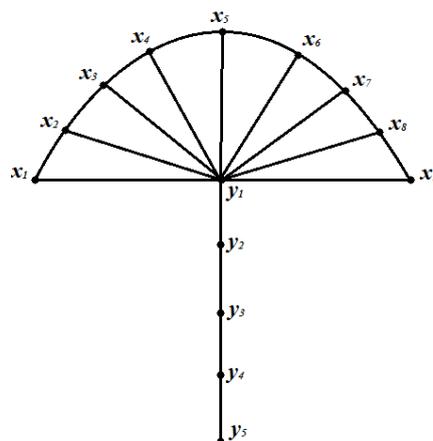


Fig. 4 Umbrella Graph for U (9, 5)

Define a labeling  $f: E(U(m, n)) \rightarrow \{1, 2, \dots, (2m + n - 2)\}$  as follow,

$$(i) \quad f(y_jy_{j+1}) = j \quad \text{for } j = 1, 2, \dots, (n - 1)$$



$$(ii) f(y_1x_i) = (n - 1) + 2i - 1 \quad \text{for } i = 1, 2, \dots, m$$

$$(iii) f(x_ix_{i+1}) = (n - 1) + 2i \quad \text{for } i = 1, 2, \dots, (m - 1)$$

Here the edge labels are distinct. The vertex prime labeling of U (9, 5) is given below

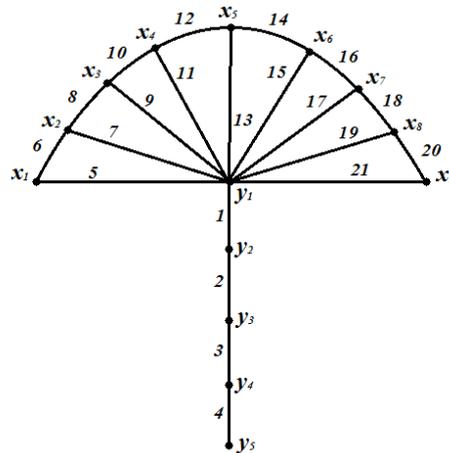


Fig. 5 Vertex Prime Labeling of Umbrella Graph U (9, 5)

$$f^*(y_1) = \gcd \text{ of labels of all edges incident at } y_1$$

$$= \gcd \{f(y_1y_2), f(y_1x_i)\} \quad \text{for } 1 \leq i \leq m \text{ and } m = 9$$

$$= \gcd \{1, f(y_1x_i)\}$$

$$= 1.$$

$$f^*(y_i) = \gcd \{f(y_{j-1}y_j), f(y_jy_{j+1})\} \quad \text{for } 2 \leq j \leq n - 1 \text{ and } n = 5$$

$$= 1$$

$$f^*(x_1) = \gcd \{f(x_1y_1), f(x_1x_2)\}$$

$$= \gcd \{(n - 1) + 2j - 1, (n - 1) + 2j\} \quad \text{for } n = 5, j = 1$$

$$= 1$$

$$f^*(x_m) = \gcd \{f(x_my_1), f(x_mx_{m-1})\} \quad \text{for } m = 9$$

$$= 1$$

$$f^*(x_j) = \gcd \{f(x_{i-1}x_i), f(x_ix_{i+1}), f(y_1x_i)\} \quad \text{for } 2 \leq i \leq m - 1 \text{ and } m = 9$$

$$= 1$$

Clearly,  $f^*(y_i) = f^*(x_1) = f^*(x_m) = f^*(x_i) = 1$  for all  $i, j$ .  
Umbrella graph U (9, 5) is a Vertex Prime Labeling.

### V. CONCLUSION

In this paper, we observe that the Vertex Prime Labeling is the dual of the Prime labeling and we investigate Vertex Prime Labeling of Umbrella graph and this work can be extended to some other standard graphs or other families of graphs.

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