



e-ISSN: 2319-8753 | p-ISSN: 2320-6710

IJRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 10, Issue 4, April 2021

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 7.512

9940 572 462

6381 907 438

ijirset@gmail.com

www.ijirset.com

Thermal Analysis of Annular Disc by effect of Point Heat Source

Badrinath E Ghonge

Department of Mathematics, S.R. Arts, Commerce and Science College Ghansawangi, Jalna, Maharashtra, India

ABSTRACT: In this work a non-homogeneous annular disc is considered to determine the temperature, displacement and quasi-static thermal stresses under an arbitrary time dependent heat flux at the outer surface and zero heat flux in the internal surface with the help of integral transform technique. The results here are obtained in a series form in terms of Bessel's functions. The mathematical model has been constructed for copper material and solved by using computational mathematical software Math-Cad 2007 and thermal stresses are discussed graphically using Origin software.

KEYWORDS: Thermal stresses, Point heat source, Annular Disc, Integral transform.

I. INTRODUCTION

Nowaki in [1] discuss the temperature distribution on upper face of circular plate, by assuming zero temperature on the lower surface and circular edge thermally insulated. Roy Choudhary [2] invented the quasi-static thermal stresses due to transient temperature along the circumference of a circle over the upper face with lower face at zero temperature and the fix circular edge of the plate thermally insulated. In [3] Phythian studied the cylindrical heat flow with arbitrary heating rates at the outer surface and zero heat flux at the internal boundary. Deshmukh et al. [4] studied the thermal stresses induced by a point heat source in a circular plate by quasi-static approach. Gaikwad and Ghadle [5-6] determine temperature, displacement and thermal stresses in an inverse transient thick annular disc and a non-homogeneous thick rectangular plate due to internal heat generation. Ghonge and Ghadle [7-12] investigated problems of thermoelastic behavior in different solids for different conditions by using Marchi-Fasulo, Marchi-Zgrablich and Laplace integral transform. Recently Ghonge [13-14] studied the temperature, deflections, displacements and thermal stresses in inverse transients circular plates, and Khobragade et al. [15] determine the an inverse transient temperature, displacement and thermal stresses in an annular disc.

In this paper we consider a non-homogeneous annular disc and determine the temperature, displacement and quasi-static thermal stresses under an arbitrary time dependent heat flux at the outer surface and zero heat flux in the internal surface with the help of integral transform technique as in Ozisik [16]. The results are obtained in a series form in terms of Bessel's functions. The mathematical model has been constructed for copper material and solved by using computational mathematical software Math-Cad 2007 and thermal stresses are discussed graphically using Origin software. To the authors knowledge, no literature on non-homogeneous thermoelastic problem in an annular disc due to internal heat generation has been published. The results presented here will be more useful in engineering problem particularly, in the determination of the state of strain in an annular disc.

II. PROPOSED FORMULATION

Consider a thin annular disc occupying the space D defined by $a \leq r \leq b$. Initially disc is at arbitrary temperature distribution $F(r)$, for time $t > 0$ heat is generated within the disc at a rate of $g(t, r)/k$. The inner circular boundary surface at $r = a$ is insulated, while the arbitrary time dependent heat flux $Q(t)$ is applied at outer circular boundary $r = b$. The arbitrary time dependent heat flux means radial heat flux prescribed on boundary surface and converges rapidly for large value of t often a rapidly convergent series is required for small value of time. Under this conditions the displacement and thermal stresses in the disc due to internal heat generation are to be determined. The differential equation governing the displacement potential function $\psi(t, r)$ as in Khobragade et.al [15] is given by

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} = (1+\nu) \alpha_i T \quad (1)$$

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = \frac{-2\mu}{r} \frac{\partial \psi}{\partial r} \quad (2)$$

$$\sigma_{\theta\theta} = 2\mu \frac{\partial^2 \psi}{\partial r^2} \quad (3)$$

While in each case the stress functions σ_{zz} , σ_{zr} and $\sigma_{\theta z}$ are zero with the disc in the plane state of stress. The temperature of the annular disc satisfies the following heat conduction equation as in Deshmukh et.al [4] as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{g(t,r)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{in } a \leq r \leq b, t > 0 \quad (4)$$

Subject to the boundary condition's

$$k \frac{\partial T}{\partial r} = 0 \quad \text{at } r = a, t > 0 \quad (5)$$

$$k \frac{\partial T}{\partial r} = Q(t) \quad \text{at } r = b, t > 0 \quad (6)$$

with initial conditio

$$T(t,r) = F(r) \quad \text{at } t = 0, a \leq r \leq b \quad (7)$$

Where k and α are the thermal conductivity and thermal diffusivity material of the annular disc respectively. Equations (1) to (7) constitute the proposed mathematical formulation of the thermoelastic problem of an annular disc.

III. ANALYSIS OF THE WORK

Determination of Temperature distribution

To obtain the expression for temperature $T(t,r)$ we introduce the finite Hankel transform over the variable r and its inverse transform defined by Ozisik [16] as

$$\bar{T}(t, \lambda_n) = \int_{r'=a}^b r' (K_0(r', \lambda_n)) T(t, r') dr' \quad (8)$$

$$T(t, r) = \sum_{n=1}^{\infty} \bar{T}(t, \lambda_n) (K_0(r, \lambda_n)) \quad (9)$$

Where

$$K_0(r, \lambda_n) = \frac{\pi}{\sqrt{2}} \frac{\lambda_m J_0'(\lambda_m b) Y_0'(\lambda_m r)}{\sqrt{1 - \frac{J_0'^2(\lambda_m b)}{J_0'^2(\lambda_m a)}}} \left[\frac{J_0(\lambda_m r)}{J_0'(\lambda_m b)} - \frac{Y_0(\lambda_m r)}{Y_0'(\lambda_m b)} \right] \quad (10)$$

And $\lambda_1, \lambda_2, \lambda_3, \dots$ are the positive roots of the equation

$$\frac{J_0'(\lambda_m a)}{J_0'(\lambda_m b)} - \frac{Y_0'(\lambda_m a)}{Y_0'(\lambda_m b)} = 0 \quad (11)$$

$J_0(x)$ and $Y_0(x)$ are the Bessel's function of first and second kind of order zero respectively. Also the transform defined in (8) satisfies the relation

$$H \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = -\lambda_n^2 \bar{T}(t, \lambda_n) \quad (12)$$

On applying the finite Hankel transform defined in (8) and its inverse transform defined in (9) to the equation (4), one obtain the expression for temperature as

$$T(t, r) = \sum_{n=1}^{\infty} K_0(r, \lambda_n) e^{-\alpha \lambda_n^2 t} \times \left\{ \begin{aligned} & \int_{r'=a}^b r' (K_0(r', \lambda_n)) F(r') dr' + \frac{\alpha}{k} \int_{t'=0}^t e^{\alpha \lambda_n^2 t'} \\ & \times \left[\int_{r'=a}^b r' (K_0(r', \lambda_n)) g(t, r') dr' + b (K_0(b, \lambda_n)) Q(t') \right] dt' \end{aligned} \right\} \quad (13)$$

Determination of Displacement function

Substituting the equation (13) in equation (1), one obtain the displacement function $\psi(t, r)$ as

$$\psi(t, r) = -(1+\nu) a_i \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} K_0(r, \lambda_n) e^{-\alpha \lambda_n^2 t} \times \left\{ \begin{aligned} & \int_{r'=a}^b r' (K_0(r', \lambda_n)) F(r') dr' + \frac{\alpha}{k} \int_{t'=0}^t e^{\alpha \lambda_n^2 t'} \\ & \times \left[\int_{r'=a}^b r' (K_0(r', \lambda_n)) g(t, r') dr' + b (K_0(b, \lambda_n)) Q(t') \right] dt' \end{aligned} \right\} \quad (14)$$

Determination of Thermal Stresses

Substituting the equation (14) in to equations (2) and (3), one obtain the radial and angular thermal stresses respectively as below

$$\sigma_{rr} = \sqrt{2}(1+\nu) \pi \mu a_i \sum_{n=1}^{\infty} \frac{\lambda_n J_0'(\lambda_n b) Y_0'(\lambda_n r)}{\sqrt{1 - \frac{J_0'^2(\lambda_n b)}{J_0'^2(\lambda_n a)}}} \left[\frac{J_0'(\lambda_n r)}{J_0'(\lambda_n b)} - \frac{Y_0'(\lambda_n r)}{Y_0'(\lambda_n b)} \right] e^{-\alpha \lambda_n^2 t} \times \left\{ \begin{aligned} & \int_{r'=a}^b r' (K_0(r', \lambda_n)) F(r') dr' + \frac{\alpha}{k} \int_{t'=0}^t e^{\alpha \lambda_n^2 t'} \\ & \times \left[\int_{r'=a}^b r' (K_0(r', \lambda_n)) g(t, r') dr' + b (K_0(b, \lambda_n)) Q(t') \right] dt' \end{aligned} \right\} \quad (15)$$

and

$$\sigma_{\theta\theta} = \sqrt{2}(1+\nu)\pi\mu a_i \sum_{n=1}^{\infty} \frac{\lambda_n J_0'(\lambda_n b) Y_0'(\lambda_n r)}{\sqrt{1 - \frac{J_0'^2(\lambda_n b)}{J_0'^2(\lambda_n a)}}} \left[\frac{J_0''(\lambda_n r)}{J_0'(\lambda_n b)} - \frac{Y_0''(\lambda_n r)}{Y_0'(\lambda_n b)} \right] e^{-\alpha \lambda_n^2 t} \times \left\{ \int_{r'=a}^b r'(K_0(r', \lambda_n)) F(r') dr' + \frac{\alpha}{k} \int_{t'=0}^t e^{\alpha \lambda_n^2 t'} \times \left[\int_{r'=a}^b r'(K_0(r', \lambda_n)) g(t, r') dr' + b(K_0(b, \lambda_n)) Q(t') \right] dt' \right\} \quad (16)$$

IV. PARTICULAR CASE AND NUMERICAL OUTCOMES

Setting $F(r) = r^2$

$$g(t, r) = g_{pi} \delta(t - \tau) \delta(r - r_1)$$

$$Q(t) = e^{-\omega t}, \quad \omega > 0$$

where r is radius measured in meter, δ is the Dirac-delta function.

The heat source $g(t, r)$ is an instantaneous point heat source of strength $g_{pi} = 50 J / m$ situated at certain circle along the radial direction of the disc and releasing its heat instantaneously at the time of $t \rightarrow \tau = 4$ sec.

The numerical calculation has been carried out for a Copper (pure) annular disc with the material and physical properties defined as bellow

Table -1 Material and physical properties copper annular disc.

Inner radius of annular disc	$a = 1m$
Outer radius of annular disc	$b = 2m$
Circular path of annular disc	$r_1 = 1.5m$
Thermal diffusivity	$\alpha = 112.34 \times 10^{-6} m^2 s^{-1}$
Thermal conductivity	$k = 386 \times 10^{-6} w / mk$
Density	$\rho = 8954 Kg / m^3$
Specific heat	$c_p = 383 J / kgK$
Poisson ratio	$\nu = 0.35$
Coefficient of linear thermal expansion	$a_t = 16.5 \times 10^{-6} 1 / K$
Lame constant	$\mu = 26.67$

$\lambda_1 = 3.1665, \lambda_2 = 6.3123, \lambda_3 = 9.4445, \lambda_4 = 12.5812$ and $\lambda_5 = 15.7199$ are the positive roots of transcendental equation

$$\frac{J_0'(\lambda_m a)}{J_0'(\lambda_m b)} - \frac{Y_0'(\lambda_m a)}{Y_0'(\lambda_m b)} = 0$$

We set for our convenience $A = \frac{\pi}{100\sqrt{2}}, B = \frac{\pi(1+\nu)a_i}{100\sqrt{2}}, C = \frac{\pi(1+\nu)\mu a_i}{100\sqrt{2}}$ here A, B and C are constants.

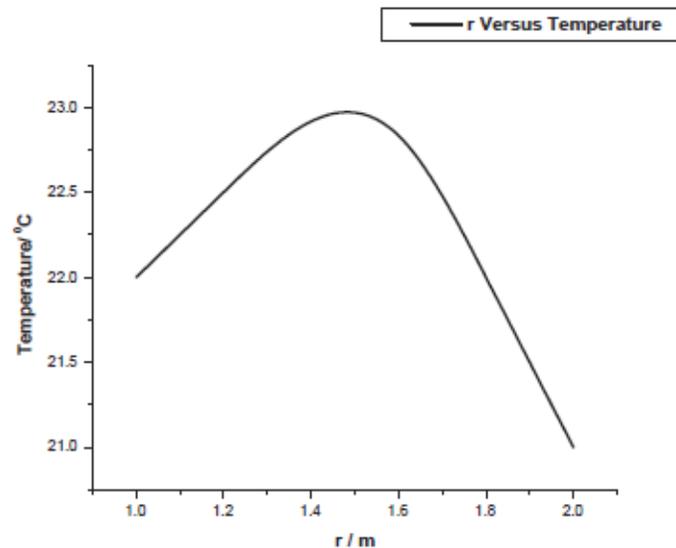


Figure 1. Temperature distribution

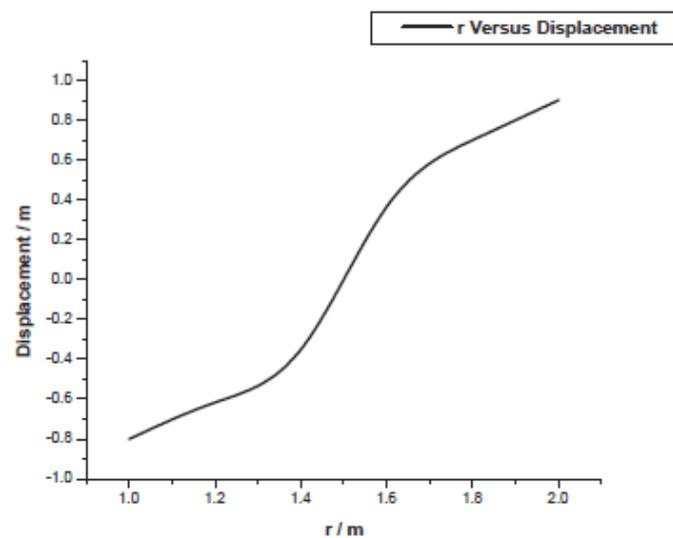


Figure 2. Displacement distribution

- From Fig.1, it is observed that temperature attained maximum at the center and then decreases towards the outer circular edge. After some time $t - \tau = 4$, due to internal heat generation, the temperature of the copper plate increases proportionally.

- From Fig.2, it is observed that there is displacement occurring around the outer circular edge which is proportional to the temperature due to point heat source of strength g_{pi} .

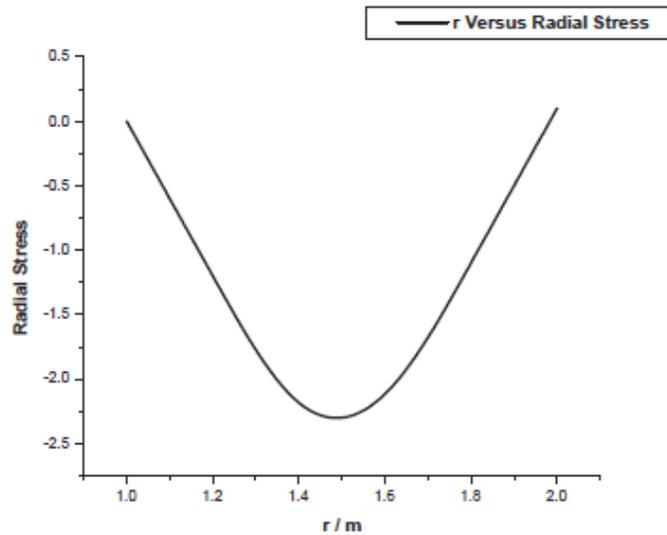


Figure 3. Radial stress distribution

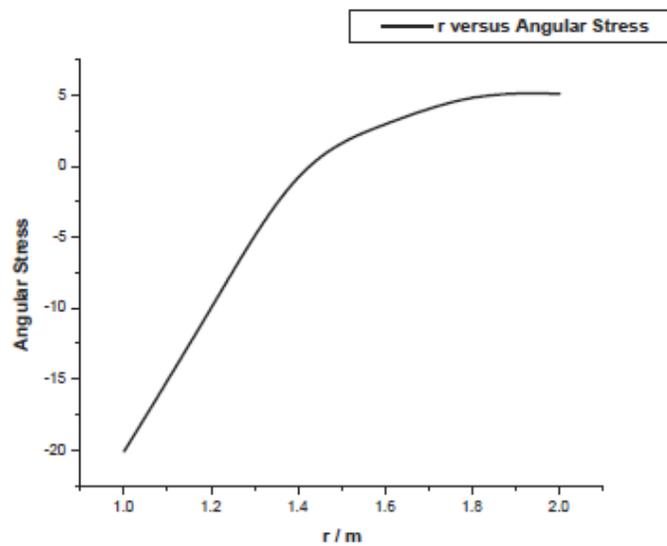


Figure 4. Angular stress distribution

- From Fig.3, it is observed that the radial stress function σ_{rr} is zero at the outer circular boundary. As the heat source $g(t, r)$ is an instantaneous point heat source situated at certain circle within the central part $r_1 = 1.5m$ of the annular disc, we can observe the radial stress developing with compressive stresses around the circle of radius $r_1 = 1.5m$ and the radial stress decreases towards the outer circular edge.
- In Fig.4, the angular stress function $\sigma_{\theta\theta}$ increases from the inner face to the outer circular edge. It reaches maximum in the region $1.4 \leq r \leq 2$ and develops a compressive stresses in the central part of $1 \leq r \leq 1.4$ with a tensile stresses in the annular region of $1.4 \leq r \leq 2$.

V. CONCLUSION

In this paper, we extend the work of [4] to one dimensional non homogeneous boundary value problem of heat conduction in an annular disc and determined the expressions of temperature, displacement and stresses due to internal heat generation. As a special case, a mathematical model is constructed for copper (pure) annular disc with specified material properties. The heat source is an instantaneous point heat source of strength g_{pi} situated at certain circle along the radial and axial direction of the plate, and releases its heat instantaneously at the time of $t = \text{Due}$ to heat generation within the annular disc, the radial stress develops as compressive stresses, whereas the angular stress develops with compressive stresses around the center and tensile stresses around the outer circular edge. Also it can be observed from the figures of temperature reaches maximum at central part and displacement increases from inner surface to outer surface of an annular disc. The results obtained here are useful for engineering problems, particularly in the determination of the state of stress in thin circular plates. Also any particular case of special interest can be derived by assigning suitable values to the parameters and functions in (13)-(16).

REFERENCES

- [1] W. Nowaki "The State of Stress in a Thick Circular Plate Due to temperature Field", Bulletin of Academic pol. Science and Technology, vol.5(1), pp.227-233, 1957.
- [2] S. K. Roy Choudhary "A Note on the Quasi-Static Stress in a Thin Circular Plate Due to Transient Temperature Applied Along the Circumference of a Circle over the Upper Face", Bulletin of Academic pol. Science and Technology, vol.5, pp.20-24, 1972
- [3] J. E. Phythian "Cylindrical Heat Flow with Arbitrary Heating Rates", American Institute of Aeronautics and Astronautics, vol.1(4), pp.925-927, 1963.
- [4] K. C. Deshmukh, Y. I. Quazi, S. D. Warbhe and V. S. Kulkarni "Thermal Stresses Induced by a Point Heat Sources in a Circular Plate by Quasi-Static Approach", The Chinese Society of Theoretical and Applied Mechanics, vol.1, pp.0310071-3, 2011.
- [5] K. R. Gaikwad and K. P. Ghadle "An Inverse Heat Conduction Problem in a Thick Annular Disc", International Journal of Applied Mathematics and Mechanics, vol.7(6), pp.27- 41, 2011.
- [6] K. R. Gaikwad and K. P. Ghadle "Three Dimensional Non-Homogeneous Thermoelastic Problem in a Thick Rectangular Plate Due to Internal Heat Generation", Southern Africa Journal of Pure and Applied Mathematics, vol.5, pp.26- 38, 2011.
- [7] B. E. Ghonge and K. P. Ghadle "An inverse transient thermoelastic problem of solid sphere", Bulletin of Pure & Applied Sciences-Mathematics and Statistics, 29 (1): 13-21, 2010. DOI: 10.5958/2320-3226.
- [8] B. E. Ghonge and K. P. Ghadle "An Inverse Transient Thermoelastic Problem of Hollow Sphere", Advances in Applied Research , 3 (1): 76-82, 2011.
- [9] B. E. Ghonge and K. P. Ghadle "Inverse transient thermoelastic behavior of annular disc by using Marchi-Zgrablich and Laplace transform technique", International Journal of Physics and Mathematical Sciences, 1(1): 68-75, 2011.
- [10] B. E. Ghonge and K. P. Ghadle "Study of exact solution of deflection of thermoelastic circular plate by using Marchi-Fasulo integral transform", Global Journal of Mathematical Science, 4(2): 115-122, 2012.



- [11] B. E. Ghonge and K. P. Ghadle “Deflection of Transient Thermoelastic Circular Plate by Marchi-Zgrablich and Laplace Transform Technique”, Theoretical and Applied Mechanics Letter, 2(2): 021004, 2012. DOI:10.1063/2.1202104
- [12] B. E. Ghonge and K. P. Ghadle “Two dimensional unsteady state non-homogeneous thermoelastic problem of circular plate”, International Mathematical Forum, 7(33): 1645-1653, 2012.
- [13] B. E. Ghonge “An Inverse Transient Thermoelastic Behavior of Circular Plate by Using Marchi-Fasulo and Laplace Integral Transform”, International Journal of Engineering Science Invention, 7(2): 1-5, 2018.
- [14] B. E. Ghonge “A note on thermal treatment in an inverse transient thin circular plate”, International Journal of Creative Research Thoughts, 9(4), 1223-1227, 2021.
- [15] K. W. Khobragade, V. Varghese and N. W. Khobragade, “An Inverse Transient Thermoelastic Problem of a Thin Annular Disc”, Applied Mathematics E-Notes, Vol.6, pp.17- 25, 2006.
- [16] N. M. Ozisik, “Boundary Value Problem of Heat Conduction”, International Textbook Company, Scranton, Pennsylvania, pp. 481-492, 1968.



INNO  **SPACE**
SJIF Scientific Journal Impact Factor

Impact Factor:
7.512

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA



INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

 **9940 572 462**  **6381 907 438**  **ijirset@gmail.com**



www.ijirset.com

Scan to save the contact details