

e-ISSN: 2319-8753 | p-ISSN: 2347-6710

International Journal of Innovative Research in SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 10, Special Issue 2, December 2021

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.569

9940 572 462

6381 907 438

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|| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569| || Volume 10, Special Issue 2, December 2021 || 4th International Conference on Emerging Trends in Science, Technology and Mathematics [ICETSTM '2021]

15th & 16th September 2021

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Department of Computer Science, Parvathy's Arts and Science College, Dindigul, Tamilnadu, India

New Similarity Measures of Trapezoidal Fuzzy Numbers

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ABSTRACT : α -cuts of trapezoidal fuzzy numbers and α -cuts of trapezoidal fuzzy number matrices are introduced and evaluated.

KEYWORDS : α-cuts of Trapezoidal Fuzzy Numbers (TrFNs), α-cuts of Trapezoidal Fuzzy Number Matrices (TrFNMs)

I. INTRODUCTION

Real world decision making problems are very often uncertain (or) vague in a number of ways. In 1965, Zadeh [4] introduced the concept of fuzzy set theory to meet those problems. The fuzzyness can be represented by different ways. One of the most useful representation is the membership function. Depending on the nature of the membership function the fuzzy numbers can be classified in different forms, such as Triangular Fuzzy Numbers (TFNs), Trapezoidal Fuzzy Numbers, Interval Fuzzy Numbers etc. Fuzzy matrices play an important role in scientific development. Fuzzy matrices were introduced by M.G.Thomson [3]. Two new operators and some properties of fuzzy number matrices are given in [1]. Some new operators on triangular fuzzy numbers and triangular fuzzy number matrices are given in [2].

Definition 2.1^[2]:

II. PRELIMINARIES

A Fuzzy set A in a universe of discourse X is defined as the following set of pairs A= {(x, $\mu_A(x)$): x \in X}. Here μ_A : X \rightarrow [0,1] is a mapping called the membership value of x \in X in a fuzzy set A.

Definition 2.2^[2]:

A **Fuzzy number** is an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each

possible values has its own weight 0 and 1. This weight is called the membership function .

Definition 2.3^[1]:

A normal fuzzy number A with shape function

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$$\mu_{A} = \begin{cases} \left(\frac{x-a}{b-a}\right)^{n} & \text{when } x \in [a, b), \\ w & \text{when } x \in [b, c], \\ \left(\frac{d-x}{d-c}\right)^{n} & \text{when } x \in (c, d] \\ 0 & \text{otherwise} \end{cases}$$

where n > 0, will be denoted by $A = (a, b, c, d)_n$. If A be non-normal fuzzy number, it will be denoted by $A = (a, b, c, d; w)_n$. If n = 1, we simply write A = (a, b, c, d), which is known as a **normal Trapezoidal fuzzy number**.



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Definition 2.4^[5]:

A **Trapezoidal Fuzzy Number (TrFN)** denoted by $\langle m, \alpha, \beta, \gamma \rangle$ has the membership function

$$\mu_{A}(x) = \begin{cases} 0, & \text{for } x \leq m \\ \frac{x-m}{\alpha-m} & m \leq x \leq \alpha \\ 1, & \alpha \leq x \leq \beta \\ \frac{\gamma-x}{\gamma-\beta}, & \beta \leq x \leq \gamma \\ 0, & x \geq \alpha \end{cases}$$

or, $\mu_A(x) = \max(\min \frac{x-m}{\alpha-m}, 1, \frac{\gamma-x}{\gamma-\beta}), 0)$

The point m, with membership grade of 1, is called the **mean value** and α , β are the **left hand spreads** of M repectively.

When $\alpha = \beta$, the trapezoidal fuzzy number coincides with triangular one.

Example 2.5:

 \widetilde{M} = < 3, 6, 7, 9 > is a Trapezoidal Fuzzy number.

Definition 2.6^[3]:

Let $M = (\tilde{M}_{ij})_{m \times n}$ and $N = (\tilde{N}_{ij})_{m \times n}$ be two Trapezoidal Fuzzy Number Matrices (TrFNM) of the same order. Then the following **operators** are defined.

(1) $M \bigoplus N = (\widetilde{M}_{ij} \bigoplus \widetilde{N}_{ij})$

(2) M V N = $(\widetilde{M}_{ij} \vee \widetilde{N}_{ij})$

(3) $M \ominus \mathbb{Z} N = (\widetilde{M}_{ij} \ominus \widetilde{N}_{ij})$

(4) $M \ge N$ iff $\widetilde{M}_{ij} \ge \widetilde{N}_{ij} \forall i = 1 \text{ to } m, j = 1 \text{ to } n.$

2.7 Trapezoidal Fuzzy Number and Its Operators

If a **regular convex fuzzy set** \widetilde{M} is based on the real domain and meets the following conditions:

(1) there exists only one element x_0 that $\mu_{\tilde{M}}(x_0)=1$,

(2) $\mu_{\widetilde{M}}(x)$ is continuous, then \widetilde{M} is a fuzzy number, which means "a real number that is approximated to x_0 ." \widetilde{M} can be expressed as follows :

$$\mu_{\widetilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & -\infty < x < m, \\ 1, & m \le x \le n, \\ R\left(\frac{x-n}{\beta}\right), & x \le m \le +\infty, \end{cases}$$

which is referred to as L-R fuzzy number denoted by $\widetilde{M} = (m, n, \alpha, \beta)$, where $m \le n$, $\alpha \ge 0$ and $\beta \ge 0$ are respectively the **left hand and right hand spreads**, and L $(\frac{m-x}{\alpha})$ and R $(\frac{x-n}{\alpha})$ are continuous and non-increasing functions satisfying

$$L(0)=R(0)=1$$
 and $L(1)=R(1)=0$.

When L and R are both linear functions, the L-R fuzzy number $\widetilde{M}=(m,n,\alpha,\beta)$ becomes a trapezoidal fuzzy number (TrFN) $\widetilde{A}=(a^l,a^m,a^u,a^n)$, and its membership function is as follows:



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$$\mu_{\widetilde{M}}(\mathbf{x}) = \begin{cases} \frac{x-a^{u}}{a^{m}-a^{l}}, & a^{l} \leq x \leq a^{m}, \\ \frac{x-a^{u}}{a^{m}-a^{u}}, & a^{m} \leq x \leq a^{u}, \\ \frac{x-a^{n}}{a^{m}-a^{n}}, & a^{u} \leq x \leq a^{n}, \\ 0, & \text{otherwise.} \end{cases}$$

,

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Let $\tilde{A} = (a^l, a^m, a^u, a^n)$ and $\tilde{B} = (b^l, b^m, b^u, b^n)$ be two trapezoidal fuzzy numbers; the **basic operators** for trapezoidal fuzzy numbers are as follows:

$$\begin{split} \tilde{A} & \oplus \tilde{B} = (a^l, a^m, a^u, a^n) \oplus (b^l, b^m, b^u, b^n) \\ &= (a^l + b^l, a^m + b^m, a^u + b^u, a^n + b^n), \\ \tilde{A} & \otimes \tilde{B} = (a^l, a^m, a^u, a^n) \otimes (b^l, b^m, b^u, b^n) \\ &= (a^l b^l, a^m b^m, a^u b^u, a^n b^n), \end{split}$$

$$=(a^{t}b^{t},a^{m}b^{m},a^{u}b^{u},a^{n})$$

 $\frac{1}{\tilde{A}} = \left(\frac{1}{a^l}, \frac{1}{a^m}, \frac{1}{a^u}, \frac{1}{a^n}\right),$

$$m\tilde{A} = m(a^l, a^m, a^u, a^n) = (ma^l, ma^m, ma^u, ma^n).$$

III. SIMILARITY OF TRAPEZOIDAL FUZZY NUMBERS

A similarity measure is an important tool for presenting a degree of similarity between two objects. The application of trapezoidal fuzzy number in different area needs different measurements of its similarity. **Chen** and **Lin** proposed a method to calculate **Similarity based on Distance of Centre and Boundary.** Let $\tilde{A} = (a^l, a^m, a^u, a^n)$ and $\tilde{B} = (b^l, b^m, b^u, b^n)$ be two trapezoidal fuzzy numbers, the similarity of them can be denoted as follows:

$$S_{MB}(\tilde{A},\tilde{B}) = 1 - \frac{|a^{l} - b^{l}| + |a^{m} - b^{m}| + |a^{u} - b^{u}| + |a^{n} - b^{n}|}{4}$$

Suppose that there are trapezoidal fuzzy numbers, and all of them match the membership function $\tilde{A}_i = (a_i^l, a_i^m, a_i^u, a_i^n)$, i=1,2,...,n. Then the **maximizing set T** and **minimizing set B** with membership function are as follows:

$$\mu_T(x) = \begin{cases} \frac{x - x_{min}}{x_{max} - x_{min}} , & x_{min} \le x \le x_{max} \\ 0, & \text{otherwise,} \end{cases}$$
$$\mu_B(x) = \begin{cases} \frac{x - x_{max}}{x_{min} - x_{max}} , & x_{min} \le x \le x_{max} \\ 0, & \text{otherwise.} \end{cases}$$

Let $\tilde{A} = (a^l, a^m, a^u, a^n)$ be a trapezoidal fuzzy number. Then its **right utility value** $\mu_T(\tilde{A}_i)$ based on maximizing set and its **left utility value** $\mu_B(\tilde{A}_i)$ based on minimizing set G are

$$\mu_T(\tilde{A}_i) = \sup(\mu_{\tilde{A}_i}(x) \wedge \mu_T(x))$$



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and

 $\mu_B(\tilde{A}_i) = \sup(\mu_{\tilde{A}_i}(x) \wedge \mu_B(x))$

And the **total utility value** $\mu_{IIV}(\tilde{A}_i)$ is :

 $\mu_{UV}(\tilde{A}_i) = \frac{\mu_T(\tilde{A}_i) + 1 - \mu_B(\tilde{A}_i)}{2}.$ Suppose $\tilde{A} = (a^l, a^m, a^u, a^n)$ and $\tilde{B} = (b^l, b^m, b^u, b^n)$ be two trapezoidal fuzzy numbers. The **utility** similarity $S_{UV}(\tilde{A}, \tilde{B})$ between them is as follows:

$$S_{UV}(\tilde{A}, \tilde{B}) = \frac{U_{UV}(\tilde{A}) \times U_{UV}(\tilde{B})}{max((U_{UV}(\tilde{A}_i))^2, (U_{UV}(\tilde{B}))^2)}$$

where $U_{UV}(\tilde{A})$ and $U_{UV}(\tilde{B})$ are the utility values of \tilde{A} and \tilde{B} respectively. And $S_{UV}(\tilde{A}, \tilde{B}) = 1$ when $U_{UV}(\tilde{A})$ and $U_{UV}(\tilde{B})$ are both 0.

Based on the idea of graded mean integration-representation distance (GMIR), Hsieh and Chen (1999) presented a similarity measure for two trapezoidal fuzzy numbers. The measurement of GMIR is as follows [12]

$$S_{GMIR}(\tilde{A}, \tilde{B}) = \frac{1}{1 + d(\tilde{A}, \tilde{B})}$$

where $\mathbf{d}(\widetilde{A}, \widetilde{B}) = |P(\widetilde{A}) - P(\widetilde{B})|$,

 $P(\tilde{A})$ and $P(\tilde{B})$ are the graded mean integeration representations of \tilde{A} and \tilde{B} , respectively.

If \tilde{A} and \tilde{B} are trapezoidal fuzzy numbers, then the graded mean integeration-representations $P(\tilde{A})$ and $P(\tilde{B})$ of \tilde{A} and \tilde{B} are defined as follows:

$$\mathbf{P}(\widetilde{A}) = \frac{a^l + 2a^m + 2a^u + a^n}{6},$$

$$\mathbf{P}(\widetilde{B}) = \frac{b^l + 2b^m + 2b^u + b^n}{6}.$$

Theorem 3.1:

Let A = (a_1, b_1, c_1, d_1) and B = (a_2, b_2, c_2, d_2) and C = (a_3, b_3, c_3, d_3) be three trapezoidal fuzzy numbers, then their distance measure satisfies the following relations

i) d(A, A) = 0,

ii) d(A,B) = d(B,A),

- iii) if $A \le B \le C$, then $d(A,B) \le d(A,C)$,
- iv) $d(A,B) \le d(A,C) + d(C,B)$.

Proof.

(i)
$$d(A,A) = |P(A) - P(A)|$$

 $= [(\frac{(a_1+d_1)}{2} + \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)] - [\frac{(a_1+d_1)}{2} + \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)]|$
 $= |[\frac{(a_1+d_1)}{2} - \frac{(a_1+d_1)}{2}] + [\frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1) - \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)]| = 0$
(ii) $d(A,B) = |P(A) - P(B)|$
 $= |[\frac{(a_1+d_1)}{2} + \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)] - [\frac{(a_2+d_2)}{2} + \frac{n}{2n+1}(b_2 - a_2 - d_2 + c_2)]|$
 $= |[\frac{(a_2+d_2)}{2} + \frac{n}{2n+1}(b_2 - a_2 - d_2 + c_2)] - [\frac{(a_1+d_1)}{2} + \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)]|$
 $= |P(B) - P(A)| = d(B,A)$
(iii) We have $B \in C$. So

(iii) We have $B \leq C$, So

 $P(B) \le P(C)$ $\implies \left[\frac{(a_2+d_2)}{2} + \frac{n}{2n+1}(b_2 - a_2 - d_2 + c_2)\right] \le \left[\frac{(a_3+d_3)}{2} + \frac{n}{2n+1}(b_3 - a_3 - d_3 + c_3)\right]$



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$$\Rightarrow \left[\frac{(a_{2}+a_{2})}{2} + \frac{n}{2n+1}(b_{2}-a_{2}-d_{2}+c_{2})\right] - \left[\frac{(a_{1}+a_{1})}{2} + \frac{n}{2n+1}(b_{1}-a_{1}-d_{1}+c_{1})\right] \\ \leq \left| \left[\frac{(a_{3}+d_{3})}{2} + \frac{n}{2n+1}(b_{3}-a_{3}-d_{3}+c_{3})\right] - \left[\frac{(a_{1}+d_{1})}{2} + \frac{n}{2n+1}(b_{1}-a_{1}-d_{1}+c_{1})\right] \right| \\ \Rightarrow \left| \left[\frac{(a_{2}+d_{2})}{2} + \frac{n}{2n+1}(b_{2}-a_{2}-d_{2}+c_{2})\right] - \left[\frac{(a_{1}+d_{1})}{2} + \frac{n}{2n+1}(b_{1}-a_{1}-d_{1}+c_{1})\right] \right| \\ \leq \left| \left[\frac{(a_{3}+d_{3})}{2} + \frac{n}{2n+1}(b_{3}-a_{3}-d_{3}+c_{3})\right] - \frac{(a_{1}+d_{1})}{2} + \frac{n}{2n+1}(b_{1}-a_{1}-d_{1}+c_{1})\right] \right| \\ Of (ii), we have \\ \Rightarrow \left| \left[\frac{(a_{1}+d_{1})}{2} + \frac{n}{2n+1}(b_{1}-a_{1}-d_{1}+c_{1})\right] - \left[\frac{(a_{2}+d_{2})}{2} + \frac{n}{2n+1}(b_{2}-a_{2}-d_{2}+c_{2})\right] \right| \\ = \left| P(A) - P(B) \right| \\ \leq \left| \left[\frac{(a_{1}+d_{1})}{2} + \frac{n}{2n+1}(b_{1}-a_{1}-d_{1}+c_{1})\right] - \left[\frac{(a_{3}+d_{3})}{2} + \frac{n}{2n+1}(b_{3}-a_{3}-d_{3}+c_{3})\right] \right| \\ = \left| P(A) - P(C) \right| \\ \Rightarrow \left| P(A) - P(B) \right| \leq \left| P(A) - P(C) \right| \Rightarrow d(A,B) \leq d(A,C) . \end{cases}$$

$$= \left| \left[\frac{a_1 + d_1}{2} + \frac{n}{2n+1} (b_1 - a_1 - d_1 + c_1) \right] - \left[\frac{a_2 + d_2}{2} + \frac{n}{2n+1} (b_2 - a_2 - d_2 + c_2) \right] \right|$$

$$= \left| \left[\frac{a_1 + d_1}{2} + \frac{n}{2n+1} (b_1 - a_1 - d_1 + c_1) \right] - \left[\frac{a_2 + d_2}{2} + \frac{n}{2n+1} (b_2 - a_2 - d_2 + c_2) \right] \right|$$

$$\implies \left[\frac{(a_1+a_1)}{2} + \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)\right] - \left[\frac{(a_2+a_2)}{2} + \frac{n}{2n+1}(b_2 - a_2 - d_2 + c_2)\right]$$

$$\Rightarrow \leq \left| \left[\frac{(a_1+d_1)}{2} + \frac{n}{2n+1} (b_1 - a_1 - d_1 + c_1) \right] - \left[\frac{(a_2+d_2)}{2} + \frac{n}{2n+1} (b_2 - a_2 - d_2 + c_2) \right] \right|$$

$$\pm \left[\frac{(a_3+d_3)}{2} + \frac{n}{2n+1}(b_3 - a_3 - d_3 + c_3)\right] |$$

$$\le \left| \left[\frac{(a_1+d_1)}{2} + \frac{n}{2n+1}(b_1 - a_1 - d_1 + c_1)\right] - \left[\frac{(a_3+d_3)}{2} + \frac{n}{2n+1}(b_3 - a_3 - d_3 + c_3)\right] \right| + \left| \left[\frac{(a_3+d_3)}{2} + \frac{n}{2n+1}(b_3 - a_3 - d_3 + c_3)\right] - \left[\frac{(a_2+d_2)}{2} + \frac{n}{2n+1}(b_2 - a_2 - d_2 + c_2)\right] \right|$$

 $\leq \mid P(A) - P(A) \mid + \mid P(C) - P(B) \mid = d(A,C) + d(C,B) \Longrightarrow d(A,B) \leq d(A,C) + d(C,B).$

Yager presented the concept of kernel of fuzzy number [6]. Let $(\tilde{A}, \mu_{\tilde{A}}(x))$ be a fuzzy number and its kernel function can be denoted as follows:

$$\mathrm{K}(\tilde{A}) = \frac{\int_0^1 x \mu_{\tilde{A}}(x) dx}{\mu_{\tilde{A}}(x) dx}.$$

S.-J. Chen and S.-M. Chen presented a simple center method to calculate the center-of-gravity (COG) points (x^*, y^*) of generalized fuzzy numbers, and put forward a similarity measure of trapezoidal fuzzy numbers based on the center-of-gravity [1].

Let $\tilde{A} = (a^L, a^M, a^U, a^N)$ be a trapezoidal number and $(x_{\tilde{A}}^* y_{\tilde{A}}^*)$ the **center-of-gravity**; its formula is shown as follows:

Let $\tilde{A} = (a^l, a^m, a^u, a^n)$ and $\tilde{B} = (b^l, b^m, b^u, b^n)$ be two trapezoidal fuzzy numbers, and let $(x_{\tilde{A}}^* y_{\tilde{A}}^*)$ and $(x_{\tilde{B}}^*, y_{\tilde{B}}^*)$ be their COGs; the similarity of \tilde{A} and \tilde{B} can be defined as follows:

$$S_{COG}(\tilde{A}, \tilde{B}) = \left(1 - \frac{|a^l - b^l| + |a^m - b^m| + |a^u - b^u| + |a^n - b^n|}{4}\right) \times \left(1 - \left|x_{\tilde{A}}^* - x_{\tilde{B}}^*\right|\right)^{B(\tilde{A}, \tilde{B})} * \frac{\min(y_{\tilde{A}}^*, y_{\tilde{B}}^*)}{\max(y_{\tilde{A}}^*, y_{\tilde{B}}^*)},$$

where $x_{\tilde{A}}^*, y_{\tilde{A}}^*, x_{\tilde{B}}^*$ and $y_{\tilde{B}}^*$ are calculated according to formula (<u>13</u>) and B(\tilde{A}, \tilde{B}) is used to determine if the COG distance is considered; the new formula is defined as follows:



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$$B(\tilde{A}, \tilde{B}) = \begin{cases} 1, & \text{if } S_{\tilde{A}} + S_{\tilde{B}} > 0\\ 0, & \text{if } S_{\tilde{A}} + S_{\tilde{B}} = 0 \end{cases}$$

and $S_{\tilde{A}} = a^n - a^l,$
 $S_{\tilde{B}} = b^n - b^l.$

where $S_{\tilde{A}}$ and $S_{\tilde{B}}$ are the lengths of the trapezoidal fuzzy numbers \tilde{A} and \tilde{B} , respectively, defined as follows:

It has been proven that the similarity based on COG has the following property: $S_{COG}(\tilde{A}, \tilde{B}) = 1$ if and only if $\tilde{A} = \tilde{B}$.

Because of the complexity of objective things and the disturbance of the measurement surroundings, there exist uncertainty and fuzziness in dealing with **Multisensor Data Fusion (MSDF)**. Wan (2011) used trapezoidal fuzzy number to express its uncertainty and fuzziness [4]. What is more, multi-attribute decision-making theory was used to solve problems in MSDF and a new multi-sensor object recognition method was proposed. Thus, another similarity of two trapezoidal fuzzy numbers is presented.

Suppose $\tilde{A} = (a^l, a^m, a^u, a^n)$ and $\tilde{B} = (b^l, b^m, b^u, b^n)$ are two trapezoidal fuzzy numbers; their **similarity** is defined as follows:

$$S_{max}(\tilde{A},\tilde{B}) = \frac{\sum_{i \in \{l,m,u,n\}} a^{l} b^{i}}{max(\sum_{i \in \{l,m,u,n\}} a^{i} a^{i}, \sum_{i \in \{l,m,u,n\}} b^{i} b^{i})}.$$

To solve **multiple criteria group decision-making** (**MCGDM**) problems, method based on vector similarity is used to measure similarity of two trapezoidal fuzzy numbers:

$$S_{VS}(\tilde{A},\tilde{B}) = \frac{2\sum_{i \in \{l,m,u,n\}} a^i b^i}{\sum_{i \in \{l,m,u,n\}} a^i a^i + \sum_{i \in \{l,m,u,n\}} b^i b^i}.$$

IV. SOME DISTANCE MEASURES OF TRAPEZOIDAL FUZZY NUMBERS

The proposed ranking method consists of five distance - based components which are the horizontal x component of fuzzy numbers, centroid point, similarity measure, height and spread of fuzzy numbers. Conceptually, the proposed method is essentially an extension of the method given by [11] and [12] where they only consider three components of distances in fuzzy numbers which are the **x-value**, centroid point and similarity measure. The two extra components i.e., the **spread and height** of the fuzzy number are included as they are crucial in discriminating some complex combination of fuzzynumbers.

Let A= $(a_1, a_2, a_3, a_4; wA)$ and B= $(b_1, b_2, b_3, b_4; wB)$ be two standardized generalized fuzzy numbers. Step 1:

Obtain the agreement between A and B by taking the average of the two fuzzy numbers denoted by δ_{AB} where

$$\delta_{AB} = \frac{a_1 + b_1}{2}, \frac{a_2 + b_2}{2}, \frac{a_3 + b_3}{2}, \frac{a_4 + b_4}{2}; \min(w_A + w_B),$$

$$=(c1, c2, c3, c4; w)$$

where $C_i \in [-1, 1]$.

Step 2:

Calculate the centroid point x^* , y^* of each standardized generalized fuzzy numbers and their agreement using formula given by

$$x_A^* = \frac{\int_{-1}^1 x f(x) dx}{\int_{-1}^1 f(x) dx},$$

$$y_A^* = \frac{\int_0^w \alpha \ A^\alpha \ d\alpha}{\int_0^w A^\alpha \ d\alpha}$$

where A^{α} is the length of the α -cut of A^{α} and x_A^* , y_A^* are the centroid of fuzzy number A with $x_A^* \in [-1,1]$ and $y_A^* \in [0,1]$.



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Step 3:

Calculate the similarity measure between the agreement and δ_{AB} and A given as

$$\varphi^* \delta_{AB}, A = \mu_d \ \delta_{AB}, A \ \left(1 - x^*_{\delta_{AB}} - x^*_A \right) \frac{\min(y^*_{\delta_{AB}}, y^*_A)}{\max(y^*_{\delta_{AB}}, y^*_A)},$$

where

 $\mu_{d} \, \delta_{AB}, A = \frac{1}{4} \Big(1 - \frac{c_1 - a_1}{0.5} + 1 - \frac{c_2 - a_2}{0.5} + 1 - \frac{c_3 - a_3}{0.5} + 1 - \frac{c_4 - a_4}{0.5} \Big)$

Essentially, μd is the distance between each point of the fuzzy numbers.

Step 4:

Calculate the spread of each fuzzy numbers using the following formula:

$$\sigma^*(\mathbf{A}) = \frac{\overline{\sum_{1=1}^4 (a_1 - x_{\mathbf{A}}^*)^2}}{4-1}$$

where $\sigma^*(A) \in [0, 1.1547]$. The value of 1.1547 is obtained by considering the maximum of $\sigma^{*}(A)$ with A = (-1, -1, 1, 1; wA).

Step 5:

Obtain the score values for A and B. Let Δ A be the score value representing A, the ranking value is calculated

 $\Delta A = x_A^* \cdot h_A^* \cdot \phi^* \delta_{AB}, A^{\sigma^*(A)}$ and $\Delta A \in -1.1$.

Step 6:

Rank A and B. If $\Delta A > \Delta B$ then A > B i.e A is ranked higher than B. If $\Delta A \leq \Delta B$ then $A \leq B$ i.e A is ranked lower than B. If $\Delta A = \Delta B$ then $A \approx B$ i.e A and B are equally ranked.

V. CONCLUSION

Trapezoidal fuzzy number has been applied in many fields such as risk analysis, decision-making, and evaluation. A new method to measure the similarity of two trapezoidal fuzzy numbers is proposed, which measures the similarity based on the shape's indifferent area and centroid of two trapezoidal fuzzy numbers. Comparison with other measurements shows that the similarity measuring method proposed in this paper can be distributed more normally.

Finally, trapezoidal fuzzy number is applied in collaborative filtering recommendation system, in which trapezoidal fuzzy number is used to express the users' comprehensive evaluation on items. Case demonstration and simulation on MoviesLens show that trapezoidal fuzzy number can express users' comprehensive evaluation on items and the collaborative filtering recommendation accuracy can be higher.

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|| e-ISSN: 2319-8753, p-ISSN: 2320-6710| www.ijirset.com | Impact Factor: 7.569| || Volume 10, Special Issue 2, December 2021 ||

4th International Conference on Emerging Trends in Science, Technology and Mathematics [ICETSTM '2021]

15th & 16th September 2021

Organized by

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Impact Factor: 7.569





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