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Department of Computer Science, Parvathy's Arts and Science College, Dindigul, Tamilnadu, India

# **Regular Sets in Generalised Topological Space**

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ABSTRACT: We define and study regular and locally closed sets in generalized topological spaces.

**KEYWORDS**: eneralized topological spaces, Strong space, Regular set, extremely disconnected space.

#### **I.INTRODUCTION**

In 1975, A. Csaszar defined and studied regular sets in generalized topological spaces. In Section 2, we define various kinds of regular sets and discuss their properties. Section 3 is devoted to the study of locally closed sets in generalized topological spaces.

# **II.PRELIMINARIES**

We recall some notion of the essential concepts needed in the following. A nonempty family  $\lambda$  of subsets of a set X is said to be a generalized topology [3] if  $\emptyset \in \lambda$  and an arbitrary union of elements of  $\lambda$  is again in  $\lambda$ . The pair  $(X, \lambda)$  is called a generalized topological space and elements of  $\lambda$  are called  $\lambda$ -open sets. A  $\subset$  X is  $\lambda$ -closed if X – A is  $\lambda$ -open. By a space  $(X, \lambda)$ , we always mean a generalized topological space. If  $X \in \lambda$ ,  $(X, \lambda)$  is called a strong generalized topological space. In a space  $(X, \lambda)$ , if  $\lambda$  is closed under finite intersection,  $(X, \lambda)$  is called a quasi-topological space [4]. Clearly, every strong quasi topological space is a topological space. For  $A \subset X$ ,  $C_{\lambda}(A)$  is the smallest  $\lambda$ -closed set containing A and  $i_{\lambda}(A)$  is the largest  $\lambda$ -open set contained in A. Moreover,  $X - C_{\lambda}(A) = i_{\lambda}(X - A)$  or every subset A of X.

**Lemma 2.1.** If  $(X, \lambda)$  is a quasi-topological space, then the following hold. (a) If A and B are  $\lambda$ -open sets, then A  $\cap$  B is a  $\lambda$ -open set [7, Theorem 3.1].

(b)  $i_{\lambda}(A \cap B) = i_{\lambda}(A) \cap i_{\lambda}(B)$  for every subsets A and B of X [5, Theorem 3.2].

(c)  $C_{\lambda}(A \cup B) = C_{\lambda}(A) \cup C_{\lambda}(B)$  for every subsets A and B of X [7, Theorem 3.3(b)].

**Lemma 2.2.** If  $(X, \lambda)$  is a quasi-topological space, then the following hold. (a)  $i_{\lambda}(A \cup F) \subset i_{\lambda}(A) \cup F$  where F is  $\lambda$ -closed [8, Theorem 2.1(c)]. (b)  $C_{\lambda}(A \cup F) = C_{\lambda}(A) \cup F$  where F is  $\lambda$ -closed [8, Theorem 2.1(d)].

#### **III. REGULAR SETS IN GTS**

Let  $(X, \lambda)$  be a space and  $A \subset X$ . Let  $\alpha, \sigma, \pi, \beta$  be defined as in [3]. A is said to be  $\alpha$ -regular (resp.  $\sigma$ -regular,  $\pi$ -regular ) if  $A = i_{\lambda}C_{\lambda}i_{\lambda}(A)$  (resp.  $A = C_{\lambda}i_{\lambda}(A)$ ,  $A = i_{\lambda}C_{\lambda}(A)$ ,  $A = C_{\lambda}i_{\lambda}C_{\lambda}(A)$ ). Clearly, if  $\xi \in \{\alpha, \sigma, \pi, \beta\}$  and  $A \subset X$ , then A is  $\xi$ -regular implies that A is  $\xi$ -open. A subset A of X is said to be  $\xi$ -clopen if A is both  $\xi$ -open and  $\xi$ -closed. The following Theorem 2.1. gives the relation between  $\pi$ -regular sets and  $\alpha$ -regular sets. Theorem 2.2 gives the relation between  $\sigma$ -regular sets.

**Theorem 3.1.** Let  $(X, \lambda)$  be a space and  $A \subset X$ . Then A is a  $\pi$ -regular set if and only if A is an  $\alpha$ -regular set. **Proof.** Suppose that A is a  $\pi$ -regular set. Then  $A = i_{\lambda}C_{\lambda}(A)$  which implies that A is  $\mu$ -open and so  $A = i_{\lambda}C_{\lambda}i_{\lambda}(A)$ . Therefore, A is an  $\alpha$ -regular set. Conversely, suppose A is an  $\alpha$ -regular set. Then  $A = i_{\lambda}C_{\lambda}i_{\lambda}(A)$ . This implies that  $i_{\lambda}C_{\lambda}(A) = i_{\lambda}C_{\lambda}i_{\lambda}(A) = A$  and so  $A = i_{\lambda}C_{\lambda}(A)$ . Therefore, A is a  $\pi$ -regular set.

**Theorem 3.2.** Let  $(X, \lambda)$  be a space and  $A \subset X$ . Then A is a  $\sigma$ -regular set if and only if A is a  $\beta$ -regular set. **Proof.** Suppose that A is a  $\sigma$ -regular set. Then  $A = C_{\lambda}i_{\lambda}(A)$  and so A is  $\lambda$ -closed. Therefore,  $A = C_{\lambda}i_{\lambda}(A) = C_{\lambda}i_{\lambda}C_{\lambda}(A)$ which implies that A is a  $\beta$ -regular set. Conversely, suppose that A is a  $\beta$ -regular set. Since A is a  $\lambda$ -closed set, we have  $A = C_{\lambda}i_{\lambda}C_{\lambda}(A) = C_{\lambda}i_{\lambda}(A)$ . Therefore A is a  $\gamma$ -regular set.

The following theorem characterizes  $\sigma$ -regular sets and  $\sigma$ -clopen sets of generalized topological spaces.



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**Theorem 3.3.** Let  $(X, \lambda)$  be a space and  $A \subset X$ . Then the following hold.

(a) A is  $\sigma$ -regular if and only if A is  $\sigma$ -open and  $\mu$ -closed if and only if A is  $\sigma$ -open and  $\pi$ -closed. (b) A is  $\sigma$ -clopen if and only if A =  $C_{\nu}i_{\nu}(A)$ .

**Proof.** (a) If A is  $\gamma$ -regular, then clearly, A is  $\sigma$ -open and  $\lambda$ -closed which implies that A is  $\gamma$ -open and  $\pi$ -closed. Suppose A is  $\gamma$ -open and  $\pi$ -closed. Then  $A \subset C_{\lambda}i_{\lambda}(A)$  and  $C_{\lambda}i_{\lambda}(A) \subset A$ . Hence  $A = C_{\lambda}i_{\lambda}(A)$  which implies that A is  $\sigma$ -regular.

(b) If A is  $\gamma$ -open and  $\gamma$ -closed, then clearly,  $A = C_{\gamma}i_{\gamma}(A)$ . Conversely, suppose  $A = C_{\gamma}i_{\gamma}(A)$ . Then A is  $\gamma$ -closed. Also  $A = C_{\gamma}i_{\gamma}(A) \subset C_{\lambda}i_{\gamma}(A) = C_{\lambda}i_{\lambda}(A)$ , by Lemma 2.4(f) of [7] and so A is  $\gamma$ -open. Hence A is  $\gamma$ -clopen.

The following theorem characterizes  $\beta$ -regular sets and  $\beta$ -clopen sets of generalized topological spaces.

**Theorem 3.4.** Let  $(X, \lambda)$  be a space and  $A \subset X$ . Then the following hold.

(a) A is  $\beta$ -regular if and only if A is  $\beta$ -open and  $\lambda$  -closed if and only if A is  $\beta$ -open and  $\alpha$ -closed.

(b) A is  $\beta$ -clopen if and only if A =  $C_{\beta}i_{\beta}C_{\beta}(A)$ .

**Proof.** (a) Similar to Theorem 2.3(a).

(b) If A is  $\beta$ -clopen, then clearly,  $A = C_{\beta}i_{\beta}C_{\beta}(A)$ . Conversely, suppose  $A = C_{\beta}i_{\beta}C_{\beta}(A)$ . Then A is  $\beta$ -closed. Also,  $A \subset C_{\lambda}i_{\beta}C_{\beta}(A) \subset C_{\lambda}i_{\beta}C_{\lambda}(A) \subset C_{\lambda}i_{\lambda}C_{\lambda}(A)$ , since  $i\beta(A) \subset A \cap C_{\lambda}i_{\lambda}C_{\lambda}(A)$  by Lemma 1.1 of [2]. Hence A is  $\beta$  open which implies that A is  $\beta$ -clopen.

Let  $(X, \lambda)$  be a space. Let  $\xi \in {\pi, \gamma, \alpha, \beta}$ . X is said to be extremally  $\xi$ -disconnected if the  $\xi$ -closure of every  $\xi$ -open set is  $\xi$ -open. The following theorem characterizes  $\pi$ -regular sets and  $\pi$ -clopen sets of generalized topological spaces. Example 2.6 below shows that the condition extremally  $\pi$ -disconnected in Theorem 2.5(b) cannot be dropped.

**Theorem 3.5.** Let  $(X, \lambda)$  be a space and  $A \subset X$ . Then the following hold.

set even if the generalized topology is strong and extremally disconnected.

(a) A is  $\pi$ -regular if and only if A is  $\pi$ -open and  $\gamma$ -closed.

(b) If (X,  $\lambda$ ) is extremally  $\pi$ -disconnected, then A is  $\pi$ -clopen if and only if A =  $i_{\pi}C_{\pi}(A)$ .

**Proof.** (a) A is  $\pi$ -regular if and only if  $A = i_{\lambda}C_{\lambda}(A)$  if and only if  $A \subset i_{\lambda}C_{\lambda}(A)$  and  $A \supset i_{\lambda}C_{\lambda}(A)$  if and only if A is  $\pi$ -open and A is  $\sigma$ -closed.

(b) If A is  $\pi$ -clopen, then clearly,  $A = i_{\pi}C_{\pi}(A)$ . Conversely, suppose  $A = i_{\pi}C_{\pi}(A)$ . Then A is  $\pi$ -open. Since X is extremally  $\pi$ -disconnected,  $C_{\pi}(A)$  is  $\pi$ -open and so  $i_{\pi}C_{\pi}(A) = C_{\pi}(A)$  which implies that  $A = C_{\pi}(A)$ . Therefore A is  $\pi$ -closed and hence A is  $\pi$ -clopen.

**Example 3.6.** Let  $X = \{a, b, c\}$  and  $\mu = \emptyset, X, \{a\}, \{b\}, \{a, b\}$ . Then  $\pi(\lambda) = \emptyset, \{a\}, \{b\}, \{a, b\}, X$ . Since  $\{a\}$  is  $\pi$ -open but  $C_{\pi}$   $\{a\} = \{a, c\}$  is not  $\pi$ -open,  $(X, \lambda)$  is not extremely  $\pi$ -disconnected. But  $\{a\} = i_{\pi}C_{\pi}\{a\}$  and is not  $\pi$ -clopen. The following theorem shows that the intersection of two  $\pi$ -regular sets is again a  $\pi$ -regular set. Corollary 3.8 follows from Theorem 3.1. Example 3.9 below shows that the intersection of  $\gamma$ -regular sets (resp.  $\beta$ -regular sets) need not be a  $\gamma$ -regular (resp.  $\beta$ -regular) set. Example 3.10 below shows that the intersection of two  $\pi$ -clopen sets is not a  $\pi$ -clopen

**Theorem 3.7.** Let  $(X, \lambda)$  be a quasi-topological space and  $A, B \subset X$ . Then, the intersection of two  $\pi$ -regular sets is a  $\pi$ -regular set.

**Proof.** Let A and B be two  $\pi$ -regular sets. Then  $A = i_{\lambda}C_{\lambda}(A)$  and  $B = i_{\lambda}C_{\lambda}(B)$ . Then A and B are  $\mu$ -open and so  $A \cap B$  is  $\mu$ -open, since  $(X, \lambda)$  is a quasi-topological space. Therefore,  $A \cap B = i_{\lambda}(A \cap B) \subset C_{\lambda}i_{\lambda}(A \cap B)$  and so  $A \cap B \subset i_{\lambda}C_{\lambda}i_{\lambda}(A \cap B) \subset i_{\lambda}C_{\lambda}(A \cap B)$ . Again,  $i_{\lambda}C_{\lambda}(A \cap B) \subset i_{\lambda}C_{\lambda}(A) \cap i_{\lambda}C_{\lambda}(B) = A \cap B$ . Thus  $A \cap B = i_{\lambda}C_{\lambda}(A \cap B)$  which implies that  $A \cap B$  is  $\pi$ -regular.

**Corollary 3.8.** Let  $(X, \lambda)$  be a quasi-topological space and  $A \subset X$ . Then, the intersection of two  $\alpha$ -regular sets is an  $\alpha$ -regular set.

**Example 3.9.** Let X = R, with the usual topology. If A = [-1, 0] and B = [0, 1], then A and B are  $\sigma$ -regular but  $A \cap B = \{0\}$  is not  $\sigma$ -regular. By Theorem 3.2, it follows that the intersection of two  $\beta$ -regular sets need not be a  $\beta$ -regular set. **Example 3.10.** Let  $X = \{1, 2, 3, ..., n\}$  and  $\mu = \{\emptyset, X\} \cup A \subset X \mid A = X - \{i\}$  for some  $i \in X$ . Then  $(X, \lambda)$  is not a quasi-topological space such that  $\pi = \{\emptyset, X\} \cup A \subset X \mid |A| > 1$ . If  $A = \{1, 2\}$  and  $B = \{1, 3\}$ , then A and B are  $\pi$ -clopen sets but  $A \cap B = \{1\}$  is not a  $\pi$ -clopen set.



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# IV. MORE ON $\Lambda$ -LOCALLY $\Lambda$ -REGULAR SETS

Let  $(X, \lambda)$  be a generalized topological space, and  $\xi \in \{\mu, \alpha, \sigma, \pi, \beta\}$ . A subset A of a generalized topological space  $(X, \lambda)$  is said to be  $\mu$ -locally  $\xi$ -regular [6] if  $A = U \cap S$  where U is  $\mu$ -open and S is  $\xi$ -regular. A is said to be  $\mu$ -locally  $\xi$ -closed if  $A = U \cap S$  where U is  $\mu$ -open and S is  $\xi$ -closed. In this section, we discuss the properties of  $\mu$ -locally  $\xi$ -regular and  $\mu$ -locally  $\xi$ -closed sets.

**Theorem 4.1.** Let  $(X, \lambda)$  be a space and  $A \subset X$ . If  $(X, \lambda)$  is strong and A is  $\xi$ -closed, then A is  $\mu$ -locally  $\xi$ -closed. **Proof.** If  $(X, \lambda)$  is strong, then X is  $\mu$ -open and so  $A = X \cap A$ , which implies that A is  $\mu$ -locally  $\xi$ -closed.

**Theorem 4.2**. Let  $(X, \lambda)$  be a space and  $A \subset X$ . Then the following hold.

(a) A is a  $\lambda$ -locally  $\gamma$ -regular set if and only if A is  $\lambda$ -locally  $\beta$ -regular.

(b) A is a  $\lambda$ - locally  $\alpha$ - regular set if and only if A is  $\lambda$ - locally  $\pi$ - regular.

**Proof.** The proof follows from Theorem 3.1 and Theorem 3.2.

It is well known that every  $\gamma$ - closed set is a  $\beta$ - closed set. The following Theorem 4.3 shows that the converse holds for  $\lambda$ - locally  $\gamma$ - closed sets in a quasi – topological space. Example 4.4 below shows that the condition quasi-topological space on the space cannot be dropped.

**Theorem 4.3**. Let  $(X, \lambda)$  be a quasi-topological space and  $A \subset X$ . If A is a  $\beta$ -closed set and a  $\lambda$ -locally  $\gamma$ -closed set, then A is a  $\gamma$ -closed set.

**Proof.** Suppose that *A* is  $\beta$ -closed and  $\lambda$ -locally  $\gamma$ -closed. Then  $i_{\lambda}c_{\lambda}i_{\lambda}(A) \subset A = U \cap S$  where *U* is  $\lambda$ - open and *S* is  $\gamma$ -closed. Therefore,  $i_{\lambda}c_{\lambda}(S) \subset S$  which implies that  $i_{\lambda}c_{\lambda}(S) \subset i_{\lambda}(S)$ . Now  $i_{\lambda}c_{\lambda}(A) = i_{\lambda}c_{\lambda}(U \cap S) \subset i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}(S) \subset i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}(S) = i_{\lambda}(c_{\lambda}(U) \cap i_{\lambda}(S))$  by lemma 2.1(b). Therefore,  $i_{\lambda}c_{\lambda}(A) \subset i_{\lambda}c_{\lambda}((U) \cap i_{\lambda}(S)) = i_{\lambda}c_{\lambda}i_{\lambda}(U \cap S) \subset A$ . Therefore, *A* is  $\gamma$ -closed.

**Example 4.4.** Let  $X = \{p, q, r\}$  and  $\lambda = \{\emptyset, \{p\}, \{q, r\}, \{p, q, r\}\}$ . Then  $(X, \lambda)$  is not a quasi-topological space and the  $\lambda$ - closed sets are  $\{\emptyset, \{q, r\}, \{p\}, \{q\}, \{p, q, r\}\}$ . Now  $\{r\} = \{p, r\} \cap \{q, r\}$  where  $\{p, r\}$  is  $\lambda$ -open and  $\{q, r\}$  is  $\gamma$ -closed and so  $\{r\}$  is  $\lambda$ -locally  $\gamma$ -closed. Also,  $\{r\}$  is  $\beta$ -closed, since  $i_{\lambda}c_{\lambda}i_{\lambda}(\{r\}) = i_{\lambda}c_{\lambda}(\emptyset) \subset \{r\}$ . Hence  $\{r\}$  is both  $\beta$ - closed and  $\lambda$ -locally  $\gamma$ -closed, but  $\{r\}$  is not  $\gamma$ -closed, since  $i_{\lambda}c_{\lambda}r\} = \{q, r\}$  is not a subset of  $\{r\}$ .

The following Theorem 4.5 gives a characterization of  $\lambda$ -locally  $\gamma$ -regular sets and Example 4.6 shows that the condition quasi-topological space on the space cannot be dropped.

**Theorem 4.5.** Let  $(X, \lambda)$  be a quasi-topological space and  $A \subset X$ . Then the following are equivalent.

(a) A is a  $\lambda$ -locally  $\gamma$ -regular set.

(b)  $A = U \cap c_{\lambda} i_{\lambda}(A)$  for some  $\lambda$ -open set U.

(c) A is  $\gamma$ -open and  $\lambda$ -locally  $\lambda$ -closed.

(d) A is  $\beta$ -open and  $\lambda$ -locally  $\lambda$ -closed.

**Proof.** (a)  $\rightarrow$  (b). Suppose that A is a  $\lambda$ -locally  $\gamma$ -regular set. Then  $A = U \cap S$  where U is  $\lambda$ -open and  $S = c_{\lambda}i_{\lambda}(S)$ .  $A \subset S$  implies that  $c_{\lambda}i_{\lambda}(A) \subset c_{\lambda}i_{\lambda}(S) = S$  which implies that  $S \cap c_{\lambda}i_{\lambda}(A) = c_{\lambda}i_{\lambda}(A)$ . Now  $A = U \cap S = U \cap c_{\lambda}i_{\lambda}(S) \subset c_{\lambda}(U \cap i_{\lambda}(S))$ , since U is  $\lambda$ -open and so  $A \subset c_{\lambda}(i_{\lambda}(U) \cap i_{\lambda}(S)) \subset c_{\lambda}i_{\lambda}(U \cap S) = c_{\lambda}i_{\lambda}(A)$ . Therefore,  $A = A \cap c_{\lambda}i_{\lambda}(A) = U \cap S \cap c_{\lambda}i_{\lambda}(A) = U \cap c_{\lambda}i_{\lambda}(A)$ .

(b)  $\rightarrow$  (a). Suppose that  $A = U \cap c_{\lambda}i_{\lambda}(A)$  for some  $\lambda$ -open set U. Let  $G = c_{\lambda}i_{\lambda}(A)$ . Then  $c_{\lambda}i_{\lambda}(G) = c_{\lambda}i_{\lambda}c_{\lambda}i_{\lambda}(A) = c_{\lambda}i_{\lambda}(A) = c_{\lambda}i_{\lambda}(A) = G$  and so G is a  $\gamma$ -regular set. Therefore, A is a  $\lambda$ -locally  $\gamma$ -regular set.

(a)  $\rightarrow$  (c). Suppose that A is  $\lambda$ -locally  $\gamma$ -regular. Then  $A = U \cap S$  where U is  $\lambda$ -open and S is  $\gamma$ -regular. By Theorem 3.3, S is  $\lambda$ -closed and so A is  $\lambda$ -locally  $\lambda$ -closed.  $A = U \cap S = U \cap c_{\lambda}i_{\lambda}(S) \subset c_{\lambda}(U \cap i_{\lambda}(S)) \subset c_{\lambda}i_{\lambda}(U \cap S) = c_{\lambda}i_{\lambda}(A)$ . Therefore, A is  $\gamma$ -open.

(c)  $\rightarrow$  (d). Since  $\gamma(\lambda) \subset \beta(\lambda)$ , the proof follows.

(d)  $\rightarrow$  (a). Assume that A is  $\beta$ -open and  $\lambda$ -locally  $\lambda$ -closed. Then  $A = U \cap S$  where U is  $\lambda$ -open and S is  $\lambda$ -closed. Since A is  $\beta$ -open,  $A \subset c_{\lambda}i_{\lambda}c_{\lambda}(A)$  which implies that  $c_{\lambda}(A) \subset c_{\lambda}i_{\lambda}c_{\lambda}(A) \subset c_{\lambda}c_{\lambda}(A) = c_{\lambda}(A)$ . Therefore,  $c_{\lambda}(A) = c_{\lambda}i_{\lambda}c_{\lambda}(A) \subset c_{\lambda}i_{\lambda}c_{\lambda}(A) \subset c_{\lambda}i_{\lambda}c_{\lambda}(A) \subset c_{\lambda}c_{\lambda}(A) \subset c_{\lambda}(A) \subset C_{\lambda}(A) = c_{\lambda}(A)$ . Therefore,  $c_{\lambda}(A) = c_{\lambda}i_{\lambda}c_{\lambda}(A) \subset U \cap S = A \subset U \cap c_{\lambda}(A)$ , since  $A \subset U$ . Hence,  $A = U \cap c_{\lambda}(A) = U \cap c_{\lambda}i_{\lambda}c_{\lambda}(A)$ . If  $S = c_{\lambda}i_{\lambda}c_{\lambda}(A)$ , then  $c_{\lambda}i_{\lambda}(S) = c_{\lambda}i_{\lambda}c_{\lambda}i_{\lambda}c_{\lambda}(A) = c_{\lambda}i_{\lambda}c_{\lambda}(A) = S$ . Therefore, S is  $\gamma$ -regular and so A is  $\lambda$ -locally  $\gamma$ -regular.



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**Example 4.6**. (a)  $\rightarrow$  (b).

Let  $X = \{p, q, r, s, t\}$  be

 $\lambda = \emptyset, X, \{p, s\}, \{s, t\}, \{p, q, r\}, \{p, q, r, s\}, \{p, q, r, t\}, \{p, s, t\}, \{q, r\}, \{q, r, s, t\}. \text{ Then } \{p, s\} \cap \{s, t\} = \{s\} \in \lambda \text{ and so } (X, \lambda) \text{ is not a quasi-topological space. The family of all } \lambda\text{-closed sets is } \emptyset, X, \{q, r, t\}, \{p, q, r\}, \{q, r, s\}, \{s, t\}, \{p\}, \{t\}, \{s\}, \{q, r\}, \{p, s, t\}. \text{ Now } \{p, q, r\} = c_{\lambda}i_{\lambda}\{p, q, r\} \text{ and so } \{p, q, r\} \text{ is } \gamma\text{-regular. Since } \{p\} = \{p, s\} \cap \{p, q, r\}, \text{ where } \{p, s\} \text{ is } \lambda\text{-open and } \{p, q, r\} \text{ is } \gamma\text{-regular.} \{p\} \text{ is } \lambda\text{-locally } \gamma\text{-regular.} \text{ Again, } c_{\lambda}i_{\lambda} (\{p\}) = c_{\lambda}(\emptyset) = \emptyset, \text{ implies that } U \cap c_{\lambda}i_{\lambda} (\{p\}) = \emptyset = \{p\} \text{ for every } \lambda\text{-open set } U \text{ and so } (a) \to (b).$ 

The following theorem gives a characterization of  $\lambda$ -locally  $\pi$ -regular sets.

**Theorem 4.7**. Let  $(X, \lambda)$  be a quasi-topological space. Then the following are equivalent.

(a) A is  $\lambda$ -locally  $\pi$ -regular.

(b) A is  $\pi$ -open and  $\lambda$ -locally  $\gamma$ -closed.

Proof.

(a)  $\rightarrow$  (b). Suppose that A is  $\lambda$ -locally  $\pi$ -regular. Then  $A = U \cap S$  where U is  $\lambda$ -open and S is  $\pi$ -regular. By Theorem 4.5, S is  $\gamma$ -closed. Therefore, A is  $\lambda$ -locally  $\gamma$ -closed. Now  $A = i_{\lambda}(U) \cap i_{\lambda}c_{\lambda}(S) = i_{\lambda}(U \cap c_{\lambda}(S)) \subset i_{\lambda}c_{\lambda}(U \cap S)$  and so  $A \subset i_{\lambda}c_{\lambda}(A)$ . Therefore, A is  $\pi$ -open.

(b)  $\rightarrow$  (a). Suppose that A is  $\pi$ -open and  $\lambda$ -locally  $\gamma$ -closed. Then  $A = U \cap S$  where U is  $\lambda$ -open and S is  $\gamma$ -closed. Since A is  $\pi$ -open,  $U \cap S \subset i_{\lambda}c_{\lambda}(U \cap S)$  which implies that  $A = U \cap S = U \cap U \cap S \subset U \cap i_{\lambda}c_{\lambda}(U \cap S) \subset U \cap i_{\lambda}c_{\lambda}(S) \subset U \cap i_{\lambda}c_{\lambda}(S) \subset U \cap i_{\lambda}c_{\lambda}(S) \subset U \cap i_{\lambda}c_{\lambda}(S)$ , since U is  $\pi$ -open. Then,  $U \cap i_{\lambda}c_{\lambda}(S) \subset U \cap S = A$ , since S is  $\gamma$ -closed. Therefore,  $A = U \cap i_{\lambda}c_{\lambda}(S)$ . Take  $B = i_{\lambda}c_{\lambda}(S)$ . Then  $i_{\lambda}c_{\lambda}(B) = i_{\lambda}c_{\lambda}i_{\lambda}c_{\lambda}(S) = i_{\lambda}c_{\lambda}(B) = B$ . Therefore, B is  $\pi$ -regular.

The following theorem gives various characterizations of  $\lambda$ -open sets in terms of  $\lambda$ -locally  $\lambda$ -closed,  $\lambda$ -locally  $\gamma$ -closed and  $\lambda$ -locally  $\gamma$ -regular sets.

**Theorem 4.8**. Let  $(X, \lambda)$  be a quasi-topological space and  $A \subset X$ . Then

the following are equivalent.

(a) A is  $\lambda$ -open.

(b) A is  $\alpha$ -open and  $\lambda$ -locally  $\gamma$ -regular.

(c) A is  $\alpha$ -open and  $\lambda$ -locally  $\lambda$ -closed.

(d) A is  $\pi$ -open and  $\lambda$ -locally  $\lambda$ -closed.

(e) A is  $\pi$ -open and  $\lambda$ -locally  $\gamma$ -closed.

(f) A is  $\pi$ -open and A is  $\lambda$ -locally  $\gamma$ -regular.

**oof.** (a)  $\rightarrow$  (b). Suppose that A is  $\lambda$ -open. Then A is  $\alpha$ -open. Since

 $X = c_{\lambda} i_{\lambda}(X), A = A \cap X$  and so A is  $\lambda$ -locally  $\gamma$ -regular.

(b)  $\rightarrow$  (a). Suppose that A is  $\alpha$ -open and  $\lambda$ -locally  $\gamma$ -regular. Then  $A \subset i_{\lambda}c_{\lambda}i_{\lambda}(A)$  and  $A = U \cap S$  where U is  $\lambda$ -open and  $S = c_{\lambda}i_{\lambda}(S)$ . Now  $U \cap S \subset i_{\lambda}c_{\lambda}i_{\lambda}(U \cap S) \subset i_{\lambda}c_{\lambda}i_{\lambda}(U) \cap i_{\lambda}c_{\lambda}i_{\lambda}(S) = i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}(S)$  and so  $U \cap S = U \cap U \cap S \subset U \cap i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}(S) \subset U \cap i_{\lambda}(S)$ . Therefore,  $A \subset U \cap i_{\lambda}(S) \subset U \cap S = A$ . Hence  $A = U \cap i_{\lambda}(S)$  which implies that A is  $\lambda$ -open as X is quasi-topological.

(a)  $\rightarrow$  (c). Suppose that A is  $\lambda$ -open. Then A is  $\alpha$ -open. Since X is  $\lambda$ -closed, we have  $A = A \cap X$  and so A is  $\lambda$ -locally  $\lambda$ -closed.

(c)  $\rightarrow$  (d). The proof follows from the fact that  $\alpha(\lambda) \subset \pi(\lambda)$ .

(d)  $\rightarrow$  (e). Suppose that A is  $\pi$ -open and  $\lambda$ -locally  $\lambda$ -closed. Then  $A = U \cap S$  where U is  $\lambda$ -open and  $S = c_{\lambda}(S)$ . Since  $i_{\lambda}c_{\lambda}(S) \subset c_{\lambda}(S) = S$ , S is  $\gamma$ -closed. Therefore, A is  $\lambda$ -locally  $\gamma$ -closed.

(e)  $\rightarrow$  (a). Suppose that A is  $\pi$ -open and  $\lambda$ -locally  $\gamma$ -closed. Then  $A = U \cap S$  where U is  $\lambda$ -open and  $i_{\lambda}c_{\lambda}(S) \subset S$ and  $A \subset i_{\lambda}c_{\lambda}(A)$ .  $U \cap S \subset i_{\lambda}c_{\lambda}(U \cap S) \subset i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}c_{\lambda}(S) \subset i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}(S)$ . Now  $U \cap S = U \cap U \cap S \subset i_{\lambda}c_{\lambda}(U) \cap i_{\lambda}(S) \cap (U) \subset U \cap i_{\lambda}(S) \subset U \cap S$ . Hence  $A = U \cap i_{\lambda}(S)$ . Therefore, A is  $\lambda$ -open.

(b)  $\rightarrow$  (f). The proof follows from the fact that  $\alpha(\lambda) \subset \pi(\lambda)$ .

(f)  $\rightarrow$  (d). Suppose that A is  $\pi$ -open and  $\lambda$ -locally  $\gamma$ -regular. Then  $A = U \cap S$  where U is  $\lambda$ -open and S is  $\gamma$ -regular. By Theorem 3.3, S is  $\lambda$ -closed.

Therefore, *A* is  $\lambda$ -locally  $\lambda$ -closed.



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