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# Connected Cut Free domination and Equal Domination Numbers Using Sensor Network

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**ABSTRACT:** The purpose of this work is to introduce the reader to several graph-theoretic concepts relevant to the analysis and synthesis of reliable or invulnerable networks. Connected Dominating Sets are popularly used for constructing virtual backbones for broadcasting operation in WSNs (wireless sensor Networks). For a finite undirected graph  $G$  on  $n$  vertices some continuous optimization problems taken over the  $n$ -dimensional cube are presented and it is proved that their optimum values equal the domination number  $\gamma(G)$ . The last category of approaches to create virtual backbone aims at creating a structure with the characteristics of a connected dominating set. This notion is a connected variation of the dominating set, i.e. a set of nodes covering a whole graph. Moreover, the sub-graph induced by the components of the backbone is connected, i.e. a path only composed of backbone nodes exists between any pair of backbone nodes. In Ad-hoc and sensor networks are rapidly growing areas of research which study the problems arising when small and thin devices build a communication infrastructure.

**KEYWORDS:** Bipartite graph, Ad-hoc networks, Geometric spanner, Median graph

## I. INTRODUCTION

In the first part we survey the fundamentals and known results in the area of domination, mainly focused on Connected Cut free Domination of graph. Beside ordinary domination, and also survey that Dominating sets and other related subsets and the corresponding graph. This is one of the fastest growing areas in graph theory and has analysis of communication networks, (etc.,). In Ad-hoc and sensor networks are rapidly growing areas of research which study the problems arising when small and thin devices build a communication infrastructure. Let  $G=(V, E)$  be a graph. A set  $S \subseteq V$  is called dominating set in  $G$ , if every vertex in  $V-S$  is adjacent to some vertex in  $S$ . A dominating set  $S$  is said to be a connected cut free dominating set. If the induced sub graph  $\langle S \rangle$  of  $G$  is connected graph having no cut vertex.  $S$  is called a block dominating set if  $\langle S \rangle$  is a block in  $G$ . In this dissertation we study about domination numbers and investigate their relationship with Wireless sensor networks. The connected dominating set is one of the earliest structures proposed as a candidate for a virtual backbone in wireless networks.

### 1.1 Origin of Graph Theory

The origin of graph theory started with the problem of Koenigsberg bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koenigsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits.

### 1.2 Over view of Graph Theory

The field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. The field graph theory started its

journey from the problem of Königsberg bridge in 1735. This paper gives an overview of the applications of graph theory in heterogeneous fields to some extent but mainly focuses on the computer science applications that uses graph theoretical concepts.

“Fundamentally, computer science is a science of abstraction.” Computer scientists must create abstractions of real-world problems that can be represented and manipulated in a computer. Sometimes the process of abstraction is simple. The basic ideas of graphs were introduced in 18th century by the great Swiss mathematician **Leonhard Euler**. He used graphs to solve the famous Königsberg bridge problem. Here is a picture (taken from the internet)

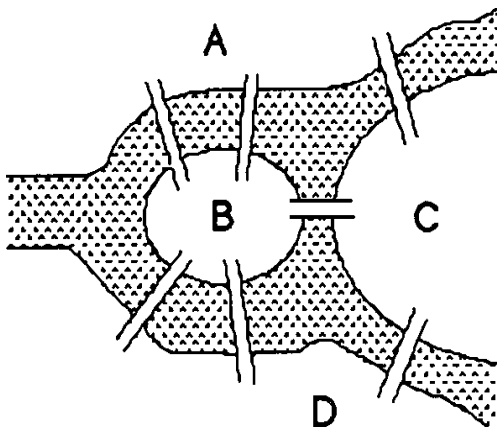


Fig 1.1 – Königsberg Bridge

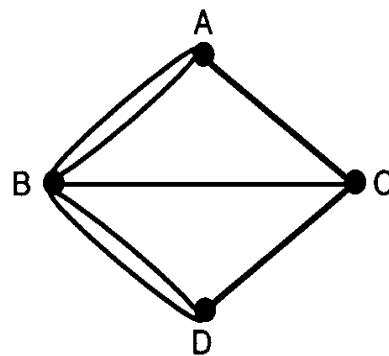


Fig 1.2 - Multigraph

The two islands A and B and the two banks C and D of the Pregel river are connected by seven bridges. V. Adamchik 1 German city of Königsberg (now it is Russian Kaliningrad) was situated on the river Pregel. It had a park situated on the banks of the river and two islands. Mainland and islands were joined by seven bridges. A problem was whether it was possible to take a walk through the town in such a way as to cross over every bridge once, and only once. Here is the graph model of the problem. A graph is a set of points (we call them vertices or nodes) connected by lines (edges or arcs).

Graph theoretical ideas are highly utilized by computer science applications. Especially in research areas of computer science such as data mining, image segmentation, clustering, image capturing, networking etc., For example a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using graph concepts.

Domination in graph theory is a natural model for many location problems in operations research. As an example, consider the following fire station problem. Suppose a county has decided to build some fire stations, which must serve all of the towns in the county. The fire stations are to be located in some towns so that every town either has a fire station or is a neighbor of a town which has a fire station. To save money, the county wants to build the minimum number of fire stations satisfying the above requirements. Domination has many other applications in the real world.

### 1.3 Graph theory relevant to ad-hoc networks

The role of graph theory related to the issues in Mobile Ad-hoc Networks (MANETS). In Ad-hoc networks, issues such as connectivity, scalability, routing, modeling the network and simulation are to be considered. Since a network can be modeled as a graph, the model can be used to analyze these issues. Graphs can be algebraically represented as matrices. Also, networks can be automated by means of algorithms. The issues such as node density, mobility among the nodes, link formation between the nodes and packet routing have to be simulated

#### Applications

A major application area for UDG models is in wireless communication. A dominating set in UDGs modeling wireless network function as a small set of nodes that can send an emergency communication to the entire graph. Domination and connected domination are also used to cluster wireless networks.

#### Connected Domination

A dominating set  $S$  is called a connected dominating set if the induces a sub graph  $\langle S \rangle$  is connected.

The connected domination number  $\gamma_c$  of  $G$  is the minimum cardinality taken over all minimal connected dominating set of  $G$ .

For example the connected domination number  $\gamma_c$  of  $G$  is  $\{v_1, v_2, v_5\}$ .

Ie.,  $\gamma_c = 3$ .

Since a dominating set must contain atleast one vertex from every component of  $G$ , it follows that a connected dominating set exists for a graph  $G$  if  $G$  is connected.

Remark:

The connected domination number of some standard graphs can be easily found, and are given as follows.

$$(i) \quad \gamma_c(K_p) = 1$$

Since every singleton set in  $K_p$  is a connected graph and also it is the dominating set of  $K_p$  with minimum cardinality

We have,  $\gamma_c(K_p) = 1$

$$(ii) \quad \gamma_c(K+G), \text{ for any graph } G.$$

This follows from the definition of join of two graph and from result (ii) above.

$$(iii) \quad \gamma_c(K_{m,n}) = \begin{cases} 1 & \text{if either } m \text{ or } n = 1 \\ 2 & \text{if } m, n \geq 2 \end{cases}$$

Clearly this follows from the definition of bipartite graph  $K_{m,n}$ .

If either  $m$  or  $n=1$ , then the minimum dominating set of  $K_{m,n}$  is a singleton set and since singleton set is connected in  $K_{m,n}$  we get

$$\gamma_c(K_{m,n})=1.$$

(iii) If  $m, n \geq 2$ , then any pair of vertices one from each partition gives raise a minimum dominating set for  $K_{m,n}$ . Also since any two vertices each of which from different partitioned subsets of  $V(K_{m,n})$  is connected, it follows that  $\gamma_c(K_{m,n})=2$ .

(iv)  $\gamma_c(C_p)=p-2$  In a cycle of order  $p$  and  $p-2$  consecutive vertices in a cycle form a minimum dominating set and they are connected, we get  $\gamma_c(C_p)=p-2$ .

(v) for any tree  $T$  of order  $p$ ,  $\gamma_c(T)=p-e$  where  $e$  is the number of pendant vertices.(i.e) vertices of degree 1 in  $T$ .

In a tree if we leave any internal vertices [the vertices which are not pendant vertices (i.e) any vertex of degree greater than one] it will affect the connectedness of the tree and all the internal vertices form a dominating set, we have  $\gamma_c(T)=p-e$ .

#### 1.4 Bounds on the Domination number

In the study of subsets of a given type in a graph, it is natural to find either a smallest or a largest such set in a graph. For instance, in an ad hoc network, a set of computers called clients are connected to a set of processors called backbone servers that collect the data together from the clients and transfers the same to the other clients in the network. It is desirable to find the minimum number of backbone servers needed to connect all the clients in the network. Since these subset problems are NP-complete, it is natural to find reasonable upper and lower bounds for these numbers. In this chapter, we discuss the bounds for the domination number  $\gamma_c(G)$  and the connected domination number  $\gamma_{c(G)}$ .

## II. RELATED WORK

### Perfect Dominating Set

A dominating set is called a perfect dominating set if every vertex in  $V-S$  is adjacent to exactly one vertex in  $S$ .

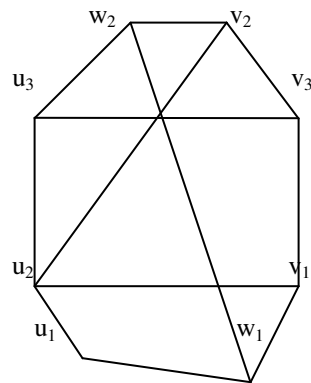
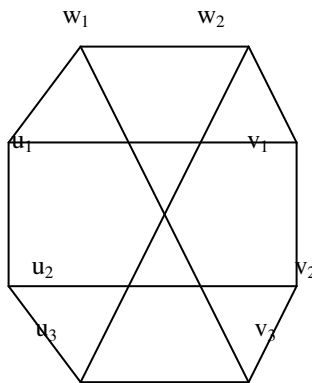
Theorem :

Let  $G$  be a connected cubic graph of order  $p$ . then  $\gamma = \gamma_c$  and only if  $G$  is isomorphic to  $K_4, C_6, K_{3,3}, G_1$  are  $G_2$  where  $G_1$  and  $G_2$  are

If  $G$  is isomorphic  $K_4$ , then  $\gamma = \gamma_c = 1$ . when  $G$  is isomorphic or  $C_6$  or  $K_{3,3}$ ,  $\gamma = \gamma_c = 2$ . when  $G$  is isomorphic  $G_1$  or  $\gamma = \gamma_c = 3$ .



Now let  $G$  be a connected regular graph of degree  $r = 3$ . let  $\gamma = \gamma_c$  and  $S$  be a minimum connected dominating set of  $G$ . If  $\gamma_c = 1$ , then  $G$  is isomorphic to  $K_4$ . If  $\gamma_c = 2$ ,  $p = 6$ . Also  $G$  is regular of degree 2. Hence  $G$  is isomorphic  $C_6$  or  $C_3 \cup C_3$  so that  $G$  is isomorphic to  $C_6$  or  $K_{3,3}$ .



If  $\gamma_c = 3$  put  $p = 8$ . let  $S = \{u_1, u_2, u_3\}$  be a minimum connected dominating set in  $G$  and let  $(S) = u_1 u_2 u_3$ . let  $V - S = \{v_1, v_2, v_3, w_1, w_2\}$  where  $u_i v_i, u_i w_i, u_3 w_2 \in E(G)$ . let  $S_1 = \{v_1, w_1\}$  and  $S_2 = \{v_3, w_2\}$ . We consider the following cases.

Case(i):

$S_1$  and  $S_2$  are independent. Since  $G$  is cubic,  $v_2$  is adjacent to one vertex in  $S_1$  and one vertex in  $S_2$ . without loss of generality. We assume that  $v_2 v_1, v_2 v_3 \in E(G)$ . Then  $v_1$  must be adjacent to  $w_2$  and  $w_1$  must be adjacent to  $v_3$  and  $w_2$  so that  $G$  is isomorphic to  $G_1$ .

Case(iii):

$(S_1) = K_2$  and  $S_2$  is independent. In this case  $v_2$  must be adjacent to  $v_3$  and  $w_3$ . Without loss of generality let  $v_1$  be adjacent to  $v_3$  and  $w_1$  be adjacent to  $w_2$ . Then  $G$  is isomorphic to  $G_2$ .

Hence the Theorem.

### III. METHODOLOGY

Theorem :

Let  $G$  be a connected cactus graph, then  $\gamma(G) = n - 2|D_1| - 2|D_2| - |L|$ .

It is obvious that every  $\gamma_c$ -set of  $G$  does not contain pendant vertices and every cut vertex is in every connected dominating set of  $G$ . Hence if a vertex is neither in any cycle nor a pendant vertex, then it must be a cut vertex and belongs to every  $\gamma_c$ -set.

Case 1:

For each  $C_i \in D_1$ ,  $C_i$  has only one cut vertex. Hence  $C_i$  is an end cycle and every  $\gamma_c$ -set must contain atleast  $|V(C_i)|-2$  vertices of  $C_i$ . That is to say, for every  $\gamma_c$ -set  $S$  of  $G$ ,  $C_i$  has atleast two vertices such that they are not in  $S$ .

Case 2: If there exists a cycle  $C_i$  such that each vertex in  $C_i$  is of degree atleast 3, then every vertex in  $C_i$  belongs to every  $\gamma_c$ -set  $S$  of  $G$ .

Case 3: For each  $C_i \in D_{21}$ , since  $C_i$  contains atleast one vertex of degree 2 but no two adjacent vertices of degree 2, it follows that every  $\gamma_c$ -set must contain atleast  $|V(C_i)|-2$  vertices of  $C_i$ . That is to say, for every  $\gamma_c$ -set  $S$  of  $G$ ,  $C_i$  has atleast one vertex, that is not in  $S$ .

Case 4: For each  $C_i \in D_{22}$ ,  $C_i$  has atleast two cut vertices and there exist such two cut vertices such that the length of a longest way between them in  $C_i$  not containing other cut vertices is atleast 3.

Hence every  $\gamma_c$ -set must contain atleast  $|V(C_i)|-2$  vertices of  $C_i$ . That is to say, for every  $\gamma_c$ -set  $S$  of  $G$ ,  $C_i$  has atleast two vertices such that they are not in  $S$ .

By Cases (1)-(4),

$$\gamma_{c(G)} \geq |V(C_i)| \sum_{C_i \in D_1} 2 - \sum_{C_i \in D_{21}} 1 - \sum_{C_i \in D_{22}} 2 - |L|$$

$$S = n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|$$

Since it is obvious that there exists a connected dominating set with cardinality  $n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|$ , it follows that  $\gamma_{c(G)} = n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|$ .

Hence the theorem.

Theorem: Let  $G$  be a connected block graph with  $1 \leq 2$ . Then  $\gamma_{c(G)} = \gamma(G)$  iff every cut vertex of  $G$  is an end block cut vertex

Suppose every cut vertex is in every connected dominating set and  $\gamma_{c(G)} = |F|$ ,  $F$  is the unique minimum connected dominating set in  $G$ .

Now, let  $\gamma_{c(G)} = \gamma(G)$ . if there exists a cut vertex  $v$  such that  $v$  is not an end block cut vertex, then  $F - \{v\}$  is a dominating set of  $G$  with cardinality  $\gamma(G) - 1$ , which is a contradiction.

Conversely, if every cut vertex  $v$  is an end block cut vertex, then  $\gamma(G) \geq |F|$  Since  $\gamma(G) \leq \gamma_{c(G)}$  and  $\gamma_{c(G)} = |F|$ , it follows that  $\gamma_{c(G)} = \gamma(G)$ .

Hence the theorem

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#### **IV. CONCLUSION**

UD graph is the most suitable model for WSNs. Virtual backbone in a WSNs is a subset of sensor nodes which can transmit and receive the messages through out the network. Generally speaking, we use graphs in two situations. Firstly, since a graph is a very convenient and natural way of representing the relationships between objects we represent objects by vertices and the relation ship between them by lines. In many situations such as pictorial representation may be all that is needed.

Many applications of graph theory exist in the form of network analysis. These split broadly into 3 categories.

First, analysis to determine structural properties of a network. Such as the distribution of vertex degrees and the diameter of the graphs. A vast number of graph theory measures exist, and the production of useful over for various domains remains an active area of research.

Second, analysis to find a measurable quantity within the network for (e.g.) for a transportation network, the level of ventricular flow within any portion of it.

Third, analysis of dynamicalm properties of network.

In graph theory, connected dominating set are useful in the computation of routing for mobile ad-hoc networks. In this application, a small connected dominating set is used as a backbone for communications and nodes that are not in this set communicate by passing messages through neighbors that are in the set. Connected dominating set is widely used in topology control backbone to improve the performance and increase the lifetime of the network. Most of the distributed algorithms for approximately connected dominating set are based on constructing maximal independent set. The performance of such algorithms highly depends on the relation between the sizes of the minimum connected dominating set in the graph G. Practical analysis of Constructing connected dominating set based on maximal independent set in wireless networks. It is shown that the theoretical bound for unit disk graph is still practically applicable for general wireless network

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