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Vertex Coloring and Chromatic Number for Self-Dual Graphs

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ABSTRACT In this paper, we derive the self-dual graphs from the dual graph and the planar graphs in graph theory. Moreover our main objective is to apply proper vertex coloring for the self-dual graphs by using some properties and examples. Finally finding the chromatic number for complete and non-complete self-dual graphs.

KEYWORDS Complete graph, Vertex coloring, Chromatic number, Planar graph, Wheel graph, Dual graph, Selfdual graph.

I. INTRODUCTION

The study of duality was one of the classical concepts in the spherical polyhedral field. First, it was identified from the research about platonic solids. That is, the cube is dual to the octahedron, the dodecahedron is dual to the icosahedrons, while the tetrahedron is dual to itself. In our research, we use the undirected graph duality. It can help explain the structures of mazes and of drainage basins. Graph duality concept also applied the field of computer vision, computational geometry, mesh generation and the design of integrated circuits. Many usual and main notions in graph theory correspond to other equally natural but distinct concepts in the duality.

The wide range wisdom of coloring is a decomposition of discrete objects into simple sub objects. In general, this notion arises in various branches of discrete mathematics and has important applications. The plane graph coloring concept is analyzed in the middle of the nineteenth century. It was inspired by the famous four color problem such that every plane graph is 4-colorable. Nowadays it is a broad area of research; so many contributors have presented a lot of contributions.

In this article, we obtain the chromatic numbers of different self dual graphs and analyses its nature. Also, the chromatic number of complete and non-complete self-dual graphs is obtained by using some theorems and properties.

Here all graphs used are undirected graphs. Throughout this paper, the notations V, E and F will be considered as

|V| = n, denotes the number of vertices in graph, also it is called the order of the graph (or) vertex ardinality.

|E| = m, denotes the number of edges in graph, also it is called the size of the graph (or) edge cardinality and

|F| = f, denotes the number of faces or regions.

The faces are taken in both the bounded and unbounded areas considered as bounded faces and unbounded

faces.

- Each face is bounded by a closed walk called the bounded face.
- > Unbounded area outside the whole graph is considered as one face that is called unbounded face.

The dual graph has a vertex for each faces $(f_1, f_2 \dots f_n)$ or regions $(r_1, r_2 \dots r_n)$ and has a edge, whenever two faces are separated by an edge.



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DEFINITION

A planar graph *G* is said to be self-dual graph G^*

- ▶ If G is isomorphic to geometric dual of G^* , $G \cong G^*$.
- > If number of vertices and edges of G and G^* are equal.
- > If cycle length and degree sequences of G and G^* are equal.
- > If G and G^* has one-to-one correspondence between the vertices.

If a graph G is dual to itself then it is called the self-dual graph G^* . Some examples of self-dual graphs are wheel graphs, skeleton of the self-dual polyhedron, pyramids, etc.

In section 1, we said the brief introduction of duality concepts with simple applications and self-dual graphs.

In section 2, we study the basic notions, definitions and theorems related to our article.

In section 3, we examine the chromatic number of complete and non-complete self-dual graphs.

In section 4, we explain the theorem in section 2 with some examples.

II. BASIC DEFINITIONS AND EXAMPLES:

COMPLETE GRAPH:

A graph without loops and parallel edges should have, every pair of vertices connected by an edge is called the complete graph.

VERTEX COLORING:

Let V(G) be a vertex set of a graph G. A vertex coloring is a labeling of the vertex set, where the labels are called colors and no two adjacent vertices gets the same colors.

A proper vertex coloring is a coloring the vertex set using minimum number of colors(say k) and also it is called k-coloring of G.

CHOROMATIC NUMBER:

Chromatic number is defined as the least number of colors(say k) needed for coloring the graph and is denoted by $\aleph(G)$ or k-colorable or k-chromatic graph.

PLANAR GRAPH:

A graph G is said to be planar, if it can be drawn on a plane without any edges crossing (no two edges intersects).

The point of intersection is called crossover.

WHEEL GRAPH:

A wheel graph of order n is a graph obtained by joining a single new vertex to each vertex of cycle graph C_{n-1} of order n-1 and it is denoted by W_n .

DUAL GRAPH:

The dual graph has a vertex for each faces $(f_1, f_2 \dots f_n)$ and has a edge, with every two faces are separated by an edge.



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Example:



SELF-DUAL GRAPH:

A dual graph G^* is said to be self-dual graph, if a planar graph G has the number of vertices and edges equal to the number of vertices and edges in G^* .

Example:



Now n = 4 & m = 6 in G and n = 4 & m = 6 in $G^* \Rightarrow$ Hence $G \cong G^*$ i.e., G^* is Self-dual graph.



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THEOREM: 1 Euler's formula:

Statement:

Let *G* be a planar graph and Let n, m and f denotes the amount of vertices, edges and faces (regions) respectively in a plane drawing of *G*. Then n - m + f = 2.

Proof:

Let us prove this by induction method.

Step 1: Take n = 2, We have, m = 1, n = 2, f = 1Now, LHS = n - m + f = 2 - 1 + 1 = 2 = RHS Therefore the theorem is true for n = 2.

> Step 2: Take n = 3, We have, n = 3, m = 3, f = 2LHS = n - m + f = 3 - 3 + 2 = 2 = RHS Therefore the theorem is true for n = 3.

Step 3: Take n = k
Consider the theorem is true for n = k,
i.e., LHS = V_k - E_k + F_k=2 ----- (1)
Let E_{k+1} be the additional edge by joining the vertices (a_{k+1}, b_{k+1}) that is added to G_k.

Case 1:

Consider the additional edge in the bounded region. $F_{k+1} = F_k + 1$, $E_{k+1} = E_k + 1$, $V_{k+1} = V_k$



Thus LHS =
$$V_{k+1} - E_{k+1} + F_{k+1}$$

$$= V_k - E_k - 1 + F_k + 1$$

 $= V_k - E_k + F_k$
 $= 2 = RHS$ (by (1))

Case 2:

Consider the additional edge in the unbounded region.



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$$F_{k+1} = F_k, E_{k+1} = E_k + 1, V_{k+1} = V_k + 1$$

Thus LHS = $V_{k+1} - E_{k+1} + F_{k+1}$

$$= V_{k} + 1 - E_{k} - 1 + F_{k}$$
$$= V_{k} - E_{k} + F_{k}$$
$$= 2 = \text{RHS} \quad (\text{by (1)})$$

Therefore the theorem is true for n = k by both the cases.

By the Principle of induction,

Hence the Euler's formula is true for all values of *n*.

Hence V - E + F = 2 or n - m + f = 2.

THEOREM: 2

If a graph G with n vertices and m edges is self-dual graph then m = 2n - 2.

Proof:

From the Euler's formula, we have V - E + F = 2, i.e., n - m + f = 2. f denotes faces or regions of G. f denotes vertices in G^* . Therefore, Take f = nHence n - m + n = 2n - m = 2m = 2n - 2

THEOREM: 3

If G^* is a self-dual graph of G. Then sum of degree of each faces equal to twice the number of edges of G^* . i.e., $\sum_{i=1}^{n} (degree \ of \ f) = 2|E|$ of $G^*, n > 3$.

Proof :

Let us prove this by induction method. Step 1: Take f = 4 (For self-dual graph vertices should be greater than 3)



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Fig:5

LHS: $\sum (degree \ of \ f) = degree \ of \ (f_1 + f_2 + f_3 + f_4) = 12.$

RHS: 2|E| = 2(6) = 12. Hence LHS = RHS

<u>Step 2:</u> Take f = 5



Fig:6

LHS: $\sum(degree \ of \ f) = degree \ of \ (f_1 + f_2 + f_3 + f_4 + f_5)$ =16 (i.e. 3+3+3+3+4) RHS: 2|E| = 2(8) = 16Hence LHS = RHS

Step 3: Take f = nConsider the theorem is true for f = ni.e., $\sum (degree \ of \ f) = 2|E|$ = 2(2n - 2) (by Theorem: 2) = 4n - 4 ------(1)

 $\begin{array}{l} \underline{\text{Step 4:}} \text{ Take } f = n+1 \\ \text{LHS: } \sum (degree \ of \ f) = 4(n+1) - 4 \\ = 4n+4-4 = 4n \ (by \ (1)) \\ \text{RHS: } 2|E| = 2(2(n+1)-2) = 4n+4-4 \\ = 4n \ (by \ \text{Theorem:2}) \\ \text{Hence } \text{LHS} = \text{RHS} \\ \text{Thus the theorem is true for all values of } f. \\ \text{Hence } \sum_{i=1}^{n} (degree \ of \ f) = 2|E| \quad \text{or} \quad \sum_{i=1}^{n} (degree \ of \ f) = 2m \ \text{for } n > 3 \ . \end{array}$



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THEOREM: 4

If G^* be a self-dual graph with $G^*(n, m)$ or $G^*(V, E)$ and every faces is bounded by equal number of edges(say p) except the unbounded(infinite) face [*P*-cycle] then

$$E = \frac{(P-1)(V-1)}{(P-2)}, P > 2 \text{ or } m = \frac{(P-1)(n-1)}{(P-2)}, P > 2$$

Proof:

Let *F* be the amount of faces of G^* . Since every face is bounded by p-edges and each edges lies on the boundary of exactly two faces, we have

 $2E = \sum_{i=1}^{F} (the number of edges in the bounded faces + the number of edges in the unbounded faces)$

 $=\sum_{i=1}^{F}$ number of edges excluding in the unbounded region +|V|-1

$$= (F - 1)P + V - 1$$

Using Euler's formula, $F = E - V + 2$
 $\Rightarrow 2E = (E - V + 2 - 1)P + V - 1$
 $\Rightarrow 2E = PE - PV + P + V - 1$
 $\Rightarrow PE - 2E = PV - V - P + 1$
 $\Rightarrow E(P - 2) = P(V - 1) - (V - 1)$
 $\Rightarrow E(P - 2) = (V - 1)(P - 1)$
 $\Rightarrow E = \frac{(P-1)(V-1)}{(P-2)}, P > 2$ or $m = \frac{(P-1)(n-1)}{(P-2)}, P > 2$

THEOREM: 5

If G^* be a self-dual graph with (V, E, F) then $E \ge \frac{3}{2}F$ and $E \le 3V - 6$ where $V \ge 3$.

Proof:

Consider a self-dual graph G^* with 4-vertices. Since G^* should have V = n > 3.

Thus we have F = 4, E = 2V - 2, V = 4. Also every edge of G^* is on the boundary of atmost two regions. Thus by the above theorem,

 $\begin{aligned} 2E &\geq \sum (the number of edges in the bounded faces + number of edges in the unbounded faces) \\ &\geq (F-1)P+V-1 \\ &\geq (F-1)3+4-1 \\ 2E &\geq 3F - 3 + 4 - 1 \\ 2E &\geq 3F \Rightarrow E &\geq \frac{3}{2}F. \end{aligned}$ Now substitute the value of *F* from the Euler's formula, i.e., F = E - V + 2Thus $E &\geq \frac{3}{2}(E - V + 2) \\ \Rightarrow 2E &\geq 3E - 3V + 6 \quad \Rightarrow -E &\geq -3V + 6 \\ \Rightarrow E &\leq 3V - 6. \end{aligned}$

CHROMATIC NUMBER OF SELF-DUAL GRAPH:

THEOREM: 6

If G^* is a self-dual graph with every face is only of Odd cycles(odd degrees) then the chromatic number of G^* is atleast 3.

Proof:

A self-dual graph G^* with every face is only of Odd cycles (odd degrees), as it contains several triangles or atleast one triangle i.e., P = 3 say.

Thus a triangle should be a 3 – Colorable

Hence $\aleph(G^*) \geq 3$.



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i.e., The chromatic number of G^* is at least 3.

Now we are considering two types of self-dual graph as:

- Complete self-dual graph
- ➢ Non-complete self-dual graph

COMPLETE SELF-DUAL GRAPH:

A self-dual graph should have, every pair of vertices connected by an edge is called the complete self-dual graph. Otherwise, if a self-dual graph is complete then it is called complete self-dual graph.

A self-dual graph which is not complete self-dual graph is called Non-complete self-dual graph.

III THE CHROMATIC NUMBER OF COMPLETE AND NON-COMPLETE SELF-DUAL GRAPHS:

(1).CHROMATIC NUMBERS OF COMPLETE SELF-DUAL GRAPH:

THEOREM: 6

A complete self-dual graph with n-vertices $G^*(K_n)$ has the chromatic number n. i.e., $\aleph[G^*(K_n)] = n$, n = 4.

Proof:

In a complete self-dual graph, each vertex is adjacent to remaining n - 1 vertices. Hence each vertex requires a new color. Hence the chromatic number is n.

Thus $\aleph[G^*(K_n)] = n, \quad n = 4.$

Result:

- A graph with vertices n > 4 cannot be a complete self-dual graph. i.e., A graph with vertices n > 4 may be a complete graph or self-dual graph both not be a complete self-dual graph.
- > Every complete self-dual graph with 4-vertices is again a self-dual graph.

(2).CHROMATIC NUMBERS OF NON-COMPLETE SELF-DUAL GRAPH:

THEOREM: 7

Every wheel graph is a non-complete self-dual graph has the chromatic number

 $\aleph(G^*) = \begin{cases} 4, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases} \text{ with always } n > 3.$

Proof:

We can show this theorem by some examples,





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From graphs G_1 , G_3 , we get the chromatic number as 4.

i.e., W_4 and W_6 has even number of vertices has the chromatic number 4.

From graphs G_2 , G_4 , we get the chromatic number as 3.

i.e., W_5 and W_7 has odd number of vertices has the chromatic number 3.

Hence $\aleph(G^*) = \begin{cases} 4, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases}$ with always n > 3.

THEOREM: 8

The chromatic number of any non-complete self-dual graph is at states 4. i.e., $\aleph(G^*) \leq 4$

Proof:

We can prove this by an example. Consider a self-dual graph G^*



Original graph (G) Fig:10 (i)



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Dual graph (\Box^*) (ii)

The chromatic number of \Box^* , $\aleph(\Box^*) = 4$ Hence $4 \le 4$ From this we conclude that $\aleph(\Box^*) \le 4$.

THEOREM: 9

Let \square^* be a self-dual graph with maximum degree of vertex of \square^* , $\Delta(\square^*) \ge 3$. Then the chromatic number of $\square^* \square \square$

$$\aleph(\square^*) \leq l + \Delta(\square^*)$$

Proof:

Given, Let \square^* be a self-dual graph with maximum degree of vertex of \square^* , $\Delta(\square^*) \ge 3$. Consider a self-dual graph with $\Delta(\square^*) \ge 3$.

For proving this, we consider non-complete self-dual graph(Wheel graphs)

We have from theorem: 7,

<u>Step 1</u>: Take $\Box = 4$ (For self-dual graph vertices should be greater than 3)

 $\aleph(\square^*) = 4 \text{ and } \Delta(\square^*) = 3$ Refer Graph $\square_1(\square_4)$ in theorem:7 Therefore, LHS = $\aleph(\square^*) = 4$ RHS = $1 + \Delta(\square^*) = 1 + 3 = 4$ Thus LHS = RHS i.e., The theorem is true for $\square = 4$ $\frac{\text{Step 2:}}{\aleph(\square^*) = 3} \text{ and } \Delta(\square^*) = 4$ Refer Graph $\square_2(\square_5)$ in theorem:7 Therefore, LHS = $\aleph(\square^*) = 3$ RHS = $1 + \Delta(\square^*) = 1 + 4 = 5$ Thus LHS \leq RHS i.e., The theorem is true for $\square = 5$ $\frac{\text{Step 3:}}{\aleph(\square^*) = 4} \text{ and } \Delta(\square^*) = 5$ Refer Graph $\square_3(\square_6)$ in theorem:7



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Therefore, LHS = $\aleph(\square^*) = 4$ RHS = $l + \Delta(\square^*) = l + 5 = 6$ Thus LHS \leq RHS i.e., The theorem is true for $\square = 6$

 Step 4:
 Consider the theorem is true
 for □ = □

 Suppose □ □ □ □ then $\aleph(□^*) = 3$ and $\Delta(□^*) = □ - 1$

 Therefore, LHS = $\aleph(□^*) = 3$

 RHS = $I + \Delta(□^*) = I + □ - I = □$, since □ > 6.

 Thus LHS ≤ RHS

 Suppose □ □ □ □ □ then $\aleph(□^*) = 4$ and $\Delta(□^*) = □ - 1$

 Therefore, LHS = $\aleph(□^*) = 4$

 RHS = $I + \Delta(□^*) = I + □ - I = □$, since □ > 6.

 Thus LHS ≤ RHS

 i.e., The theorem is true for □ = □

Similarly, the theorem is true for all self-dual graphs of $\Box > 3$.

Result:

- 1) Every self-dual graph should not be,
 - A tree
 - A non-planar graph
 - A graph with vertices less than or equal to 3
- 2) Every self-dual graph is a planar graph.
- 3) If any planar graph □ with every face is only of Even cycle (Even degree) then the chromatic number of □ is exactly equal to 2. But every self-dual graph should have atleast one odd (triangle) face, so the chromatic number is not equal to 2.

IV. EXAMPLES

Example for theorem: 2

Consider a graph \Box of Fig:5 (Dual \Box^*) is Planar Graph in which we have, $|\Box| = 4, |\Box| = 6; |\Box| = 4, |\Box| = 6$

In
$$\Box$$
, $\Box = 4$, $\Box = 6 \Rightarrow \Box = 2\Box - 2$
 $\Rightarrow 6 = 2(4) - 2$
 $\Rightarrow 6 = 6$

In \square^* , $\square = 4$, $\square = 6 \Rightarrow 6 = 6$ Hence $\square = 2 \square - 2$ is satisfied. \square^* is self-dual of \square .

Also $\square \cong \square^*$. Hence the theorem: 2 is satisfied.

Example for theorem: 9

Consider the graph,



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Original graph (\Box)- Fig:11 (i)



Dual graph (\square^*) (ii)

From this graph, we have, $\Delta(\Box^*) = \Box_6 = 5$ ---(1) Now the proper vertex coloring of \Box^* is $\Re(\Box^*) = 4$ ---- (2) From (1) and (2), we get $\Re(\Box^*) \le 1 + \Delta(\Box^*)$.

V. CONCLUSION

In this paper, we provided an overview of the family of self-dual graphs and showing that how to calculate the number of edges of the self-dual graph in different ways. We obtain the chromatic number of self-dual graphs by giving proper vertex coloring in two ways such as chromatic number of complete and non-complete self-dual graph. Hence we conclude the result from theorem:7, 8 and Result (3) that the chromatic number of self-dual graph is only either 3 or 4.

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