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# Dynamics of a Quasiperiodically Forced Brusselator Chemical Reaction

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**ABSTRACT:** The dynamics of a quasiperiodically forced Brusselator chemical reaction is studied numerically. With the variation of the amplitudes and frequency of the quasiperiodic force, period-doubling, quasiperiodic, periodicbubble and chaotic oscillations are found to occur. Parametric regimes where suppression of chaos occurs are predicted. The basic properties of the dynamics of the system are analyzed by bifurcation diagram, maximal Lypunov exponent, Phase portrait, Poincaré map and trajectory plot.

KEYWORDS: Brusselator chemical system, quasiperiodic force, bifurcation, chaos, periodic bubble.

#### I. INTRODUCTION

Nonlinear chemical systems are capable of exhibiting variety of dynamics ranging from simple periodic to complicated irregular behaviour. The study of complex dynamics and chemistry of oscillating reactions under the influence of external perturbation have received a considerable interest in recent years [1-11]. The reaction mechanism to be studied, commonly called the Brusselator is an example of an autocatalytic oscillating chemical reaction. An autocatalytic reaction is one in which a species acts to increase the rate of its producing reaction. In many autocatalytic systems complex dynamics are seen, including multiple steady-states and periodic orbits [12-16].

The classical Brusselator chemical reaction consists of four irreversible steps given by

$$A \xrightarrow{K_1} X$$

$$B + X \xrightarrow{K_2} Y + D \qquad (1)$$

$$2X + Y \xrightarrow{K_3} E$$

$$X \xrightarrow{K_4} E$$

Where A and B are constant components, D and E are products and X and Y are the two components variable in time and space. The dynamics of the Brusselator chemical reaction can be described by a system of two ordinary differential equations (ODE's). In dimensionless form, they are,

$$\dot{x} = a - (b+1)x + x^2y + f\sin\omega t + g\sin\Omega t$$
$$\dot{y} = bx - x^2y$$
(2)

Where *a* and *b* are constants with *a*, b > 0, *x* and *y* represent the dimensionless concentrations of two reactants. *f* and *g* are the amplitudes and  $\omega$  and  $\Omega$  are the frequencies of the external driving force. Very recently, Kavitha et al [18] studied the effect of frequency modulated signal on chaotic motion in Brusselator chemical reaction. The main objective of the present paper is to numerically study the dynamics of quasiperiodically forced Brusselator chemical



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reaction (Eq.(2)). The paper is organised as follows. In Section 2 we study the dynamics of the Brusselator under the influence of quasiperiodic force. First we study the dynamics of single force such as  $f \sin \omega t$  and  $g \sin \Omega t$  separately. Then we analyze the dynamics of Brusselator with quasiperiodic force. Finally Section 3 contains the conclusion of the present work.

#### **II. DYNAMICS OF THE BRUSSELATOR SYSTEM**

In this section for some specific sets of values of control parameters, we show the occurrence of different routes to chaos and suppression of chaos in the presence of quasiperiodic force.

We fix the values of the parameters as a=0.4, b=1.2,  $\omega=1.0$  and  $\Omega=(\sqrt{5}-1)/2$ , the reciprocal of the golden-mean. Eq.(2) is solved by fourth-order Runge kutta method with time step size  $2\pi/\omega$ . Numerical solution corresponding to first 500 drive cycle is left as transient. We analyse the response of the system (Eq.(2)) by varying the amplitudes f and g of the force for the cases separately, (i) f = 0 and g is varied, (ii) g = 0 and f is varied (iii) g is fixed and f is varied and (iv) f is fixed and g is varied. Fig.1(a) shows the bifurcation diagram where g is set to zero while f is varied. The corresponding maximal Lyapunov exponent ( $\lambda_m$ ) diagram is shown in Fig. 1(b). The Maximal Lyapunov exponent ( $\lambda_m$ ) is computed using the algorithm given in ref. [17]. If  $\lambda_{max} < 0$ , the disturbed trajectory is attracted eventually to a stable periodic orbit.  $\lambda_{max} > 0$  reveals an unstable as chaotic trajectory, at  $\lambda_{max} = 0$  means that the disturbed oscillation stay apart be a constant mean distance for an indefinite duration until perturbed again. As f is increased from - 0.5, a stable period-T (= $2\pi/\omega$ ) orbit occurs, which persists upto f = -0.48126 and then it loses its stability giving birth to a period-2T orbit. System (Eq.(2)) then undergoes period-2 bubble orbit as the control parameter f is smoothly varied. For example at f = -0.23821, period-2T orbit becomes unstable and gives birth to period-2 bubble orbit. Further increase of f, quasiperiodic, reverse periodic orbits are developed.



Fig.1 Bifurcation diagram and the corresponding maximal Lyapunov exponent diagram of the system (Eq.(2)) with g = 0 and  $f \in [-0.5, 0.5]$  The other parameters values are a = 0.4, b = 1.2,  $\omega = 1.0$  and  $\Omega = (\sqrt{5}-1)/2$ .

For f = 0.2 and g = 0 the system (Eq.(2)) shows the periodic behaviour. Fig. 2(a) is the bifurcation diagram obtained by varying g from -5 to +5. As g is increased from -5, the period-T orbit persists upto g = -0.24607 and then a period-2T solution is developed. This is followed by bifurcation to periodic bubbles, reverse period doubling bifurcation and so on. The corresponding maximal Lypunov exponent diagram is shown the Fig. 2(b). From Fig. 2, we can clearly observe that the parameter g can be used to suppress quasiperiodic motion by choosing its value in the above interval. An example of period-4 attractor and quasiperiodic attractor is shown in Fig. 3. Example of time evolution of the system (Eq (2)) with different f values is shown in Fig. 4. The Brusselator system (Eq.(2)) exhibits qualitatively different behaviour with periodic oscillations (Period-2, Period-4 and Period-T) in Figs. 4(a),4(b),4(d) and quasiperiodic orbit in Fig. 4(c).



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Fig. 2 Bifurcation diagram and the corresponding maximal Lyapunov exponent diagram of the system (Eq.(2)) with f = 0.2 and  $g \in [-0.5, 0.5]$  The other parameters values are a = 0.4, b = 1.2,  $\omega = 1.0$  and  $\Omega = (\sqrt{5}-1)/2$ .



Fig.3 Phase portraits and the corresponding Poincaré maps of the system (Eq.(2)) for two values of f with g = 0. The corresponding parameters are the same as shown in Fig.2.



Fig.4 Time evolution of the system (Eq.(2)). (a) Period-2T (b) Period-4T (c) Quasiperiodic orbit and (d) Period-T orbit.

We show the effect of the control parameter g by fixing the values of f. Fig 5(a) is the bifurcation diagram obtained by varying g from - 0.5 to + 0.5 and f is set to zero. As g is increased from - 0.5, period–T orbit persists upto g = -0.30895 and then periodic bubble orbit is developed. This is followed by bifurcation to chaotic orbit. Onset of chaos takes place at f = -0.11064. Further increase of f, a quasiperiodic orbit, reverse period-doubling orbit occurs and the long time



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motion settles to a periodic behaviour. Then we study the effect of f by fixing the value of g in a regular region. For g = 0.1 and f = 0, the motion of the system is periodic with period-2T. Figure 6(a) is the bifurcation diagram obtained by varying f from -1 to +1. The effect of f can be clearly seen in the bifurcation diagram is shown in Fig. 6(a). Figure 7 shows the phase portrait and corresponding Poincaré maps at g = 0.02 and g = -0.088 and f is set to zero. From this figure we can clearly confirm the occurrence of quasiperiodic orbit and chaotic orbit in the system (Eq (2)). The time evolution of the quasiperiodic orbit and chaotic orbit of the system (Eq (2)) is shown in Fig. 8. The system exhibits quasiperiodic oscillation in Fig 8(a) at g = 0.017 and chaotic oscillation in Fig 8(b) at g = -0.088.



Fig. 5 Bifurcation diagram and the corresponding maximal Lyapunov exponent diagram of the system (Eq.(2)). The other parameters values are a = 0.4, b = 1.2,  $\omega = 1.0$  and  $\Omega = (\sqrt{5}-1)/2$ .



Fig.6 Bifurcation diagram and the corresponding maximal Lyapunov exponent diagram of the system (Eq.(2)). The other parameters values are a = 0.4, b = 1.2,  $\omega = 1.0$  and  $\Omega = (\sqrt{5}-1)/2$ .



Fig.7 Phase portraits and the corresponding Poincaré maps of the system (Eq.(2)) for two values of g with f = 0. The corresponding parameters are the same as shown in Fig.2.



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Finally we show the effect of the control parameter  $\Omega$  by fixing the values of f and g in a regular region. Figure 9 is the bifurcation diagram obtained by varying  $\Omega$  from 0 to 1. The values of the other parameters fixed as f = 0.2, g = 0.1 and  $\omega = 1.0$ . As  $\Omega$  is increased from 0, period-2T orbit occurs, which persists upto  $\Omega = -0.06167$ . At  $\Omega = -0.06167$ , period-2T orbit suddenly disappears and the long time motion settles to a periodic behaviour.



Fig.8 Time evolution of the system (Eq.(2)). (a) Quasiperiodic orbit and (b) Chaotic orbit



Fig.9 Bifurcation diagram and the corresponding maximal Lyapunov exponent diagram of the system (Eq.(2)) with f = 0.2, g = .0.1. The other parameters values are a = 0.4, b = 1.2,  $\omega = 1.0$  and  $\Omega = (\sqrt{5}-1)/2$ 

#### **III. CONCLUSION**

In the present paper we considered the quasiperiodically forced Brusselator chemical reaction. We numerically studied the effect of quasperiodic force on chaotic motion in the Brusselator system (Eq (2)). We demonstrated the effect of the control parameters f g and  $\Omega$  in the dynamics of the system (Eq (2)). For a particular set of parameters values, we have shown the occurrence of periodic, quasiperiodic, periodic bubbles, reverse-periodic and chaotic oscillations. It is also found that the quasiperiodic force suppresses the critical chaotic behaviour in some parameter ranges. The additional features of the system in terms of vibrational resonance, ghost vibrational resonance, parametric resonance etc, observe for features study

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