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Families of Disjoint Sets Colouring Technique and Concept of Common Set and Non Common Set

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ABSTRACT: Families of disjoint sets colouring technique is a trial to generalize all type of colouring, edges, vertices, faces and any things can be contrast by colours. This is our ninth paper about families of disjoint sets colouring technique, previous papers deal with set as a member families of disjoint sets (colour classes). In this paper we introduce the concept of common set to establish some results for families of disjoint sets colouring technique. The concept of common set is essential concept for families of disjoint setstechnique, and we have to modify some definitions due to common set concept.

KEYWORDS: families of disjoint sets colouring technique, common set, non common set, proper intersection, improper intersection.

I. INTRODUCTION

Colouring of families of disjoint sets techniquegenerally depend on the number n , in case of sets n is degree of intersection, for colouring edges of the graph n is the largest degree of vertex (maximum number of edges such that any two are adjacent). Forcolouring vertices of the graph, n is the clique number (maximum number of vertices such that any two are adjacent) [8].

In paper [1] we introduced the concept of families of disjoint setscolouring technique as a topic of set theory. We also introduce definitions of degree of intersection, meeting and standard of meeting, minimum number of families of disjoint sets and maximum number of families of disjoint sets. In paper [2] we introduce some concepts in graph theory to introduce new method for colouring edges and vertices of graph. This method has two types first related to vertices of the graph and second related to cycles. This method depend on the theory in our paper "Family of disjoint sets and Its Applications", with suitable changes. In paper [3] we prove minimum number of families of disjoint sets with known degree of intersection (minimum number of colours) and we introduce another proof for maximum number of families of disjoint sets (maximum number of colours). Also in that paper we introduce the concept types of families of disjoint sets (types of colouring) and prove some results related to this concept. In paper[4] we assume both sets and graphs are finite. Results in that paper use families of disjoint setscolouring technique for finite sets and finite graphs. Using results in that paper we can compare between families of disjoint setscolouring technique and colouring of graphs in graph theory.

In paper [5] we prove some results in graph theory using colouring of families of disjoint sets technique. We introduced the concept of set labeled by families of disjoint sets and the degree of this set, as generalization to the degree of intersection and proved some results related to this concept. In paper [6] we prove some results of edge'scolouring (trivial colouring and nontrivialcolouring). We prove some results of edge'scolouring for many graphs connected by bridges. In many examples we determine minimum colours to colour edges of the graphs only by calculation when we know number of the edges , number of the vertices and largest degree of vertex, whatever the graph is huge. We used the concept of trivial colouring and nontrivialcolouring to prove resultsof edge'scolouring for trees, bipartitegraphs and complete graphs.

In paper [7] we modify the definitions of local maximum families of disjoint sets and local minimum families of disjoint sets, which we introduced in paper [1], and prove somepropositions related to these two concepts.

In paper we [8] used families of disjoint sets colouring technique, to colour vertices of the graph depend on the clique number, and we introduce some results of vertex's colouring (trivial colouring and nontrivial colouring) depend on clique number. We introduce the concept maximum and minimum number of edges in the graph to colour k vertices by m colours, and we used this concept to prove a result about number of all possible ways to colour k vertices by m colours, and a result about the range of edges in which we can colour k vertices by m colours.

In this paper (section three) we introduce the concepts of proper intersection and common set at number of intersections or between two intersections and we introduce some results related to this concept.

In section four we used the concept of common set to introduce some results of families of disjoint sets colouring technique.

In section five we introduce a method to construct minimum number of families of disjoint sets, and introduce some results related to this method.

In section six we introduce some results related to addition and removal of common sets and non common sets.

In section seven we introduce the main results.

II. RELATED WORK

In this section we recall some definitions and results related to colouring in graph theory and families of disjoint sets colouring technique.

Definition 2.1. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and X is nonempty set, $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, then the intersection X is called represent the families $\chi_1, \chi_2, \chi_3, \dots, \chi_r$, and we used the notation $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, or $X = (\chi_1, \chi_2, \chi_3, \dots, \chi_r)$ [1].

Remark 2.2. As abbreviation in state of the notation $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, and $X = (\chi_1, \chi_2, \chi_3, \dots, \chi_r)$ we used the notation $X = (1, 2, 3, \dots, r)$ [1].

Remark 2.3. In this paper we will labelled a vertex using its degree and edges. If v is vertex of degree three we can used the notation $v = (e_i, e_j, e_k)$ pointing to v as common point and any two of edges e_i, e_j, e_k are adjacent. The notation $v = (i, j, k)$ pointing to colours i, j, k , where i, j, k are integers. If v of degree n we write $v = (e_1, e_2, e_3, \dots, e_n)$ for pointing to edges $e_1, e_2, e_3, \dots, e_n$, or we can used notation $v = (1, 2, 3, \dots, n)$ for pointing to colours (or colour classes) $1, 2, 3, \dots, n$, and we say v represented (or labelled) by $1, 2, 3, \dots, n$. [2].

Example 2.4. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_7$ be 7 families of disjoint sets and X, Y, Z are nonempty different intersections each of degree 4. $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \chi_5$, $Y = \chi_2 \cap \chi_3 \cap \chi_5 \cap \chi_7$ and $Z = \chi_1 \cap \chi_4 \cap \chi_6 \cap \chi_7$. We can used the notation $X = (1, 2, 3, 5)$, $Y = (2, 3, 5, 7)$, $Z = (1, 4, 6, 7)$. At intersections X, Y, Z , the family of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_5$ and χ_7 each appear twice at X, Y, Z, χ_4 , and χ_6 each appear once. Also we can say at intersections X, Y, Z , the family of disjoint sets $1, 2, 3, 5, 7$ each appear twice, and $4, 6$ each appear once. As application to following proposition, each of intersections X, Y, Z the is of degree four we have $3(4) = 12 = 2 + 2 + 2 + 1 + 2 + 1 + 2$ [1].

Definition 2.5. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, and $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections of these m families of disjoint sets, each intersection of degree n , the family of disjoint sets χ_i is said to be meet the family of disjoint sets χ_j at these w intersections, if whatever we exchange families of disjoint sets at these w intersections, there exists one intersection the two families of disjoint sets χ_i and χ_j appear at this intersection [1].

Proposition 2.6. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections each of degree n , and t_i number of times the family of disjoint sets χ_i appear at these w intersections, then we have

$$\sum_{i=1}^m t_i = wn \quad [1].$$

Definition 2.7. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections of these m families of disjoint sets, each intersection of degree n . The family of disjoint sets χ_i is said meets family

of disjoint sets χ_j at these w intersections, if whatever we exchange families of disjoint sets at these w intersections, there exists at least one intersection the two families of disjoint sets χ_i and χ_j appear at this intersection [1].

Proposition 2.8. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections, the family of disjoint sets χ_i appear w_i times at w intersections and the family of disjoint sets χ_j appear w_j time at w intersections, then we say the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these w intersections if and only if $w + 1 \leq w_i + w_j$ [1].

Definition 2.9. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ each of degree n , and w be the minimum number of nonempty intersections, such that any two families of disjoint sets meet each other at these w intersections, then these w intersections called standard of meeting for these m families of disjoint sets [1].

III. COMMON SETS AND NONCOMMON SETS

In this section we introduce the concept of common set at number of intersections or between more than two intersections as essential concepts for families of disjoint sets colouring technique and we introduce some results of families of disjoint sets colouring technique related to this concept.

Definition 3.1. Suppose we have the sets $A_1, A_2, A_3, \dots, A_k$, and we have the intersections $X_1, X_2, X_3, \dots, X_w$ of these k sets with maximum degree of intersection equal n , an intersection X of degree of intersection equal n_1 is called proper intersection, if there is no any intersection Y of degree of intersection greater than n_1 , where $n_1 \leq n$, such that $Y \subset X$. An intersection Z of degree of intersection less than n is called improper intersection, if there exists proper intersection X such that $X \subset Z$. The set A_i is called common set between the two proper intersections X_j and X_l if $X_j \subset A_i$ and $X_l \subset A_i$ such that $X_j \neq X_l$, where $1 \leq i \leq k$, and $1 \leq j < l \leq w$.

Remark 3.2. In papers [1] and [2] we used the notations $X = (1, 2, 3, \dots, r)$, $v = (1, 2, 3, \dots, n)$, $X = (1, 2, 3, 5)$, see Remark 2.2, Remark 2.3. and Example 2.4. From this paper we modify these notations, we will write these notations as follows: $X \equiv (1, 2, 3, \dots, r)$, $v \equiv (1, 2, 3, \dots, n)$, $X \equiv (1, 2, 3, 5)$. If edge e incident with vertices v_1, v_2 , and we use the notation $e = \{v_1, v_2\}$.

Example 3.3. Let A_1, A_2, A_3, A_4, A_5 , be 5 sets and X_1, X_2, X_3 are nonempty intersections each of degree 3. $X_1 = A_1 \cap A_2 \cap A_3$, $X_2 = A_2 \cap A_4 \cap A_5$ and $X_3 = A_1 \cap A_4$. We use the notation $X_1 \equiv (A_1, A_2, A_3)$, $X_2 \equiv (A_2, A_4, A_5)$, $X_3 \equiv (A_1, A_4)$, as abbreviation we use the notation $X_1 \equiv (1, 2, 3)$, $X_2 \equiv (2, 4, 5)$, $X_3 \equiv (1, 4)$. The set A_2 is common set between the two intersections X_1 and X_2 , the set A_4 is common set between the two intersections X_2 and X_3 , the set A_1 is common set between the two intersections X_1 and X_3 .

Definition 3.4. Suppose we have the sets $A_1, A_2, A_3, \dots, A_k$, and the proper intersections $X_1, X_2, X_3, \dots, X_w$ of these k sets, with maximum degree of intersection equal n , where $n \leq k$, A_i is called common set between c intersections $X_1, X_2, X_3, \dots, X_t$ at $X_1, X_2, X_3, \dots, X_w$ where $t \leq w$, if for all j we have $X_j \subset A_i$, where $1 \leq j \leq t$.

Example 3.5. Suppose we have the sets $A_1, A_2, A_3, \dots, A_k$, and we have the intersections $X_1, X_2, X_3, \dots, X_w$ of these k sets, with maximum degree of intersection equal n , where $n \leq k$, the set A_i is called common set between the three intersections of X_1, X_2, X_3 at $X_1, X_2, X_3, \dots, X_w$ if each of X_1, X_2, X_3 contained in the set A_i .

Definition 3.6. Suppose we have the sets $A_1, A_2, A_3, \dots, A_k$, and we have the intersections $X_1, X_2, X_3, \dots, X_w$ of these k sets, with maximum degree of intersection equal n , where $n \leq k$, the set A_i is called non common set at the intersections $X_1, X_2, X_3, \dots, X_u$ if for any two intersections X_j and X_l we have $X_j \subset A_i, X_l \not\subset A_i$, or $X_j \not\subset A_i, X_l \subset A_i$, where $1 \leq j < l \leq u$, $u \leq w$, $1 \leq i \leq k$.

Proposition 3.7. Suppose we have the sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets, with maximum degree of intersection equal n , if these k sets partition into m families of disjoint sets

$\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, (i) two non common sets (disjoint) at u intersections, $u \geq 2$ can have the same labels of families of disjoint sets. (ii) any common set A between the two intersections X_i and X_j , the label represents the set A in X_i is same label represents the set A in X_j . (iii) let A_i be a common set between c_i intersections, if appearance of A_i at $X_1, X_2, X_3, \dots, X_w$ equal w_i , then for all $1 \leq i \leq k$ we have $c_i = w_i$ for all $1 \leq i \leq k$. (iv) if we add p non common sets, such that maximum degree of intersection remains n , then the number m of families of disjoint sets unchanged.

Proof: (i) suppose we have two different sets, which are non adjacent, they can belong to same colour class, therefore they can have the same label. (ii) suppose we have a common set A is common set between the two intersections X_i and X_j , suppose A at X_i labeled by 1 and suppose A at X_j labeled by 2, a contradiction since one set has two labels, one set can not belong to two colour classes, therefore the label represent the set A in X_i is same label represent the set A in X_j . (iii) from

definition of common set between intersections, for all $1 \leq i \leq k$ let A_i appear at w_i intersections out of w intersections $X_1, X_2, X_3, \dots, X_w$, therefore A_i common between w_i intersections $X_1, X_2, X_3, \dots, X_w$ then for all $1 \leq i \leq k$ we have $w_i = c_i$. (iv) for each intersections X_j of degree p we can write $X_j \equiv (1, 2, 3, \dots, p)$, if we add $(n-p)$ non common sets such that X_j to be of degree n and each of these $(n-p)$ sets adjacent to each set labeled by $1, 2, 3, \dots, p$, such that X_j to be of degree n , then we can write $X_j \equiv (1, 2, 3, \dots, p, p+1, p+2, \dots, n)$, or $X_j \equiv (1, 2, 3, \dots, n)$, if the intersections X_j of degree p labeled by $m, m-1, m-2, \dots, m-p+1$, we can write X_j as $X_j \equiv (m, m-1, m-2, \dots, m-p+1)$, therefore the number m of families of disjoint sets for $m=n$ or $n < m$ we have the number m unchanged.

Example 3.8. From intersection of four sets A_1, A_2, A_3, A_4 we can find many intersections such as $A_1 \cap A_2 \cap A_3 \cap A_4$, $A_1 \cap A_2$, $A_1 \cap A_4$, $A_2 \cap A_3 \cap A_4$, $A_1 \cap A_2 \cap A_3$, for colouring we take only $A_1 \cap A_2 \cap A_3 \cap A_4$, because it is proper intersection. If beside the sets A_1, A_2, A_3, A_4 exists the set A_5 , and A_5 intersect only the set A_3 , for colouring we take $A_1 \cap A_2 \cap A_3 \cap A_4$ and $A_3 \cap A_5$, therefore if the sets $A_1, A_2, A_3, \dots, A_m$ can intersect each other with maximum degree of intersection equal n (proper intersection) or degree of intersection less than n (less than n can be proper intersection or improper intersection) for colouring we take any proper intersection, and we do not take improper intersection.

Proposition 3.9. Suppose we have the sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets, with maximum degree of intersection equal n , if all sets (common sets and non common sets) partition into m families of disjoint sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, and $A_1, A_2, A_3, \dots, A_p$ belong to one family of disjoint sets \mathcal{X}_j , then $t_j = w_1 + w_2 + w_3 + \dots + w_p \leq w$ where w_i be number of times the set A_i appear at these w intersections, $1 \leq i \leq p$, and t_j be number of times the family of disjoint sets \mathcal{X}_j appear at these w intersections.

Proof: Suppose $A_1, A_2, A_3, \dots, A_p$ belong to one family of disjoint sets \mathcal{X}_j and \mathcal{X}_j labelled by j , then each of $A_1, A_2, A_3, \dots, A_p$ labelled by j and the label j appear $w_1 + w_2 + w_3 + \dots + w_p$ times at $X_1, X_2, X_3, \dots, X_w$, then whatever we exchange partition of the sets $A_1, A_2, A_3, \dots, A_k$ into m families of disjoint sets, and for any sets $A_1, A_2, A_3, \dots, A_p$ belong to one family of disjoint sets, any two sets of $A_1, A_2, A_3, \dots, A_p$ are non adjacent, then the label j not appear twice at any intersection, if j appear once at any one of $X_1, X_2, X_3, \dots, X_w$ then $t_j = w_1 + w_2 + w_3 + \dots + w_p = w$, if j appear once at some of $X_1, X_2, X_3, \dots, X_w$ then $t_j = w_1 + w_2 + w_3 + \dots + w_p < w$, therefore we have $t_j = w_1 + w_2 + w_3 + \dots + w_p \leq w$.

Remark 3.10. For intersection $X \equiv (1, 2, 3, \dots, n)$ the label i represent one family of disjoint sets, where $1 \leq i \leq n$, each i at the intersection X either it is common set belong to this family of disjoint sets or non common set belong to this family of disjoint sets. If it is common set, the label i depend on two factor (1) the labels $1, 2, 3, \dots, i-1, i+2, \dots, n$,

as each label represent a set intersect the set i . (2) t proper intersections, as common set between t intersections, means this common set appear at t intersections and with same label. If it is non common set, the label i depend on one factor (1) the labels $1, 2, 3, \dots, i-1, i+2, \dots, n$, as each label represent a set intersect the set i . Therefore when we partition k sets into m families of disjoint sets, to deal with non common set is easy than common set.

Example 3.11. Suppose all of $X_1, X_2, X_3, \dots, X_w$ are proper intersections of the sets $A_1, A_2, A_3, \dots, A_{10}$, such that each one of the sets A_1, A_2, A_3, A_4, A_5 is common set between two proper intersections, and each one of the sets $A_6, A_7, A_8, A_9, A_{10}$ is non common set between two proper intersections. $X_1 \equiv (A_1, A_2, A_8)$, $X_2 \equiv (A_1, A_4, A_{10})$, $X_3 \equiv (A_7, A_4, A_5)$, $X_4 \equiv (A_6, A_3, A_5)$, $X_5 \equiv (A_2, A_3, A_9)$. From these five intersection we can partition these ten sets into five families of disjoint sets as follows: $\chi_1 \equiv (A_1, A_6)$, $\chi_2 \equiv (A_2, A_7)$, $\chi_3 \equiv (A_3, A_8)$, $\chi_4 \equiv (A_4, A_9)$, $\chi_5 \equiv (A_5, A_{10})$. We use the label a for χ_1 , b for χ_2 , c for χ_3 , d for χ_4 , e for χ_5 , and we write these intersections as follows: $X_1 \equiv (a, b, \bar{c})$, $X_2 \equiv (a, d, \bar{e})$, $X_3 \equiv (\bar{b}, d, e)$, $X_4 \equiv (\bar{a}, c, e)$, $X_5 \equiv (b, c, \bar{d})$, we use dash for non common set, to differentiate between common set and non common set. Using Remark 3.10. we first deal with common sets, since non common sets is easy, the common set A_1 doesn't intersect the common set A_5 , so we consider A_1, A_5 belong one family of disjoint sets, and we label this family of disjoint sets by 1, the common set A_2 doesn't intersect the common set A_4 , so we consider A_2, A_4 belong one family of disjoint sets, and we label this family of disjoint sets by 2, we label family of disjoint sets contained the set A_3 by 3, the non common set A_6 intersect the common set A_3, A_5 , so we label A_6 by $\bar{2}$, continue in this manner we label A_7 by $\bar{3}$, we label A_8 by $\bar{3}$, we label A_9 by $\bar{1}$, we label A_{10} by $\bar{3}$, so we write these proper intersections as $X_1 \equiv (1, 2, \bar{3})$, $X_2 \equiv (1, 2, \bar{3})$, $X_3 \equiv (\bar{3}, 2, 1)$, $X_4 \equiv (\bar{2}, 3, 1)$, $X_5 \equiv (2, 3, \bar{1})$. Therefore these ten sets partition into three families of disjoint sets. The label 1 points to a common set belongs to the family of disjoint sets labeled by 1, and the label $\bar{1}$ points to a non common set belongs to the family of disjoint sets labeled by 1.

Here we modify Definition 2.5 and Definition 2.9. using concept of proper intersection.

Definition 3.12. Let $A_1, A_2, A_3, \dots, A_k$, be k sets and $X_1, X_2, X_3, \dots, X_w$ be proper intersections of these k sets,

$\chi_1, \chi_2, \chi_3, \dots, \chi_n$ be n families of disjoint sets and X_j is intersection, $X_j = A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$, where $1 \leq j \leq w$, $A_1 \subseteq \chi_1$, $A_2 \subseteq \chi_2$, $A_3 \subseteq \chi_3, \dots, A_n \subseteq \chi_n$, then the intersection X is called intersection of degree n .

Definition 3.13. Let $A_1, A_2, A_3, \dots, A_k$, be k sets and $X_1, X_2, X_3, \dots, X_w$ be proper intersections of these k sets, all sets (common sets and non common sets) partitioned into m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, the family of disjoint sets χ_i is said to be meet the family of disjoint sets χ_j at these w intersections, if whatever we exchange any set B_i by a set B_j , where $B_i \in \chi_i$, $B_j \in \chi_j$, $\chi_i \neq \chi_j$, $\chi_j \neq \chi_i$, and B_i intersects at least one member of the family of disjoint sets χ_j , such that the number t_i not changed (t_i appearance of χ_i at $X_1, X_2, X_3, \dots, X_w$) and χ_i remains a family of disjoint sets, then B_j intersects at least one member of the family of disjoint sets

Definition 3.14. Let $A_1, A_2, A_3, \dots, A_k$, be k sets and $X_1, X_2, X_3, \dots, X_w$ be proper intersections of these k sets, all sets (common sets and non common sets) partitioned into m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, the family of disjoint sets χ_i is said to be not meet the family of disjoint sets χ_j at these w intersections, where $1 \leq i < j \leq m$, if for any two sets A_r, A_s such that $A_r \in \chi_i$, $A_s \in \chi_j$, where $1 \leq r < s \leq k$, then A_r and A_s are disjoint, and if $t_i + t_j \leq w$.

Definition 3.15. Let $A_1, A_2, A_3, \dots, A_k$ be k sets, $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets, with maximum degree of intersection equal n , then these w intersections called standard of partition all sets (common sets and

non common sets) into m families of disjoint sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, then if for any two families of disjoint sets \mathcal{X}_i and \mathcal{X}_j we have $w + 1 \leq t_i + t_j$ is and $t_i \leq w, t_j \leq w$.

Definition 3.16. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be w proper intersections of these k sets (standard of partition), if we partition these k sets into m families of disjoint sets where $n + 1 \leq m$, then m is called minimum number families of disjoint sets, if we try to partition all sets (common sets and non common sets) into $m - 1$ families of disjoint sets, there exists a family of disjoint sets \mathcal{X}_i such that $w + 1 \leq t_i$, where $1 \leq i \leq m - 1$, and t_i appearance of \mathcal{X}_i at $X_1, X_2, X_3, \dots, X_w$.

Definition 3.17. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be w proper intersections (standard of partition) of these k sets (standard of partition), and we partition all sets (common sets and non common sets) into m families of disjoint sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, then m is called maximum number families of disjoint sets, if we partition these k sets into any $m + 1$ families of disjoint sets, there exist two families of disjoint sets $\mathcal{X}_i, \mathcal{X}_j$ such that $t_i + t_j \leq w$, where $1 \leq i < j \leq m + 1$, and t_i, t_j appearance of $\mathcal{X}_i, \mathcal{X}_j$ at $X_1, X_2, X_3, \dots, X_w$, respectively.

Example 3.18. X_1, X_2, X_3, X_4, X_5 are proper intersections of the sets $A_1, A_2, A_3, A_4, A_5, A_6$, where $X_1 = A_1 \cap A_2 \cap A_7 \cap A_8 \cap A_9$, $X_2 = A_1 \cap A_3 \cap A_4$, $X_3 = A_3 \cap A_6$, $X_4 = A_6 \cap A_5$, $X_5 = A_2 \cap A_4 \cap A_5$, and A_7, A_8, A_9 are non common sets. $d(X_1) = 5$, $d(X_2) = d(X_5) = 3$, $d(X_3) = d(X_4) = 2$. The set $\{X_1, X_2, X_3, X_4, X_5\}$ can not form standard of partition to $\{A_1, A_2, A_3, A_4, A_5, A_6\}$ and any subset of $\{X_1, X_2, X_3, X_4, X_5\}$ can not form standard to partition a subset of $\{A_1, A_2, A_3, A_4, A_5, A_6\}$.

Proposition 3.19. Let $A_1, A_2, A_3, \dots, A_k$ be k sets and $X_1, X_2, X_3, \dots, X_w$ be w proper intersections of these k sets, if all sets (common and non common) partition into m families of disjoint sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, labelled by

$1, 2, 3, \dots, m$, and if t_i be number of times the family of disjoint sets \mathcal{X}_i appear at w intersections, if $1 \leq i \leq m$,

$$1 \leq l \leq k, 1 \leq j \leq w \text{ and } k_1 \text{ is number of non common sets then } \sum_{i=1}^m t_i = k_1 + \sum_{l=1}^k c_l = \sum_{j=1}^w d(X_j).$$

Proof: We labelled sets by labels of families of disjoint sets, such that two adjacent sets have two different labels, and labelled any proper intersection $X_j (1 \leq j \leq w)$ by number of labels equal its degree of intersection and same labels of all sets their intersection equal X_j , let A_l be common set between c_l proper intersections, or c_l number of times the set A_l appear at w proper intersections, since $d(X_j)$ equal number of sets at proper intersection X_j , therefore

$$k_1 + \sum_{l=1}^k c_l = \sum_{j=1}^w d(X_j), \text{ since all sets (common and non common) partition into } m \text{ families of disjoint sets}$$

$\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, labelled by $1, 2, 3, \dots, m$, and if t_i be number of times the family of disjoint sets \mathcal{X}_i appear at w

$$\text{intersections, then we have then } k_1 + \sum_{l=1}^k c_l = \sum_{i=1}^m t_i, \text{ therefore } \sum_{i=1}^m t_i = k_1 + \sum_{l=1}^k c_l = \sum_{j=1}^w d(X_j).$$

Definition 3.20. A walk of proper intersections and common sets is an alternating sequence

$X_1, S_1, X_2, S_2, X_3, S_3, \dots, X_t, S_t, X_{t+1}$ is an alternating sequence of proper intersections and common sets where $S_1, S_2, S_3, \dots, S_t$ are common sets and $X_1, X_2, X_3, \dots, X_{t+1}$ are such that $S_i \neq S_{i+1}, S_1 \neq S_t$ where $1 \leq i \leq t$. The walk of proper intersections and common sets, as abbreviation can also be denoted by $X_1, X_2, X_3, \dots, X_t, X_{t+1}$. A walk of proper intersections and common sets $X_1, S_1, X_2, S_2, X_3, S_3, \dots, X_t, S_t, X_{t+1}$ is called cycle of proper intersections and common sets if $X_1 = X_{t+1}$. A walk is called path if common sets are distinct and intersections are distinct.

Proposition 3.21. Suppose we have common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets, and each intersection is intersection of at least two common sets, then there is at least one cycle include all the sets $A_1, A_2, A_3, \dots, A_k$, or some of them.

Proof: Suppose $Y_1, B_1, Y_2, B_2, Y_3, B_3, \dots, Y_{t-1}, B_{t-1}, Y_t$ is an alternating sequence of proper intersections and common sets is path and not cycle, where $\{Y_1, Y_2, Y_3, \dots, Y_t\} = \{X_1, X_2, X_3, \dots, X_w\}$ and $\{B_1, B_2, B_3, \dots, B_{t-1}\} = \{A_1, A_2, A_3, \dots, A_k\}$, with origin Y_1 and terminus Y_t . Since each intersection is intersection of at least two common sets definition of proper intersection, then Y_1 is not origin and Y_t is not terminus, since the sequence is finite, then $Y_1, B_1, Y_2, B_2, Y_3, B_3, \dots, Y_{t-1}, B_{t-1}, Y_t$ is cycle.

Proposition 3.22. Suppose we have common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets, with maximum degree of intersection equal n , suppose these k sets and W intersections form a path $Y_1, B_1, Y_2, B_2, Y_3, B_3, \dots, Y_{t-1}, B_{t-1}, Y_t$ of proper intersections and common sets and not cycle, where $\{Y_1, Y_2, Y_3, \dots, Y_t\} = \{X_1, X_2, X_3, \dots, X_w\}$ and $\{B_1, B_2, B_3, \dots, B_{t-1}\} = \{A_1, A_2, A_3, \dots, A_k\}$, if we partition all sets (common and non common) into m minimum families of disjoint sets, then $m = n$.

Proof: From definition of path any two sets of $\chi_1 = \{B_1, B_3, B_5, \dots, B_{t-2}\}$ are disjoint, and same thing for $\chi_2 = \{B_2, B_4, B_6, \dots, B_{t-1}\}$, when t is odd, or any two of $\chi_1 = \{B_1, B_3, B_5, \dots, B_{t-1}\}$ are disjoint, and same thing for $\chi_2 = \{B_2, B_4, B_6, \dots, B_{t-2}\}$, when t is even. Let the family of disjoint sets χ_1 be labeled by 1. and let χ_2 be labeled by 2. Therefore maximum number of families of disjoint sets each consist of non common set only, equal $n - 2$, therefore of remaining non common sets can be labeled by one element of the set $\{3, 4, 5, \dots, n\}$. Therefore $m = n$.

Proposition 3.23. Suppose we have common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets, with maximum degree of intersection equal n , suppose these k sets and W intersections form two paths $Y_1, B_1, Y_2, B_2, Y_3, B_3, \dots, Y_{t-1}, B_{t-1}, Y_t$ and $Z_1, R_1, Z_2, R_2, Z_3, R_3, \dots, Z_{s-1}, R_{s-1}, Z_s$ of proper intersections and common sets and not cycle, where $\{Y_1, Y_2, Y_3, \dots, Y_t\} \cup \{Z_1, Z_2, Z_3, \dots, Z_s\} = \{X_1, X_2, X_3, \dots, X_w\}$ and $\{B_1, B_2, B_3, \dots, B_{t-1}\} \cup \{R_1, R_2, R_3, \dots, R_{s-1}\} = \{A_1, A_2, A_3, \dots, A_k\}$, if we partition all sets (common and non common) into m minimum families of disjoint sets, then $m = n$.

Proof: If there is no common set common between the two paths, using Proposition 3.22. we have $m = n$. If there is common set common between the two paths, from definition of path any two sets of $\chi_1 = \{B_1, B_3, B_5, \dots\}$, $\chi_2 = \{B_2, B_4, B_6, \dots\}$, $\chi_3 = \{R_1, R_3, R_5, \dots\}$ and $\chi_4 = \{R_2, R_4, R_6, \dots\}$, where $\chi_1 \cup \chi_2 \cup \chi_3 \cup \chi_4 = \{A_1, A_2, A_3, \dots, A_k\}$. Let the family of disjoint sets χ_1 be labeled by 1, χ_2 be labeled by 2, χ_3 be labeled by 3, and χ_4 be labeled by 4. Two of $\chi_1, \chi_2, \chi_3, \chi_4$ are can form one family of disjoint sets. Let us consider χ_1 and χ_4 form one family of disjoint sets, then the family of disjoint sets χ_4 be labeled by 1. Let the family of disjoint sets χ_1, χ_2, χ_3 be labeled by 1, 2, 3 respectively. Therefore maximum number of families of disjoint sets each consist of non common set only, equal $n - 3$, therefore remaining non common sets can be labeled by one element of the set $\{4, 5, 6, \dots, n\}$. Therefore $m = n$. If there is common proper intersection between the four families of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_4$ therefore maximum number of families of disjoint sets each consist of non common sets only, equal $n - 4$, therefore remaining non common sets can be labeled by one element of the set $\{5, 6, 7, \dots, n\}$. Therefore $m = n$. Then the result holds.

Proposition 3.24. Suppose we have common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets with maximum degree of intersection equal n , suppose these k sets and W intersections form three paths if we partition all sets (common and non common) into m minimum families of disjoint sets, then $m = n$.

Proposition 3.25. Suppose we have common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , suppose these

k sets and w intersections form q paths, if we partition all sets (common and non common) into m minimum families of disjoint sets, then $m = n$.

Remark 3.26. Proof of Proposition 3.24. and Proposition 3.25. is the same as proof of Proposition 3.22. and proof of Proposition 3.23.

Proposition 3.27. Let $A_1, A_2, A_3, \dots, A_k$ be k sets and $X_1, X_2, X_3, \dots, X_w$ be w intersections of these k sets, and each intersection of $X_1, X_2, X_3, \dots, X_w$ is proper intersection, if these k sets partition into m families of disjoint sets where $m = n + 1$ it is not necessary $X_1, X_2, X_3, \dots, X_w$ is standard of partition.

Proof: Example 3.18. is enough to prove this proposition.

Definition 3.28. Let $A_1, A_2, A_3, \dots, A_k$ be common sets and $X_1, X_2, X_3, \dots, X_w$ are proper intersections of these k sets, then union of all sets (common sets and non common sets) is called connected, if there exists a path between any two proper intersections.

Remark 3.29. Through this paper any definition or result about common sets and proper intersections of these common sets, then union of all sets (common sets and non common sets) considered connected.

IV. SOME RESULTS OF FAMILIES OF DISJOINT TECHNIQUE USING COMMON SETS

In this section we used the concept of common set at number of intersections or between more than two intersections to introduce some results of families of disjoint sets colouring technique.

Remark 4.1. When we partition all sets (common sets and non common sets) into m families of disjoint sets, by words change and exchange we mean change a set from a families of disjoint sets to another and exchange two different sets between two different families of disjoint sets, such that any two joint sets belong to two different families of disjoint set, and two disjoint sets belong to two different families of disjoint sets or belong to one family of disjoint sets.

Proposition 4.2. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets and $X_1, X_2, X_3, \dots, X_w$ be w proper intersections of these k sets, with maximum degree of intersection equal n , and each intersection of $X_1, X_2, X_3, \dots, X_w$ is proper intersection, each intersection is common between number of sets equal its degree of intersection, if all sets (common sets and non common sets) partition into m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, for any two families of disjoint sets χ_i and χ_j (i) if χ_i meets χ_j at these w intersections, then $\chi_i \cup \chi_j$ and can't form one family of disjoint sets. (ii) if χ_i dose not meet χ_j at these w intersections, then $\chi_i \cup \chi_j$ can form a family of disjoint sets.

Proof: (i) Let χ_i meet χ_j at these w intersections, suppose $\chi_i \cup \chi_j$ can form one family of disjoint sets, from definition of χ_i meet χ_j there exist two adjacent sets belong to the family of disjoint sets $\chi_i \cup \chi_j$, a contradiction therefore $\chi_i \cup \chi_j$ and can't form one family of disjoint sets. (ii) Let χ_i not meet χ_j at these w intersections, from definition of χ_i not meet χ_j therefore any two adjacent sets A_i and A_j where $A_i \in \chi_i$ and $A_j \in \chi_j$, we can exchange A_i by A_j (not adjacent), where $A_i \notin \chi_i$, after we exchange A_i by A_j we consider $A_i \in \chi_i$ and $A_i \notin \chi_i$. Let $\{B_1, B_2, B_3, \dots, B_s\} = \chi_i$, and $\{R_1, R_2, R_3, \dots, R_t\} = \chi_j$, where $\{B_1, B_2, B_3, \dots, B_s\} \subset \{A_1, A_2, A_3, \dots, A_k\}$ and $\{R_1, R_2, R_3, \dots, R_t\} \subset \{A_1, A_2, A_3, \dots, A_k\}$. From definition of χ_i dose not meet χ_j any two sets of $\{B_1, B_2, B_3, \dots, B_s, R_1, R_2, R_3, \dots, R_t\}$ are disjoint, then we have $s + t \leq w$. Therefore $\chi_i \cup \chi_j$ can form one family of disjoint sets.

Proposition 4.3. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets and $X_1, X_2, X_3, \dots, X_w$ be w proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if these k sets partition into m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, for any two families of disjoint sets χ_i meet χ_j at these w intersections, if t_i, t_j be

number of times the families of disjoint sets $\mathcal{X}_i, \mathcal{X}_j$ appear at these w intersections respectively, where these w intersections is standard of partition all sets (common sets and non common sets) into m families of disjoint sets then we have (i) $w+1 \leq t_i + t_j$ for w odd or even. (ii) $\frac{w+1}{2} \leq t_i \leq w$ for w odd. (iii) $\frac{w+2}{2} \leq t_i \leq w$ for w even. (iv)

For the family of disjoint sets \mathcal{X}_i if there is t_i such that $t_i \leq \frac{w-1}{2}$ for w odd or $t_i \leq \frac{w}{2}$ for w even, it will be only one family of disjoint sets.

Proof: (i) Since \mathcal{X}_i meet \mathcal{X}_j at these w intersections, whatever we exchange any two adjacent sets from \mathcal{X}_i and \mathcal{X}_j so as to be nonadjacent, there exist at least one pair adjacent, if there exist one pair adjacent therefore $w+1 = t_i + t_j$, if there exist two pairs adjacent therefore $w+2 = t_i + t_j$, if there exist three pairs adjacent therefore $w+3 = t_i + t_j$, continue in this process we have $w+1 \leq t_i + t_j$ for w odd or even.

(ii) Suppose $t_i = t_j < \frac{w+1}{2}$ for w odd, then $t_i = t_j \leq \frac{w-1}{2}$ for w odd, therefore $t_i + t_j \leq w-1$ for w odd, this contradict (i) therefore $\frac{w+1}{2} \leq t_i$ for w odd, from Proposition 4.1. we have $t_i \leq w$ for w odd or even, therefore

$$(iii) \frac{w+1}{2} \leq t_i \leq w \text{ for } w \text{ odd.}$$

(iii) Suppose $t_i = t_j < \frac{w+2}{2}$ for w even, then $t_i = t_j \leq \frac{w}{2}$ for w even, therefore $t_i + t_j \leq w$ for w even, this contradict (i) therefore $\frac{w+2}{2} \leq t_i$ for w even, from Proposition 4.1. we have $t_i \leq w$ for w odd or even, therefore

$$\frac{w+2}{2} \leq t_i \leq w \text{ for } w \text{ even.}$$

(iv) Suppose we have only one t_i such that $t_i \leq \frac{w-1}{2}$ for w odd or $t_i \leq \frac{w}{2}$ for w even, therefore for all other families of disjoint sets \mathcal{X}_j there is t_j such that $\frac{w+3}{2} \leq t_j \leq w$ for w odd or $\frac{w}{2} \leq t_j \leq w$ for w even, we have

$$w+1 \leq t_i + t_j \leq \frac{3w-1}{2} \text{ for } w \text{ odd, and } w+1 \leq t_i + t_j \leq \frac{3w}{2} \text{ for } w \text{ even, therefore we have } w+1 \leq t_i + t_j \text{ for } w$$

odd or even. Suppose we have t_i and t_j such that $t_i = t_j \leq \frac{w-1}{2}$ for w odd or $t_i = t_j \leq \frac{w}{2}$ for w even, therefore

$$t_i + t_j \leq w-1 \text{ for } w \text{ odd and } t_i + t_j \leq w \text{ for } w \text{ even, this contradict } w+1 \leq t_i + t_j \text{ therefore (iv) holds}$$

Proposition 4.4. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets and $X_1, X_2, X_3, \dots, X_w$ be w intersections of these k sets, and each intersection of $X_1, X_2, X_3, \dots, X_w$ is proper intersection (standard of partition), each intersection is common between number of sets equal its degree of intersection, if all sets (common sets and non common sets) partition into m families of disjoint sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$, if for all $i, 1 \leq i \leq k$ we have $\frac{w+1}{2} \leq w_i$ for w odd, or $\frac{w+2}{2} \leq w_i$ for w even, where w_i appearance of the set A_i at $X_1, X_2, X_3, \dots, X_w$, then these k sets partition into k families of disjoint sets.

Proof: Suppose the sets $A_1, A_2, A_3, \dots, A_k$ partition into m families of disjoint sets

$\chi_1, \chi_2, \chi_3, \dots, \chi_m$, and suppose $m < k$, then there exists a colour class χ_i at least consists of two sets, therefore $w+1 \leq t_i$ for w odd, $w+2 \leq t_i$ for w even, which is a contradiction to Proposition 3.9. therefore $k \leq m$, since k can't be less than m , therefore $k = m$.

Proposition 4.5. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be w proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if we partition all sets (common sets and non common sets) into m families of disjoint sets, (i) maximum value of m equal $2n-1$. (ii) if $\sum_{j=1}^w d(X_j) = m(\frac{w+1}{2}) + r$ where

$0 \leq r \leq \frac{w-1}{2}$, and w is odd, or if $\sum_{j=1}^w d(X_j) = (m-1)(\frac{w+2}{2}) + \frac{w}{2} + r$ where $0 \leq r \leq \frac{w}{2}$, and w is even, then m is maximum number families of disjoint sets.

Proof: (i) Suppose m is maximum number families of disjoint sets, let $m = 2n$, and w is odd, suppose minimum value of

$t_i, 1 \leq i \leq m, t_1 = t_2 = t_3 = \dots = t_m = \frac{w+1}{2}$, since $\sum_{j=1}^w d(X_j) = \sum_{i=1}^m t_i \leq nw$ then we have $\sum_{j=1}^w d(X_j) = \sum_{i=1}^m t_i = 2n(\frac{w+1}{2}) \leq nw$ is a

contradiction, therefore $m < 2n$, and suppose $m = 2n-1$, then we have $\sum_{j=1}^w d(X_j) = \sum_{i=1}^m t_i = (2n-1)(\frac{w+1}{2}) \leq nw$,

therefore maximum value of m equal $2n-1$. For w even odd, then we have $\sum_{j=1}^w d(X_j) = \sum_{i=1}^m t_i = (2n-1)(\frac{w+2}{2}) + \frac{w}{2} \leq nw$ is a

contradiction, therefore $m < 2n$, then we have $\sum_{j=1}^w d(X_j) = \sum_{i=1}^m t_i = (2n-1)(\frac{w+2}{2}) \leq nw$, therefore maximum value of m equal

$2n-1$.

(ii) First suppose we partition these k sets into number of families of disjoint sets equal $m+1$, and let w is odd, such that there are $m-r_1$ families of disjoint sets each family appears $\frac{w+1}{2}$ times, and there are r_1 families of disjoint sets each family

appears $\frac{w-1}{2}$ times, and there is one family of disjoint sets appears $r+r_1$ times, where $1 \leq r_1$, such that $1 \leq r+r_1 \leq \frac{w-1}{2}$,

therefore $\sum_{j=1}^w d(X_j) = m(\frac{w+1}{2}) + r = (m-r_1)\frac{w+1}{2} + r_1(\frac{w-1}{2}) + (r+r_1)$ and there exist two families of disjoint sets χ_i, χ_j such

that $t_i + t_j \leq w$, where $1 \leq i < j \leq m+1$, and t_i, t_j appearance of χ_i, χ_j at $X_1, X_2, X_3, \dots, X_w$, respectively, hence m is maximum number families of disjoint sets.

Second suppose we partition these k sets into number of families of disjoint sets equal $m+1$, and let w is even, such that

there are $m-r_1$ families of disjoint sets each family appears $\frac{w}{2} + 1$ times, and there are r_1 families of disjoint sets each family

appears $\frac{w}{2} - 1$ times, and there is one family of disjoint sets appears $r+r_1$ times, where $1 \leq r_1$, such that $1 \leq r+r_1 \leq \frac{w}{2} - 1$,

therefore $\sum_{j=1}^w d(X_j) = m(\frac{w}{2} + 1) + r = (m-r_1)(\frac{w}{2} + 1) + r_1(\frac{w}{2} - 1) + (r+r_1)$ and there exist two families of disjoint sets χ_i, χ_j such

that $t_i + t_j \leq w$, where $1 \leq i < j \leq m+1$, and t_i, t_j appearance of χ_i, χ_j at $X_1, X_2, X_3, \dots, X_w$, respectively, hence m is maximum number families of disjoint sets. Therefore the result holds.

V. CONSTRUCTION MINIMUM NUMBER OF FAMILIES OF DISJOINT SETS METHOD

In this section we introduce a method to construct minimum number of families of disjoint sets. Here we give some examples to explain this method, and introduce some results related to this method.

Definition 5.1. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if all sets (common sets and non common sets) partitioned into m families of disjoint sets, $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, using the following method: In first step we try partition maximum number of families of disjoint sets, such that each family of disjoint has appearance equal w , (we partition till there is no family of disjoint sets its appearance equal w), suppose maximum number of families of disjoint sets each has appearance w is q_0 . In second step we try partition maximum number of families of disjoint sets, such that each family of disjoint has appearance equal $w-1$, suppose maximum number of families of disjoint sets each has appearance $w-1$ is q_1 . In third step we try partition maximum number of families of disjoint sets, such that each family of disjoint has appearance equal $w-2$, suppose maximum number of families of disjoint sets each has appearance $w-2$ is q_2 . Continue in this process, till in last step suppose maximum number of families of disjoint sets each has appearance $w-s$ is q_s , where

$$0 \leq q_i, \quad 2 \leq w - q_i \leq w, \quad \text{and} \quad \sum_{i=0}^s (w-i)q_i = wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^w d(X_j) \quad \text{this}$$

method is called construction of minimum number of families of disjoint sets.

Remark 5.2. From Definition 5.1. we have all $q_0 + q_1 + q_2 + q_3 + \dots + q_s = m$.

Remark 5.3. Generally when we partition k common sets $A_1, A_2, A_3, \dots, A_k$ into m families of disjoint sets, we have four cases: (1) These m families of disjoint sets, are maximum number of families of disjoint sets. (2) These m families of disjoint sets, are minimum number of families of disjoint sets. (3) These m families of disjoint sets, are neither satisfy maximum number of families of disjoint sets, nor satisfy minimum number of families of disjoint sets. (4) These m families of disjoint sets, satisfying both maximum number of families of disjoint sets, and minimum number of families of disjoint sets.

Proposition 5.4. Suppose we have k common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , where $2 < n$, and there is an intersection of degree two, all sets (common sets and non common sets) partitioned into m families of disjoint sets, then the number of families of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$ not more than two.

Proof: Suppose the number of families of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$ equal three or more than three, let χ_1, χ_2, χ_3 be three families of disjoint sets each appear w times at $X_1, X_2, X_3, \dots, X_w$, from definition of family of disjoint sets, each one of χ_1, χ_2, χ_3 appear at all of intersections $X_1, X_2, X_3, \dots, X_w$, then there is no intersection of degree less than three, a contradiction since there is one intersection out of degree two, it is also a contradiction for more than three families of disjoint sets each appear w times at $X_1, X_2, X_3, \dots, X_w$, therefore the number of families of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$ not more than two.

We generalized Proposition 5.4. as follows:

Proposition 5.5. Suppose we have k common sets $A_1, A_2, A_3, \dots, A_k$, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , and minimum degree of intersection equal q , where $q < n$, all sets (common sets and non common sets) partitioned into m families of disjoint sets, then the number of families of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$ not more than q .

Proof of Proposition 5.5. is the same as in Proposition 5.4.

Proposition 5.6. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if we partition all sets (common sets and non common sets) into m families of disjoint sets, using method introduced by Definition 5.1. then m is minimum number of families of disjoint sets.

Proof: Step I: Let $\sum_{j=1}^w d(X_j) = wq_0 + r$ then we have $\sum_{j=1}^w d(X_j) = wm \leq wn$, or $\sum_{j=1}^w d(X_j) = w(m-1) + r \leq wn$, then $m \leq n$, since $m < n$ is not true, therefore $m = n$ and m is minimum number offamilies of disjoint sets.

Step II: Let $\sum_{j=1}^w d(X_j) = wq_0 + (w-1)q_1 + r$ then we have $\sum_{j=1}^w d(X_j) = wq_0 + (w-1)q_1 + r \leq wn$, and $q_0 + q_1 + 1 = m$ if $r \neq 0$ and $q_0 + q_1 = m$ if $r = 0$, and supposewe partition these k sets into $m-1$ families of disjoint sets, such that $q_0 + q_1 + 1 = m-1$ if $r \neq 0$ and $q_0 + q_1 = m-1$ if $r = 0$, then either there is $q_0 + x$ families of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$ or there is $q_1 + x$ families of disjoint sets appear $w-1$ times at $X_1, X_2, X_3, \dots, X_w$ where x is an integer $1 \leq x$ which is a contradiction, or there is at least onefamily of disjoint sets χ_i such that $w+1 \leq t_i$, therefore m is minimum number offamilies of disjoint sets.

Step III: Let $\sum_{j=1}^w d(X_j) = wq_0 + (w-1)q_1 + (w-2)q_2 + r$ then we have $\sum_{j=1}^w d(X_j) = wq_0 + (w-1)q_1 + (w-2)q_2 + r \leq wn$

and $q_0 + q_1 + q_2 + 1 = m$ if $r \neq 0$ and $q_0 + q_1 + q_2 = m$ if $r = 0$, and supposewe partition these k sets into $m-1$ families of disjoint sets, such that

$r = 0$, and supposewe partition these k sets into $m-1$ families of disjoint sets, such that $q_0 + q_1 + q_2 + 1 = m-1$ if $r \neq 0$ and $q_0 + q_1 + q_2 = m-1$ if $r = 0$, then either there is $q_0 + x$ families of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$ or there is $q_1 + x$ families of disjoint sets appear $w-1$ times or there is $q_2 + x$ families of disjoint sets appear $w-2$ times at $X_1, X_2, X_3, \dots, X_w$ where x is an integer $1 \leq x$ which is a contradiction, or there is at least onefamily of disjoint sets χ_i such that $w+1 \leq t_i$, therefore m is minimum number offamilies of disjoint sets. For the remaining steps where in last step

we have $\sum_{j=1}^w d(X_j) = wq_0 + (w-1)q_1 + (w-2)q_2 + (w-3)q_3 + \dots + (w-s)q_s + r$, in each step, the proof is same as in step

III. Therefore the result holds.

Proposition 5.7. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if we partition all sets (common sets and non common sets) into m families of disjoint sets, any family of disjoint sets appear not more than $w-1$ times at $X_1, X_2, X_3, \dots, X_w$ and if $n(w-1) + r = \sum_{j=1}^w d(X_j)$ where $2 \leq r \leq w-2$, then $n+1 \leq m$.

Proof: Let these k sets partitioned into number offamilies of disjoint sets equal m , if we consider appearance for anyfamily of disjoint sets at w intersections $X_1, X_2, X_3, \dots, X_w$, equal $w-1$, and since $n(w-1) + r = \sum_{j=1}^w d(X_j)$ therefore

$m = n+1$ and any two families of disjoint sets meet each other at $X_1, X_2, X_3, \dots, X_w$ and if we consider maximum appearance for anyfamily of disjoint sets less than $w-1$ and equal $w-t$, where $\frac{w+1}{2} \leq w-t < w-1$ for w be odd, and

$\frac{w}{2} + 1 \leq w-t < w-1$ for w be even, and since $n(w-1) + r = \sum_{j=1}^w d(X_j)$ we have $n(w-t) + n(t-1) + r = n(w-1) + r = \sum_{j=1}^w d(X_j)$

since maximum appearance for any family of disjoint sets equal $w-t$ therefore $m = n+1$ if $1 \leq n(t-1) + r \leq w-t$, and $n+1 \leq m$ if $(w-t) + 1 \leq n(t-1) + r$. Therefore $n+1 \leq m$.

Proposition 5.8 Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if we partition all sets (common sets and non common sets) into m families of disjoint sets, such that m is minimum number families of disjoint sets. Let q_0 be number

offamilies of disjoint sets appear w times at $X_1, X_2, X_3, \dots, X_w$, where $0 \leq q_0 \leq 2$ and $n - q_0$ be maximum number

offamilies of disjoint sets appear $w-1$ times at $X_1, X_2, X_3, \dots, X_w$, if $wn - n + q_0 + r = \sum_{j=1}^w d(X_j)$ where

$2 \leq r < w-1$, then $n+1 = m$.

Proof: Let these k sets partitioned into number offamilies of disjoint sets equal m , using Proposition 3.19. we can write

$$wq_0 + (w-1)(n - q_0) + r = \sum_{i=1}^k c_i = \sum_{j=1}^w d(X_j) \quad \text{where } c_i \text{ number of times the set } A_i \text{ appear at } w \text{ proper}$$

intersections, since $wq_0 + (w-1)(n - q_0) + r = wn - n + q_0 + r$ then for minimum number offamilies of disjoint sets we have to take maximum appearance, suppose we have q_0 families each one appear w times, and $n - q_0$ families each one appear $w-1$ times, and one family appear r times, since $q_0 + (n - q_0) + 1 = n + 1$ then $n+1 = m$.

Using method of Definition 5.1. $m = n + 1$ is minimum number offamilies of disjoint sets. To prove m is minimum number of families of disjoint sets, suppose we partition these k sets into $m-1$ families of disjoint sets, suppose we distribute the family of disjoint sets appear r times at $X_1, X_2, X_3, \dots, X_w$ to families of disjoint sets each one appear w times, it is a contradiction, since there is no family of disjoint sets appear more than w times. Suppose we distribute a family of disjoint sets appear r times to families of disjoint sets each one appear $w-1$ times, it is a contradiction, since number of families of disjoint sets each one appear w times more than not more than q_0 , and there is no family of disjoint sets appear more than w times. Therefore m is minimum number of families of disjoint sets.

VI. ADDITION OF COMMON SETS AND NON COMMON SETS

In this section we suppose k common sets $A_1, A_2, A_3, \dots, A_k$, and w proper intersections $X_1, X_2, X_3, \dots, X_w$, of these k sets (standard of partition), with maximum degree of intersection equal n , these k sets partitioned into m families of disjoint sets. Here we introduce some results of addition of common sets and non common sets, such that the proper intersections of these k sets (standard of partition), and additional sets remains unchanged.

Proposition 6.1. Suppose we have k common sets $A_1, A_2, A_3, \dots, A_k$, and w intersections $X_1, X_2, X_3, \dots, X_w$, (all of these intersections are proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if all sets (common sets and non common sets) partitioned into m minimum families of disjoint sets, if we add only one non common sets, without changing maximum degree of intersection, and without addition of any proper intersection, such that the standard of partition of these $k+1$ sets remains $X_1, X_2, X_3, \dots, X_w$, only degree of one proper intersection change, then these $k+1$ sets partitioned into m minimum families of disjoint sets.

Proof: : Let these m minimum families of disjoint sets labeled by $1, 2, 3, \dots, m$, since standard of partition these $k+1$ sets remains $X_1, X_2, X_3, \dots, X_w$, and the additional noncommon sets appears once at one proper intersection X_j where $1 \leq j \leq w$, $d(X_j) = x$, $2 \leq x \leq n-1$, if $X_j \equiv (1, 2, 3, \dots, x)$ then after addition of the noncommon set we labelled X_j by $X_j \equiv (1, 2, 3, \dots, x, y)$ where $x+1 \leq y \leq m$, and additional noncommon set belongs to family of disjoint sets \mathcal{X}_y and $x+1 \leq y \leq m$, since $d(X_j) \leq n-1$ the additional noncommon set can belongs to one of the following families of disjoint sets $\mathcal{X}_{x+1}, \mathcal{X}_{x+2}, \mathcal{X}_{x+3}, \dots, \mathcal{X}_m$. Then these $k+1$ sets partitioned into m minimum families of disjoint sets.

Proposition 6.2. Suppose we have k common sets $A_1, A_2, A_3, \dots, A_k$, and w intersections $X_1, X_2, X_3, \dots, X_w$, (all of these intersections are proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if all sets (common sets and non common sets) partitioned into m minimum families of disjoint sets, if we remove only one non common sets, without changing maximum degree of intersection, and without removal of any proper intersection, such that the standard of partition after removal of a non common set remains $X_1, X_2, X_3, \dots, X_w$, only degree of one proper intersection changed, then after removal of a non common set these sets partitioned into m minimum families of disjoint sets.

Proof: : Let these m minimum families of disjoint sets labeled by $1, 2, 3, \dots, m$, since after removal a non common set standard of partition remains $X_1, X_2, X_3, \dots, X_w$, suppose belongs to family of disjoint sets χ_y , the removal of one non common set changes only one proper intersection. Let this proper intersection denoted by $X_j \equiv (1, 2, 3, \dots, x, y)$ where $2 \leq x \leq n-1$,

$1 \leq j \leq w$, $d(X_j) = x+1$ if $X_j \equiv (1, 2, 3, \dots, x, y)$ then after removal of the non common set w of disjoint sets χ_y and $x+1 \leq y \leq m$, then after removal of the non common set $d(X_j) \leq n-1$ and if $w+1 \leq d(X_j) + d(X_i)$ where $i \neq j, 1 \leq i \leq m$ then the result holds. Suppose after removal of the non common set $d(X_j) + d(X_i) \leq w$, a contradiction since before removal $w+1 \leq d(X_j) + d(X_i)$ where holds for common sets, see remark 3.30. Then after removal of a non common set $w+1 \leq d(X_j) + d(X_i)$. Therefore after removal of a non common set these sets partitioned into m minimum families of disjoint sets.

Proposition 6.3. Suppose we have k common sets $A_1, A_2, A_3, \dots, A_k$, and w intersections $X_1, X_2, X_3, \dots, X_w$, (all of these intersections are proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , if all sets (common sets and non common sets) partitioned into m minimum families of disjoint sets, then addition of q non common sets, without changing maximum degree of intersection, and without addition of any proper intersection, such that the standard of partition of these $k+q$ sets remains $X_1, X_2, X_3, \dots, X_w$, only degree of some proper intersection change, then these $k+q$ sets partitioned into m minimum families of disjoint sets.

Proof: : Let these m minimum families of disjoint sets labeled by $1, 2, 3, \dots, m$, since standard of partition these $k+q$ sets remains $X_1, X_2, X_3, \dots, X_w$, and any non common sets appear once at one proper intersection then any additional non common sets appear at the intersection X_j where $1 \leq j \leq w$, $d(X_j) = x$, $2 \leq x \leq n-1$, if $X_j \equiv (1, 2, 3, \dots, x)$ then any additional non common set labelled by one element of the set $\{x+1, x+2, x+3, \dots, m\}$. Case one: we add q non common set appear at X_j such that any two non common sets out of these q non common sets have two different labels, where $1 \leq q \leq y$, $1 \leq y \leq n-2$, $3 \leq x+y \leq n$, we denote X_j after addition of q sets by X_q , from definition of non common set, any non common set related only to one proper intersection, we can label X_q by $X_q \equiv (1, 2, 3, \dots, x, x+1, x+2, x+3, \dots, x+q)$, since $n \leq m$ then these $k+q$ sets partitioned into m families of disjoint sets. Second let $y < q$, and let $q_1 + q_2 + q_3 + \dots + q_s = q$, such that for all i we have $q_i \leq y$ where $1 \leq i \leq s$, which satisfies for all i conditions in case one, we can label X_{q_i} by $X_{q_i} \equiv (1, 2, 3, \dots, x, x+1, x+2, x+3, \dots, x+q_i)$, since $n \leq m$ then these $k+q_i$ sets partitioned into m minimum families of disjoint sets. Therefore the result holds.

Proposition 6.4. Suppose we have k common sets $A_1, A_2, A_3, \dots, A_k$, and w intersections $X_1, X_2, X_3, \dots, X_w$, all of these intersections are proper intersections of these k sets (standard of partition), with maximum degree of intersection equal n , and there is at least one intersection only of common sets with degree of intersection equal n , if there are q non common sets, if these $k+q$ sets partitioned into m minimum families of disjoint sets, then after removal of these q non common sets, all sets (common sets and non common sets) partitioned into m minimum families of disjoint sets.

Proof: Step I: Here conditions of Proposition 6.2. satisfied, suppose we remove one non common set, using Proposition 6.2, these k sets partitioned into m minimum families of disjoint sets.

Step II: Conditions of Proposition 6.2. satisfied, suppose we remove the second non common set, using Proposition 6.2, these k sets partitioned into m minimum families of disjoint sets. Continue in this process till last step (Step q)

Step q : Conditions of Proposition 6.2. satisfied, suppose we remove the last non common set, using Proposition 6.2, these k sets partitioned into m minimum families of disjoint sets.

VII. THE MAIN RESULT

Remark 7.1 Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets, with maximum degree of intersection equal n , if we partition these k sets into $n+1 = m$ minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, if there are two families of disjoint sets χ_i, χ_j such that $t_i + t_j \leq w$, then will not be standard of partition these $X_1, X_2, X_3, \dots, X_w$ will not be standard of partition these k sets into $n+1 = m$ minimum families of disjoint sets.

Proposition 7.2 Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets, with maximum degree of intersection equal n , if we partition all sets (common sets and non common sets) into $n+1 = m$ minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, and $X_1, X_2, X_3, \dots, X_w$ is not standard of partition these k sets into $n+1 = m$ minimum families of disjoint sets. If $X_1, X_2, X_3, \dots, X_t$ is standard of partition (with maximum degree of intersection equal n) $A_1, A_2, A_3, \dots, A_r$ into $n+1 = m$ minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ where $\{A_1, A_2, A_3, \dots, A_r\} \subset \{A_1, A_2, A_3, \dots, A_k\}$, and $\{X_1, X_2, X_3, \dots, X_t\} \subset \{X_1, X_2, X_3, \dots, X_w\}$, then the sets $A_1, A_2, A_3, \dots, A_k$ partition into $n+1 = m$ minimum families of disjoint sets.

Proof: Since $X_1, X_2, X_3, \dots, X_t$ be standard of partition $A_1, A_2, A_3, \dots, A_r$ into

$n+1 = m$ minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ such that

$$\chi_1 = \{A_{11}, A_{12}, A_{13}, \dots, A_{1r_1}\}$$

$$\chi_2 = \{A_{21}, A_{22}, A_{23}, \dots, A_{2r_2}\}$$

$$\chi_3 = \{A_{31}, A_{32}, A_{33}, \dots, A_{3r_3}\}$$

$$\chi_m = \{A_{m1}, A_{m2}, A_{m3}, \dots, A_{mr_m}\}$$

From definition of standard of partition we have $t+1 \leq r_i + r_j$, $r_l \leq t$ where $r_1 + r_2 + r_3 + \dots + r_m = r$, $1 \leq i < j \leq m$, $1 \leq l \leq m$.

Since $\{A_1, A_2, A_3, \dots, A_r\} \subset \{A_1, A_2, A_3, \dots, A_k\}$ we can write

$$\chi_1 = \{A_{11}, A_{12}, A_{13}, \dots, A_{1r_1}\} \subseteq \{A_{11}, A_{12}, A_{13}, \dots, A_{1k_1}\}$$

$$\chi_2 = \{A_{21}, A_{22}, A_{23}, \dots, A_{2r_2}\} \subseteq \{A_{21}, A_{22}, A_{23}, \dots, A_{2k_2}\}$$

$$\chi_3 = \{A_{31}, A_{32}, A_{33}, \dots, A_{3r_3}\} \subseteq \{A_{31}, A_{32}, A_{33}, \dots, A_{3k_3}\}$$

$$\chi_m = \{A_{m1}, A_{m2}, A_{m3}, \dots, A_{mr_m}\} \subseteq \{A_{m1}, A_{m2}, A_{m3}, \dots, A_{mk_m}\}$$

$X_1, X_2, X_3, \dots, X_w$ is not standard of partition these k sets, therefore $w+1 \leq k_i + k_j$ not holds for all i, j where

$$1 \leq i < j \leq m, k_l \leq w, 1 \leq l \leq m, \text{ and } k_1 + k_2 + k_3 + \dots + k_m = k.$$

Then the sets $A_1, A_2, A_3, \dots, A_k$ partition into $n+1 = m$ minimum families of disjoint sets.

Proposition 7.3 Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_w$ be all proper intersections of these k sets, with maximum degree of intersection equal n , if we partition all sets (common sets and non common sets) into m minimum families of disjoint sets, then $n+1 = m$ if and only if either $X_1, X_2, X_3, \dots, X_w$ be standard of partition for these

k sets into m minimum families of disjoint sets, and $wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^w d(X_j)$

such that $q_0 + q_1 + q_2 + \dots + q_s = m$ or $X_1, X_2, X_3, \dots, X_t$ is standard of partition for $A_1, A_2, A_3, \dots, A_r$ with maximum degree of intersection equal n , where $\{A_1, A_2, A_3, \dots, A_r\} \subset \{A_1, A_2, A_3, \dots, A_k\}$,

$\{X_1, X_2, X_3, \dots, X_t\} \subset \{X_1, X_2, X_3, \dots, X_w\}$, $wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^t d(X_j)$ and

$$q_0 + q_1 + q_2 + \dots + q_s = m = n + 1.$$

Proof: First let $n + 1 = m$, we have two cases: (I) suppose $X_1, X_2, X_3, \dots, X_w$ form standard of partition k common sets into $n + 1 = m$ minimum families of disjoint sets, therefore

$$wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^w d(X_j) \text{ such that } q_0 + q_1 + q_2 + \dots + q_s = m$$

where $n + 1 = m$, therefore the result holds. (II) suppose $X_1, X_2, X_3, \dots, X_w$ can not form standard of partition k common sets into $n + 1 = m$ minimum families of disjoint sets, using Proposition 7.2. there exists $X_1, X_2, X_3, \dots, X_t$ is standard of partition for $A_1, A_2, A_3, \dots, A_r$ with maximum degree of intersection equal n , where $\{A_1, A_2, A_3, \dots, A_r\} \subset \{A_1, A_2, A_3, \dots, A_k\}$, $\{X_1, X_2, X_3, \dots, X_t\} \subset \{X_1, X_2, X_3, \dots, X_w\}$,

$$wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^t d(X_j) \text{ and } q_0 + q_1 + q_2 + \dots + q_s = m = n + 1. \text{ Therefore}$$

the result holds.

Conversely suppose $X_1, X_2, X_3, \dots, X_w$ or $\{X_1, X_2, X_3, \dots, X_t\}$ form standard of partition common sets into m minimum

families of disjoint sets, and $wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^w d(X_j)$ such that

$$q_0 + q_1 + q_2 + \dots + q_s = m \text{ where } n + 1 = m, \text{ or}$$

$$wq_0 + (w-1)q_1 + (w-2)q_2 + \dots + (w-s)q_s = \sum_{j=1}^t d(X_j) \text{ and } q_0 + q_1 + q_2 + \dots + q_s = m = n + 1.$$

Therefore $X_1, X_2, X_3, \dots, X_w$ or $\{X_1, X_2, X_3, \dots, X_t\}$ satisfy method of construction minimum number of families of disjoint sets, using Proposition 5.6. therefore $n + 1 = m$ is minimum families of disjoint sets.

VIII. CONCLUSION

This paper about general type of colouring, it is generalization of edge colouring, vertex colouring, face colouring. The concept of common set differentiate between edge colouring and other types of colouring (vertex colouring and face colouring). It explain edge colouring is special case of colouring. More detail in papers [18,19,20].

REFERENCES

- [1] Hassan, M.E., "Family of Disjoint Sets and its Applications". IJIRSET Vol. 7. Issue 1, Jan 2018, 362-393.
- [2] Hassan, M.E., "Colouring of Graphs Using Colouring of Families of Disjoint Sets Technique". IJIRSET Vol. 7. Issue 10, October 2018, 10219-10229. October
- [3] Hassan, M.E., "Types of Colouring and Types of Families of Disjoint Sets". IJIRSET Vol. 7. Issue 11, Nov 2018, 362-393.
- [4] Hassan, M.E., "Colouring of finite Sets and Colouring of Edges finite Graphs". IJIRSET Vol. 7. Issue 12, Dec 2018, 11663-11675.
- [5] Hassan, M.E., "Sorts of Colour Classes and Sorts of Families of Disjoint Sets". IJIRSET Vol. 8. Issue 1, Jan 2019, 56-63.
- [6] Hassan, M.E., "Trivial Colouring and Non-Trivial Colouring for Graph's Edges". IJIRSET Vol. 8. Issue 2, Feb 2019, 1014-1024.
- [7] Hassan, M.E., "Some Results of Edge Colouring Using Family of Disjoint Colouring Technique". IJIRSET Vol. 8. Issue 4, April 2019, 4667-4675.



- [8] Hassan, M.E., " Trivial Colouring and Non-Trivial Colouring for Graph's Vertices".IJIRSET Vol. 8. Issue 6, June2019, 7398-7410.
- [9] Bondy, J. A., Murty, S. R., "Graph Theory with Applications". Macmillan Press 1967 & Elsevier, 1967.
- [10] Wilson, R., "Introduction to Graph Theory". Fourth Edition, Addison Wesley Longman Limited, England, (1996).
- [11] Wilson, R., " Four Colors Suffice : How the Map Problem Was Solve"., Princeton, NJ: Princeton University Press, 2004.
- [12] Reinhard Diestel, " Graph Theory". Electronic 2000, Springer-Verlag New York 1997, 2000.
- [13] Frank Harary, " Graph Theory". Addison-Wesley Publication Company, Inc. 1969.
- [14] Douglas B West, "Introduction to Graph Theory". Second Edition, Department of Mathematics Illinois University, (2001).
- [15] Thomas, R. "An Update on the Four-color Theorem" Notices Amer. Math. Soc. (45) no. 7, pp.848-859, (1998).
- [16] Robertson, N., Sanders, D., Seymour, P., and Thomas, R., "A New Proof of the Four-Colour Theorem", Electronic Research Announcements of the American Mathematical Society 2(1) pp.84-96, (1996).
- [17] Gonthier, G., "Formal Proof-The Four Color Theorem", Notices of the AMS, Vol. (55) No.11, pp.1382-1393, (2008).
- [18] Hassan, M.E., " Families of Disjoint Sets Colouring Technique and Concept of Common and Non Common edge". (To appear).
- [19] Hassan, M.E., " Families of Disjoint Sets Colouring Technique and Concept of Common and Non Common vertex". (To appear).
- [20] Hassan, M.E., " Families of Disjoint Sets Colouring Technique and Concept of Common and Non Common face". (To appear).



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