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Free Convection Laminar flow of Viscoelastic Fluid Past a Porous Vertical Wall with Exponentially Decreasing Time Dependent Suction

Dr. Lakhbir Singh

Associate Professor of Mathematics, Govt. P. G. College, Hisar, India

ABSTRACT: In the present study, the two-dimensional incompressible visco-elastic fluid (Rivlin-Ericksen) flow has been considered by applying the time dependent suction to free convection past a vertical wall. A case of exponentially decreasing suction has been discussed in detail as a particular case. The expressions for velocity distribution, temperature distribution, skin friction and the rate of heat transfer from the wall to the fluid have been obtained.

Nomenclature

k	Permeability parameter
g	Acceleration due to gravity
N	Nusselt number
P	Pressure
q	Heat flux per unit area
T	Temperature in the boundary layer
T_w	Temperature at the wall
T_∞	Temperature of the free stream
t	Time
τ	Skin friction
u	Velocity in the x -direction
U_∞	Velocity of the free stream
v	Velocity in y -direction
w	Velocity in z -direction
x, y	Coordinate system
X, Y, Z	Component of the body force in x, y and z directions respectively
ρ	The density
σ	Prandtl number
θ	Dimensionless temperature
ν	The kinematic cross viscosity
β	Coefficient of Kinematics visco-elasticity

Formulation of the problem

The equations describing the unsteady free convection flow of a visco-elastic incompressible fluid are given by

$$\frac{\partial \bar{u}}{\partial t} + \bar{v} \frac{\partial \bar{u}}{\partial y} = g \beta_1 (\bar{T} - \bar{T}_\infty) + \alpha \frac{\partial^2 \bar{u}}{\partial y^2} + \beta \frac{\partial^2}{\partial y^2} \left(\frac{\partial \bar{u}}{\partial t} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) \quad (1)$$

$$\frac{\partial \bar{v}}{\partial t} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} + 2(2\beta + \nu) \frac{\partial \bar{u}}{\partial y} \frac{\partial^2 \bar{u}}{\partial y^2} \quad (2)$$

$$\frac{\partial \bar{v}}{\partial y} = 0 \tag{3}$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{v} \frac{\partial \bar{T}}{\partial y} = k \frac{\partial^2 \bar{T}}{\partial y^2} \tag{4}$$

Here the equations (1) and (2) are the equations of motion. The equations (3) and (4) are the equation of continuity and energy respectively.

Let the suction velocity normal to the wall be function of time as

$$\bar{v} = -\bar{v}_0 (1 + \varepsilon A e^{-nt}), \quad \varepsilon A \leq 1 \tag{5}$$

Introducing the following non-dimensional quantities

$$y = \frac{\bar{y}}{L}, \quad t = \frac{\alpha \bar{t}}{L^2}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{\bar{v}L}{\alpha} = \frac{|\bar{v}|}{|\bar{v}_0|}$$

$$p = \frac{\bar{p}L^2}{\rho\alpha^2}, \quad T = \frac{g\beta_1 L^3 (\bar{T} - \bar{T}_\infty)}{\alpha^3} \tag{6}$$

The equations (1) and (4) with the help of (5) and (6) take the following form

$$\frac{\partial U}{\partial t} - (1 + \varepsilon A e^{-nt}) \frac{\partial U}{\partial y} = T + \frac{\partial^2 U}{\partial y^2} + S_1 \frac{\partial^2}{\partial y^2} \left\{ \frac{\partial U}{\partial t} - (1 + \varepsilon A e^{-nt}) \frac{\partial U}{\partial y} \right\} \tag{7}$$

$$\frac{\partial T}{\partial t} - (1 + \varepsilon A e^{-nt}) \frac{\partial T}{\partial y} = \frac{1}{\sigma} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

respectively, where σ is the Prandtl number and $S_1 = \beta / L^2$.

Solution of the Problem

The relevant boundary conditions of the problem are

$$U(0, t) = 0, \quad V(0, t) = -F(t), \quad T = \theta(t) \text{ when } y = 0$$

and

$$U(\infty, t) \rightarrow 1, \quad \frac{\partial U}{\partial y} \rightarrow 0, \quad T \rightarrow 0, \text{ when } y = \infty \tag{9}$$

For solving equation (7) and (8) subject to boundary conditions (9), we assume that

$$U(y, t) = U_0(y) + U_1(y) \varepsilon e^{-nt} \tag{10}$$

and

$$T(y, t) = T_0(y) + T_1(y) \varepsilon e^{-nt} \tag{11}$$

After putting (10) and (11) in equation (7) and (8), and equating to zero the exponential term, we get

$$S_1 U_0''' - U_0'' - U_0' = T_0 \tag{12}$$

$$\frac{T_0''}{\sigma} = -T_0' \tag{13}$$

$$S_1 U_1''' - (1 - nS_1)(U_1'' - U_1' - nU_1) = T_1 - S_1 A U_0'' + A U_0 \tag{14}$$

$$\frac{T_1''}{\sigma} + T_1' + nT_1 = -AT_0' \tag{15}$$

The boundary conditions (9) reduce to

$$\left. \begin{aligned} U_0 = U_1 = 0 & \quad \text{when } y = 0 \\ U_0 \rightarrow 1, U_1 \rightarrow 0 \\ U_0' \rightarrow 0, U_1' \rightarrow 0 \end{aligned} \right\} \text{ when } y = \infty \tag{16}$$



and

$$\begin{aligned} T_0 &= \theta_0, \quad T_1 = \theta, \quad \text{when } y = 0 \\ T_0 &\rightarrow 0, \quad T_1 \rightarrow 0, \quad \text{when } y = \infty \end{aligned} \tag{17}$$

The solution of equation (13) with their relevant boundary conditions are

$$T_0(y) = \theta_0 e^{-\sigma y} \tag{18}$$

Since S_1 is negative, so substituting $S_1 = -k_0$, then the equation (12) takes the form

$$k_0 U_0''' + U_0'' + U_0' = -\theta_0 e^{-\sigma y} \tag{19}$$

The solution of equation (19) with boundary conditions (16) and $0 \leq k_0 \leq \frac{1}{4}$ is

$$U_0(y) = 1 + B_1 e^{-H_1 y} + B e^{-\sigma y} \tag{20}$$

where

$$\begin{aligned} H_1 &= \frac{1}{2} \left[\frac{1 - \sqrt{1 - 4k_0}}{k_0} \right], \quad B = \frac{\theta_0}{\sigma(k_0 \sigma^2 - \sigma + 1)}, \\ B_1 &= -[1 + B] \end{aligned} \tag{21}$$

Now solution of equation (15) subject to boundary conditions (17) is

$$T_1(y) = (\theta_1 - M) e^{-H_2 y} + M e^{-\sigma y} \tag{22}$$

where

$$H_2 = \frac{\sigma}{2} - \left\{ 1 + \sqrt{1 - \frac{4n}{\sigma}} \right\}, \quad M = \frac{A\sigma\theta_0}{n} \tag{23}$$

with these equations (14) leads to

$$k_0 U_1''' + (1 + nk_0) U_1'' + U_1' + n U_1 = k_1 e^{-H_1 y} + k_2 e^{-H_2 y} + k_3 e^{-\sigma y} \tag{24}$$

where

$$\begin{aligned} k_1 &= k_0 A H_1^3 B_1 + A H_1 B_1, \\ k_2 &= M - \theta_1, \\ k_3 &= k_0 A \sigma^3 B + A \sigma B - M. \end{aligned} \tag{25}$$

The solution of equation (24) subject to equation (16) is given by

$$U_1(y) = N e^{-m_1 y} + P e^{-H_1 y} + Q e^{-H_2 y} + R e^{-\sigma y} \tag{26}$$

where

$$\begin{aligned} N &= -(P + Q + R), \quad P = \frac{k_1}{k_0 H_1^3 - (1 + nk_0) H_1^2 + H_1 - n} \\ Q &= -\frac{k_2}{k_0 H_2^3 - (1 + nk_0) H_2^2 + H_2 - n}, \\ R &= -\frac{k_3}{k_0 \sigma^3 - (1 + nk_0) \sigma^2 + \sigma - n}. \end{aligned} \tag{27}$$

where $-m_1$ being the root of the following cubic equation

$$k_0 m^3 + (1 + nk_0) m^2 + m + n = 0.$$

The expression for velocity and temperature distribution under the boundary layer are given by

$$U(y,t) = 1 + B_1 e^{-H_1 y} + B e^{-\sigma y} + \varepsilon e^{-nt} [N e^{-m_1 y} + P e^{-H_1 y}] + Q e^{-H_2 y} + R e^{-\sigma y} \quad (28)$$

and

$$T(y,t) = \theta_0 e^{-\sigma y} + \varepsilon e^{-nt} [(\theta_1 - M) e^{-H_2 y} + M e^{-\sigma y}] \quad (29)$$

Skin Friction and Rate of Heat Transfer

Here we shall have the expressions for skin friction and rate of heat transfer

$$\begin{aligned} D &= \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0} \\ &= \frac{\mu \alpha}{L^2} \left(\frac{\partial U}{\partial y} \right)_{y=0} \\ D &= \frac{\mu \alpha}{L^2} [B_1 H_1 + \sigma B + \varepsilon e^{-nt} (m_1 N + P H_1 + Q H_2 + R \sigma)] \end{aligned} \quad (30)$$

$$\begin{aligned} q &= -\lambda \left(\frac{\partial \bar{T}}{\partial \bar{y}} \right)_{\bar{y}=0} \\ &= -\frac{\lambda \alpha^2}{g \beta_1 L^4} \left(\frac{\partial T}{\partial y} \right)_{y=0} \\ &= \frac{\lambda \alpha^2}{g \beta_1 L^4} \left\{ \theta_0 \sigma + \varepsilon e^{-nt} [(\theta_1 - M) H_2 + M \sigma] \right\} \end{aligned} \quad (31)$$

Conclusion

It is evident from the above expression that the rate of heat transfer decreases from the wall to the fluid and becomes a finite quantity when $t \rightarrow \infty$.

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