

e-ISSN: 2319-8753 | p-ISSN: 2320-6710

DIA

International Journal of Innovative Research in SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

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IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 10, Issue 7, July 2021

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.569

9940 572 462

6381 907 438

0

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| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569|

|| Volume 10, Issue 7, July 2021 ||

| DOI:10.15680/IJIRSET.2021.1007070 |

Analysis of Stiffened Rectangular Plate

Purnima M. Bawankar¹, Prof. H. Rangari²

PG Student, Department of Civil Engineering, V M Institute of Engineering & Technology, Nagpur, Maharashtra,

India¹

Assistant Professor, Department of Civil Engineering, V M Institute of Engineering & Technology, Nagpur,

Maharashtra, India²

ABSTRACT- Observations of structures created by nature indicate that in most cases strength and rigidity depend not only on the material but also upon its form. This fact was probably noticed long ago by some shrewd observers and resulted in the creation of artificial structural elements having high bearing capacity mainly due to their form such as girders, arches and shells. Adding stiffeners to the plate complicates the analysis and several assumptions must be made in order to facilitate a solution to the problem. In the present study, a simulation model for large deflection static analysis of a thin rectangular stiffened plate under transverse loading has been presented. The mathematical formulation is based on variational form of energy principle, where the displacement fields are assumed as finite linear combination of admissible orthogonal functions, satisfying the corresponding flexural and membrane boundary conditions of the plate. Geometric nonlinearity has been incorporated through consideration of nonlinear straindisplacement relationship. The resulting nonlinear set of governing equations is solved through a numerical procedure involving direct substitution method using relaxation parameter. Entire computational work is carried out in a normalized square domain, which is divided into subdomains depending on the number, orientation and location of the stiffeners. The domain decomposition technique is specially developed to introduce adequate number of computation points around the location of stiffeners and boundaries of the plate. Validation of the present method and solution technique has been carried out successfully with available results. Various results are presented using different types of stiffeners attached to plate. Different results have also been furnished for variation of stiffener location.

KEYWORDS- Rectangular Plate, Stiffeners, Nonlinear Set

I. INTRODUCTION

1.1. INTRODUCTION TO STIFFENED PLATE

Man has always been inspired from the nature be it art or engineering. Perhaps one of the derivatives of such inspiration is stiffened engineering structures. Sea shells, leaves, trees, vegetables - all of these are in fact stiffened structures. Observations of structures created by nature indicate that in most cases strength and rigidity depend not only on the material but also upon its form. This fact was probably noticed long ago by some shrewd observers and resulted in the creation of artificial structural elements having high bearing capacity mainly due to their form such as girders, arches and shells.



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(Fig.1.1: Girder, Stiffener, Plate)



(Fig.1.2 : Rectangular Stiffeners Placed Along Rectangular Plate)

1.2. ADVANTAGE

Stiffeners in a stiffened plate make it possible to sustain highly directional loads, and introduce multiple load paths which may provide protection against damage and crack growth under the compressive and tensile loads. The biggest advantage of the stiffeners is the increased bending stiffness of the structure with a minimum of additional material, which makes these structures highly desirable for loads and destabilizing compressive loads. In addition to the advantages already found in using them, there should be no doubt that stiffened plates designed with different techniques bring many benefits like reduction in material usage, cost, better performance, etc.

1.3. APPLICATIONS

The wide use of stiffened structural elements in engineering start in the nineteenth century: mainly with the application of steel plates for the hulls of ships and with development of steel bridges and aircraft structures. Stiffened plates



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mostly find applications in the modern industry. Stiffened plates have wide applications in different field, like civil engineering, aerospace and marine structures. They are used in cube girders, plate's girders, ship hulls and wing structures. In aerospace and marine constructions, where minimization of weight of the components is of paramount interest, stiffened plates find extensive application. They are used in off-shore constructions like oil rigs, marine constructions such as ship and submarine hulls, decks and bridges of ships. In aerospace applications, aircraft wings are made out of stiffened plates. Apart from that they are also utilized in making advanced rocket launching structures. Stiffened plates are of common occurrence in the field of highway bridges and elevated roadways, where they are generally employed to build the bridge decks. Among land based structures, lock gates, box girders, plate girders etc. are also some of the fields of application for such plates. In some building structures floor slab systems are often made out of these components in order to avoid uncomfortable vibrations. There are some examples of stiffened plates being used in railway wagons, cargo containers, goods vehicle sidewalls etc.

II. LITERATURE REVIEW

Research into stiffened plates has been a subject of interest for many years. Extensive efforts by many researchers were devoted to investigate the response of the structure. However, due to its complexity and the many parameters involved, a complete understanding of all aspects of behavior is still not fully realized. The research done on stiffened plates can be classified into two categories, analysis and design. The objectives of researchers in the first category are to develop numerical or analytical formulations to predict the global and local buckling load of the structure. In doing so, several assumptions are postulated in idealizing the structure in order to facilitate a solution. In the following paragraphs a brief review of some of the relevant research work has been provided.

Klitchief and Belgrade [1] analyzed the stability of infinitely long, simply supported, transverse stiffened plates under uniform compression and lateral load. An extreme motivation of the work was to assess for design rules used in naval architecture. Even though their objective was to analyze eccentric stiffeners. Their approach appears to be valid only for the concentric case. In the eccentric case, difficulties appear over the concentric configuration in the coupling between the in-plane and out-of-plane displacements, by which results in an increase of the order of the differential equations for the structure which has been ignored in the analysis of their solution. Hoppmann [2] and Hoppmann and Baltimore [3] used an orthotropic plate approach for analyzing simply supported orthogonally stiffened plates under static and dynamic loading. The plate rigidities and stiffness were determined experimentally.

Soper [4] investigated a large deflection analysis for laterally loaded orthotropic plates using Levy's approach. An approximated the stress functions by a trigonometric series and solved the resulting set of non-linear equations numerically. He presented numerically results for simply supported and clamped edges beam. The beam model presented in [3] investigated for the ydirection strains which are neglected in conventional type modeling [4]. These models are compatible and most preferable for the analysis of stiffened laminated plates. This journal paper is concerned with the application of plate and beam finite elements in the analysis of the nonlinear response of eccentrically stiffened laminated plates. For large deformations, the von Karman kinematic relations of the plate and stiffener are considered in the formulation. The formulation analyze for transverse shear deformation effects of the plate and stiffener, and the model is approximately designed for the analysis of both thin and thick stiffened laminated plates. The finite element model consisting of representing plate by nine-noded isoparametric quadrilateral elements and the stiffener by three-noded isoparametric beam elements. Numerical evaluation and examples are presented for the non-linear responses of stiffened laminated plates.

Wah [5] used an energy model approach to analyze equally spaced, concentric stiffeners with identical crosssectional properties. At a first stage, a numerical procedure for the computational evaluation of the fundamental frequency is presented. The strain energy of the designed plate/stiffener elements is derived in terms of generalized in- and out-of-plane displacement functions and Mathematical Programming is used to determine the lowest natural frequency. The prediction of a described algorithm is verified with other numerical procedures like finite-element, finite-strip and finite-difference methods. Results are then presented, by showing the influence of the plate/stiffener geometric parameters on the fundamental frequency of structure with different concentric and eccentric stiffening configurations.

Wittrick [6] pointed out an alternative procedure for modeling the structure. In this approach, the plate and the stiffeners are treated as a series of long, flat strips, rigidly connected at their edges. A sinusoidal distribution of



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forces and moments is assumed in the longitudinal direction for each plate/stiffener. By solving the imposing differential equation of each plate element, a sinusoidal stiffness matrix of undetermined coefficient for each plate is obtained. By equating the edges displacement of each panel, the problem again reduces to finding the solution of the determinant which provides the natural frequency.

Kagan and Kubo [7] presented an elasto-plastic approach to analyze perfect, laterally loaded, orthogonally stiffened plates. The plate was idealized as an elastic perfectly plastic and yielding was considered to initiate and extend in the stiffeners. The equilibrium differential and coupled compatibility equations were solved numerically using a finite difference scheme for the elastic and elasto-plastic ranges. They evaluated plate rigidities by smearing out the stiffeners. Reduced values of the plate rigidities were used for each load increment in the post-yielding stage. Several finite element formulation have been presented for stability analysis of stiffened plates. In this case the plate panels are idealized by a series of inter-connected elements of plate. The Accuracy of the solution depends upon the shape functions derived and on the boundary conditions simulating compatibility along the element boundaries configuration. Several modeling strategies were used depending upon kinematical assumptions relates the displacements of the stiffeners to the middle surface of the plate and on the stiffeners are not equally spaced or have different profiles since discretization of the whole structure becomes unavoidable. Finite element applications for stability analysis of stiffened plates are reported by several researchers.

Long [8, 9] used the stiffness method of structural analysis for the computational evaluation of the natural frequency of simply supported plates stiffened in the longitudinal direction. They illustrated the method for the analysis of a plate with one longitudinal stiffener and a plate with one longitudinal and one transverse stiffeners.

Beside the orthotropic plate model several numerical algorithms have been formulated for the analysis of stiffened plates. Asku and Ali [10, 11] presented a numerical algorithm procedure for equally spaced stiffeners. The method is based on the variational principles in addition with finite difference techniques to determine the natural frequency of the structure. Free vibration characteristics of rectangular stiffened plates with a single stiffener have been examined by using the finite difference method. The variational technique has been formulated to minimize the total energy of the stiffened plate and the derivatives appearing in the energy functional are interchanged by finite difference equations. The energy functional is minimized with respect to discretized displacement components and natural frequencies and mode shapes of the stiffened plate have been determined as the solutions of a linear algebraic eigenvalue problem. The analysis takes into consideration inplane deformation of the plate and the stiffener and the effect of in plane inertia on the natural frequencies and mode shapes function. The effect of the ratio of stiffener depth to plate thickness for the natural frequencies of the stiffened plate has also been examined. Avent and Bounin [12] presented an discrete element approach to compute the elastic buckling of stiffened plates subjected to uniform longitudinal compression. These formulations were connected to simply supported plates with the equally spaced and equally sized stiffeners. In that analysis, using the equilibrium and compatibility conditions between the plate panel and the stiffener, a set of differential equations were obtained. Using the double Fourier series approximation, the buckling load was calculated by solving the resulting eigenvalue problem. The main advantage of this approach is the size of the eigenvalue problem is not depending of the number of ribs compared with other numerical methods. This is because of the confinement of the derivation to equally spaced and geometrically identical stiffeners.

III. PROPOSED METHODOLOGY

3.1 ANALYSIS OF STIFFENED RECTANGULAR PLATES

A stiffened plate with a single stiffener attached parallel to y-axis along with the notations for significant dimensions and coordinate system used for the present analysis is shown in Figure 4.1. It is assumed that the stiffeners are always parallel to the edges of the plate and they are rigidly connected to the plate. The mathematical formulation is further based on the following assumptions:

1. Plate and stiffener materials are isotropic, homogeneous, and linearly elastic.

2. Thicknesses of the plate and stiffener are uniform.

3. The thickness of the plate is sufficiently small compared to the lateral dimensions, so that the effect of shear deformation and rotary inertia may be neglected.



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569|

|| Volume 10, Issue 7, July 2021 ||

| DOI:10.15680/IJIRSET.2021.1007070 |



(Fig.3 : Geometry and dimension of Rectangular Plate with (a) Flat, (b) T Stiffener)

Figure 3: shows the geometry of flat stiffener and inverted-T stiffener. For the present analysis both the stiffeners are placed parallel to the Y-axis of the plate. Dimensions shown in Figure 3(a) are used for the validation of the result, which is carried out by comparing the generated results with those of Sheikh and Mukhopadhyay [25]. The dimensions of the plate are as follows, length (a) = 0.6, width (b) = 0.41, thickness (tp) = 0.0633. Inverted-T stiffener dimensions are as follows Flange dimension are Height (hf) = 0.008, Width (bf) = 0.017, web dimensions are Height (hw) = 0.028, Width (bw) = 0.005, all dimensions are in meter.

In the present formulation the physical domain is converted to computational domain by normalizing the mid-plane coordinates to dimensionless form as $\xi = x / a$, $\eta = y / b$. In this $\xi - \eta$ plane, gauss points are generated along the two orthogonal coordinate directions and the intersection of constant ξ and constant η lines passing through the gauss points provide the reference points for computation. In the present work 24 gauss points are generated along the two directions, these points are not evenly spaced. Such a grid does not ensure maximum number of computation points around the stiffener, which may be placed at any location inside the domain. Hence, in the present work a new method is employed to decompose the domain into sub-domains depending on the number, orientation and location of the stiffener. The sub-domains have their own gauss points around the location of stiffeners and boundaries so that the displacement field in these important locations can be captured accurately.

3.2 MATHEMATICAL FORMULATION

It is known from the principle of minimum potential energy that for a conservative system, of all the kinematically admissible displacement fields, the one corresponding to the stable equilibrium minimizes the total potential energy of the system. The above statement is expressed mathematically as, $\delta (U+V)=0$ (4.1)

Here, U is total strain energy stored in the system, which comprises of two components -strain energy of the plate (Up) and total strain energy stored in all the stiffeners (Us), and V is work function or potential of the external forces. In case of large displacement analysis with geometric nonlinearity both bending and stretching effects are taken into consideration. It is also well known that geometric nonlinearity in structural mechanics arises from nonlinear strain-displacement relations. So, strain energy of the plate (Up) comprises of two parts - strain energy due to pure bending (Ub) and strain energy due to stretching (Um) of its midplane. The expressions of Ub and Um are well known for rectangular plates and are indicated here for ready reference.



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$$U_{b} = \frac{D}{2} (ab) \int_{1}^{0} \int_{0}^{1} \left[\left\{ \frac{1}{a^{2}} \left(w_{,\xi\xi} \right) + \frac{1}{b^{2}} \left(w_{,\eta\eta} \right) \right\}^{2} + 2(1-\mu) \frac{1}{a^{2}b^{2}} \left\{ \left(w_{,\xi\xi} \right)^{2} + \left(w_{,\xi\xi} \right) (w_{,\eta\eta} \right) \right\} \right] d\xi d\eta$$

$$(4.2)$$

$$U_{m} = \frac{E_{p}t_{p}}{2(1-\mu^{2})} (ab) \int_{0}^{1} \int_{0}^{1} \left[\frac{1}{a^{2}} (u_{,\xi})^{2} + \frac{1}{a^{3}} (u_{,\xi}) (w_{,\xi})^{2} + \frac{1}{b^{2}} (v_{,\eta})^{2} + \frac{1}{b^{2}} (v_{,\eta}) (w_{,\eta})^{2} + \frac{1}{b^{2}} (w_{,\eta})^{2} \right]^{2} + 2\mu \left\{ \frac{1}{ab} (u_{,\xi}) (v_{,\eta}) + \frac{1}{2a^{2}b} (v_{,\eta}) (w_{,\eta})^{2} + \frac{1}{2b^{2}a} (u_{,\xi}) (w_{,\eta})^{2} \right\}^{2} + \frac{1-\mu}{2} \left\{ \frac{1}{b^{2}} (u_{,\eta})^{2} + \frac{2}{ab} (u_{,\eta}) (v_{,\xi}) + \frac{1}{a^{2}} (v_{,\xi})^{2} + \frac{2}{b^{2}a} (u_{,\eta}) (w_{,\xi}) (w_{,\eta}) + \frac{2}{a^{2}b} (v_{,\xi}) (w_{,\eta})^{2} \right\} + \frac{1-\mu}{2} \left\{ \frac{1}{b^{2}} (v_{,\xi}) (w_{,\xi}) (w_{,\eta}) \right\} d\xi d\eta$$

$$(4.3)$$

Here, Ep, μ and D = {Ept 3 p /12(1- μ 2)} are the elastic modulus, Poisson's ratio and the flexural rigidity of the plate, respectively. u, v and w represent the displacement fields of the plate along x, y and z directions, respectively. It is to be noted that u and v are in-plane deformation displacement whereas w is transverse body displacement or deflection. In the present analysis these three displacements (u, v and w) are the basic unknown variables. The expression for total strain energy stored in the stiffeners (Us) can be written as,

$$U_s = \sum_{p=1}^{n_{s_x}} U_{s_x}^p + \sum_{q=1}^{n_{s_y}} U_{s_y}^q \tag{4.4}$$

where, $U_{sx}^p U_{sy}^q$ are strain energies stored in *p*-th stiffener along *x*-direction and *q*-th stiffener along *y*-direction, respectively. In order to derive the expressions for strain energies of the individual stiffeners a compatible strain distribution at the line joining the plate and the stiffener is assumed.

Hence, the axial strain of a stiffener along x-direction is derived from the expression given by,

$$\varepsilon_{sx}^{p} = \varepsilon_{px} \mid_{z=t_{p}/2} - \left(z - \frac{t_{p}}{2}\right) w_{xx} - y_{s} v_{xx}$$

$$\tag{4.5}$$

Where, y_s denotes distance from the local minor axis. The plate strain at $z = t_p/2$ is denoted by $\varepsilon_{px}|_{z=t_p/2}$ and it is expressed as follows.

$$\varepsilon_{px}|_{z=t_p/2} = u_{x} + 0.5(w_{x})^2 - (t_p/2)w_{xx}$$
(4.6)

Substituting the expression for plate strain at $z = t_p/2$ into Eq. (4.5), the expression for total strain of an *x*-direction stiffener is obtained as,

$$\varepsilon_{sx}^{p} = u_{,x} + 0.5(w_{,x})^{2} - zw_{,xx} - y_{s}v_{,xx}$$
(4.7)



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So, the axial strain of a stiffener along x-direction includes axial strain due to bending action about major axis, stretching of the neutral axis and bending action about minor axis. It should be point out that the effect of shear deformation due to direct shear and torsion has not been taken into consideration while calculating the total axial strain. Substituting the total strain in the generalized expression of strain energy, $Usx p = (ES/2) \int (exp) 2 \, dV \cdot vol$ the final expression of the strain energy stored in an x-direction stiffener is obtained. Similarly, the strain energy expression of a y-direction stiffener can be obtained.

$$U_{sx}^{p} = \frac{E_{s}a}{2} \int_{0}^{1} \left[\frac{(l_{y}^{p} + A_{y}^{p}e_{x}^{p^{2}})}{a^{4}} (w_{\imath\xi\xi})^{2} + \frac{(l_{yz}^{p} + A_{y}^{p}(a\eta_{stf}^{p})^{2})}{a^{4}} (v_{\imath\xi\xi})^{2} \left\{ \frac{2}{a^{3}} (u_{\imath\xi}) (w_{\imath\xi\xi}) (w_{\imath\xi\xi}) + \frac{1}{a^{4}} (w_{\imath\xi\xi}) (w_{\imath\xi\xi})^{2} \right\} + A_{y}^{p} \left\{ \frac{1}{a^{2}} (u_{\imath\xi})^{2} + \frac{1}{4a^{4}} (w_{\imath\xi})^{4} + \frac{1}{a^{3}} (u_{\imath\xi}) (w_{\imath\xi})^{2} \right\} \right] d\xi \qquad (4.8)$$

$$U_{sy}^{q} = \frac{E_{s}b}{2} \int_{0}^{1} \left[\frac{(l_{x}^{p} + A_{x}^{q}e_{y}^{q^{2}})}{b^{4}} (w_{\eta\eta})^{2} + \frac{(l_{xz}^{q} + A_{x}^{q}(a\eta_{stf}^{q})^{2})}{b^{4}} (u_{\eta\eta\eta})^{2} - Q_{x}^{q} \left\{ \frac{2}{b^{3}} (v_{\eta\eta}) (w_{\eta\eta}) + \frac{1}{b^{4}} (w_{\eta\eta\eta}) (w_{\eta\eta})^{2} \right\} \right] d\eta \qquad (4.9)$$

Here, E_s is the elastic modulus of the stiffener material. $I_x^q = b_{sy}^q t_{sy}^{q^3}/12$, $I_y^p = b_{sx}^p t_{sx}^{p^3}/12$ $I_{xz}^q = b_{sx}^{q^3} t_{sy}^q/12$ and $I_{yz}^p = b_{sy}^p t_{sx}^p/12$ $I_{xIT}^q = (b_{sw}^q t_{sw}^{q^3} + b_{sf}^q t_{sf}^{q^3})/12$ $I_{yIT}^p = (b_{sxw}^p t_{sxw}^{p^3} + b_{sxf}^p t_{sxf}^{p^3})/12$ $I_{xIT}^q = (b_{sxw}^q t_{syw}^q + b_{sxf}^q t_{syf}^q)/12$ $I_{yZIT}^p = (b_{syw}^p t_{sxw}^p + b_{syf}^p t_{sxf}^p)/12$

These are moment of inertia about the major and minor axis of the stiffener cross-section, while the inertia terms containing 'IT' represent the moment of inertia of inverted-T stiffener and w, b represents dimensions of web and flange respectively. Qy p = Ax q ey, Qx q = Ay p ex p are the first moment of area about the plate mid-plane and Ax q, Ay p are the cross-sectional areas of the p-th x- and q-th y-direction stiffeners, respectively.

The total potential energy (V) due to externally applied uniformly distributed load of intensity p is expressed as follows,



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$$V = -(ab) \int_0^1 \int_0^1 (pw) d\xi d\eta$$
(4.10)

It should be mentioned here that the total potential energy corresponding to other type of transverse loading pattern p(x,y) expressible mathematically by analytical or numerical functions, can be determined from the above expression. Hence the applicability of the method is not limited to only uniformly distributed loading. In the expressions of the energy functions ξ and η are the dimensionless form of the mid-plane coordinates which are associated with the computational domain.

3.3 APPROXIMATE DISPLACEMENT FIELD

The plate displacement fields (w, u and v) can be depicted as linear combinations of orthogonal functions and unknown coefficients (di) as shown below.

$$w(\xi,\eta) = \sum_{i=1}^{nw} d_i \phi_i(\xi,\eta) \tag{4.11}$$

$$u(\xi,\eta) = \sum_{i=nw+1}^{nw+nu} d_i \alpha_{i-nw}(\xi,\eta)$$

$$(4.12)$$

$$v(\xi,\eta) = \sum_{i=nw+nu+1}^{nw+nu+nv} d_i \beta_{i-nw-nu}(\xi,\eta)$$
(4.13)

Here, (ξ, η) , $\alpha(\xi, \eta)$ and $\beta(\xi, \eta)$ are sets of *nw*, *nu* and *nv* numbers of orthogonal functions for *w*, *u* and *v*, respectively. The functions $\phi_i(\xi, \eta)$ are associated with displacements due to bending, whereas, $\alpha_i(\xi, \eta)$ and $\beta_i(\xi, \eta)$ describe stretching of the midplane of the plate. Appropriate start functions for these orthogonal sets are selected in such a way that they satisfy the flexural and membrane boundary conditions of the plate. The two dimensional (2-D) start functions are generated from one dimensional (1-D) functions corresponding to the two coordinate directions. The 1-D starting functions for transverse displacement are taken to be the beam deflection functions, derived from static deflection shape of the beam, corresponding to the boundary

condition of the plate along the particular coordinate axis. The higher-order functions are generated from the selected start functions following a two-dimensional implementation of the Gram-Schmidt orthogonalization scheme. It should be noted that the generated functions are for the total domain and they need to be broken down in terms of the subdomains. The sub-domains are represented by the sets of functions $\phi i m$, $\alpha i mn$ and $\beta i mn$ derived through interpolation of the whole field functions, where m = 1,...., nsy + 1 and n = 1,...., nsx + 1. The displacement fields associated with the plate presented in Eq. (4.11-4.13) are two dimensional in nature. But the stiffeners are one



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dimensional elements and hence the energy functional expressions related to them include single integrations (as shown in Eqs. (4.8) and (4.9). So, the plate displacement fields in their original form cannot be used in these expressions. To make the functions compatible with the stiffeners, plate displacement function is evaluated at the stiffener location (ξ stf, η stf) depending on the orientation of the stiffener. For example, in case of a y-direction stiffener the transverse displacement function is taken as, $(\xi, \eta) | \xi stf q = w(\xi stf q, \eta)$.

Flexural Boundary Condition	Start Function For $w(\phi_1(\xi,\eta))$
SSSS	${\sin(\pi\zeta)}{\sin(\pi\eta)}$
SCSS	$\{\sin(\pi\xi)\}$ { η^2 (3-5 η +2 η^2)}
CSCS	$\{\xi(1-\xi)\}^2\{\sin(\pi\eta)\}$
CCSS	$\{\xi^2(3-5\xi+2\xi^2)\}\{\eta^2(3-5\eta+2\eta^2)\}$
CCCS	$\{\xi(1-\xi)\}^2 \{\eta^2(3-5\eta+2\eta^2)\}$
CCCC	$\{\xi(1-\xi)\}^2 \{\eta(1-\eta)\}^2$

Table 4.1. Start Function for Displacement w for rectangular plate

3.4 GOVERNING SYSTEM OF EQUATION

Substituting the complete energy expressions along with the approximate displacement fields in Eq. (4.1) the set of system governing equations in matrix form is obtained as,

$$[(K\{d\})]\{d\} = \{f\}$$
(4.14)

Here, [K], {d} and {f} are stiffness matrix, vector of unknown coefficients and load vector, respectively. The dimensions of all the matrices and vectors are (nw + nu + nv). The total stiffness

matrix [K] of the system is given by,
$$[K] = [K_b] + [K_m] + \sum_{p=1}^{n_{S_x}} [K_{sx}]^p + \sum_{q=1}^{n_{S_y}} [K_{sy}]^q$$

Where, $[K_b]$ and $[K_m]$ are contributions from bending and stretching action of the plate, whereas $[K_{sx}]^p$ and $[K_{sy}]^q$ represent stiffness matrices of the *p*-th stiffener along *x*-direction and *q*-th stiffener along *y*-direction, respectively.



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| DOI:10.15680/IJIRSET.2021.1007070 |

$$\begin{bmatrix} K_b \end{bmatrix} = \begin{bmatrix} k_{11}^b & [0] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [0] \end{bmatrix}, \quad \begin{bmatrix} K_m \end{bmatrix} = \begin{bmatrix} k_{11}^m & k_{12}^m & k_{13}^m \\ k_{21}^m & k_{22}^m & k_{23}^m \\ k_{31}^m & k_{32}^m & k_{33}^m \end{bmatrix}, \quad \begin{bmatrix} K_{sx} \end{bmatrix} = \begin{bmatrix} k_{11}^{sx} & k_{12}^{sx} & k_{13}^{sx} \\ k_{21}^{sx} & k_{22}^{sx} & k_{23}^{sx} \\ k_{31}^{sx} & k_{32}^{sx} & k_{33}^{sx} \end{bmatrix} \text{ and } \begin{bmatrix} K_{sy} \end{bmatrix} = \begin{bmatrix} k_{11}^{sy} & k_{12}^{sy} & k_{13}^{sy} \\ k_{31}^{sy} & k_{32}^{sy} & k_{33}^{sy} \\ k_{31}^{sy} & k_{32}^{sy} & k_{33}^{sy} \end{bmatrix}$$

$$\begin{split} [K_{11}^{b}] &= \frac{E_{p}t_{p}^{3}(ab)}{12(1-\mu^{2})} \sum_{j=1}^{nw} \sum_{i=1}^{1} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{1}{a^{4}} (\phi_{i,\xi\xi}) (\phi_{j,\xi\xi}) + \frac{1}{b^{4}} (\phi_{i,\eta\eta}) (\phi_{j,\eta\eta}) + \frac{1}{a^{2}b^{2}} (\phi_{i,\xi\xi}) (\phi_{j,\eta\eta}) (\phi_{j,\xi\xi}) \right\} - \frac{(1-\mu)}{a^{2}b^{2}} (\phi_{i,\xi\xi}) (\phi_{j,\eta\eta}) + (\phi_{i,\eta\eta}) (\phi_{j,\xi\xi}) - 2(\phi_{i,\xi\eta}) (\phi_{j,\xi\eta}) \right] d\xi d\eta \end{split}$$

$$\begin{split} \left[K_{11}^{m}\right] &= \frac{E_{p}t_{p}^{3}(ab)}{12(1-\mu^{2})} \sum_{j=1}^{nw} \sum_{i=1}^{nw} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{1}{a^{4}} \left(\sum_{i=1}^{nw} d_{i} \phi_{i,\xi} \right)^{2} (\phi_{i,\xi}) (\phi_{j,\xi}) + \right. \\ &\frac{1}{b^{4}} \left(\sum_{i=1}^{nw} d_{i} \phi_{i,\eta} \right)^{2} (\phi_{i,\eta}) (\phi_{j,\eta}) + \frac{1}{a^{2}b^{2}} \left(\sum_{i=1}^{nw} d_{i} \phi_{i,\xi} \right)^{2} (\phi_{i,\eta}) (\phi_{j,\eta}) + \\ &\frac{1}{a^{2}b^{2}} \left(\sum_{i=1}^{nw} d_{i} \phi_{i,\eta} \right)^{2} (\phi_{i,\xi}) (\phi_{j,\xi}) \right\} - \frac{(1-\mu)}{a^{2}b^{2}} (\phi_{i,\xi\xi}) (\phi_{j,\eta\eta}) + (\phi_{i,\eta\eta}) (\phi_{j,\xi\xi}) - 2(\phi_{i,\xi\eta}) (\phi_{j,\xi\eta}) + \\ &\frac{2}{a^{3}} \left(\sum_{i=nw+1}^{nw+nu} d_{i} \alpha_{i-nw,\xi} \right)^{2} (\phi_{i,\xi}) (\phi_{j,\xi}) + \frac{2}{b^{3}} \left(\sum_{i=nw+nu+1}^{nw+nu+nv} d_{i} \beta_{i-nw-nu,\eta} \right)^{2} (\phi_{i,\xi}) (\phi_{j,\xi}) + \\ &\frac{2\mu}{a^{2}b} \left(\sum_{i=nw+1}^{nw+nu+1} d_{i} \beta_{i-nw-nu,\eta} \right)^{2} (\phi_{i,\xi}) (\phi_{j,\xi}) + \frac{2\mu}{b^{2}a} \left(\sum_{i=nw+1}^{nw+nu} d_{i} \alpha_{i-nw,\xi} \right)^{2} (\phi_{i,\eta}) (\phi_{j,\eta}) + \\ &\frac{(1-\mu)}{b^{2}a} \left(\sum_{i=nw+1}^{nw+nu} d_{i} \alpha_{i-nw,\eta} \right)^{2} (\phi_{i,\xi}) (\phi_{j,\eta}) + \frac{(1-\mu)}{b^{2}a} \left(\sum_{i=nw+1}^{nw+nu} d_{i} \alpha_{i-nw,\eta} \right)^{2} (\phi_{i,\eta}) (\phi_{j,\xi}) \right] d\xi d\eta \\ &\left[K_{12}^{m} \right] = \left[K_{13}^{m} \right] = 0 \end{split}$$

| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569| || Volume 10, Issue 7, July 2021 ||

| DOI:10.15680/IJIRSET.2021.1007070 |

$$\begin{split} \left[K_{21}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+1}^{nw+nu} \sum_{i=1}^{nw} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{1}{a^{3}} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\xi} \right) (\phi_{i,\xi}) (\phi_{j-nw,\xi}) + \frac{\mu}{ab^{2}} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\xi} \right) (\phi_{i,\eta}) (\alpha_{j-nw,\eta}) \right\} \right] d\xi d\eta \\ \left[K_{22}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+1}^{nw+nu} \sum_{i=1}^{nw} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{2}{a^{2}} (\phi_{i-nw,\xi}) (\phi_{j-nw,\xi}) + \frac{\mu}{ab^{2}} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\eta} \right) (\phi_{i,\eta}) (\alpha_{j-nw,\xi}) + \frac{(1-\mu)}{b^{2}} (\phi_{i-nw,\eta}) (\alpha_{j-nw,\eta}) \right\} \right] d\xi d\eta \\ \left[K_{31}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+nu+1}^{nw+nu+nv} \sum_{i=1}^{nw} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{1}{b^{3}} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\eta} \right) (\phi_{i,\eta}) (\beta_{j-nw-nu,\eta}) + \frac{\mu}{a^{2}b} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\eta} \right) (\phi_{i,\xi}) (\alpha_{j-nw-nu,\eta}) + \frac{\mu}{a^{2}b} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\eta} \right) (\phi_{i,\xi}) (\alpha_{j-nw-nu,\eta}) + \frac{(1-\mu)}{a^{2}b} \left(\sum_{i=1}^{nw} d_{i}\phi_{i,\eta} \right) (\phi_{i,\xi}) (\alpha_{j-nw-nu,\xi}) \right\} \right] d\xi d\eta \\ \left[K_{32}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+nu+1}^{nw+nv} \sum_{i=1}^{nw} \int_{0}^{1} \int_{0}^{1} \left[\left\{ 2\mu (\alpha_{i-nw,\xi}) (\beta_{j-nw-nu,\eta}) + (1-\mu) (\alpha_{i-nw,\eta}) (\beta_{j-nw-nu,\xi}) \right\} \right] d\xi d\eta \\ \left[K_{33}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+nu+1}^{nw+nv+nv} \sum_{i=nw+nu+1}^{nw+nv+nv} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{2}{b^{2}} (\beta_{i-nw-nu,\eta}) (\beta_{j-nw-nu,\eta}) + \frac{(1-\mu)}{a^{2}} (\beta_{i-nw-nu,\eta}) (\beta_{j-nw-nu,\eta}) \right\} \right] d\xi d\eta \\ \left[K_{33}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+nu+1}^{nw+nv+nv+nv+nv+nv+nv+nv} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{2}{b^{2}} (\beta_{i-nw-nu,\eta}) (\beta_{j-nw-nu,\eta}) + \frac{(1-\mu)}{a^{2}} (\beta_{i-nw-nu,\xi}) (\beta_{j-nw-nu,\xi}) \right\} \right] d\xi d\eta \\ \left[K_{33}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+nu+1}^{nw+nv+nv+nv+nv+nv+nv+nv+nv+nv+nv} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{2}{b^{2}} (\beta_{i-nw-nu,\eta}) (\beta_{j-nw-nu,\eta}) + \frac{(1-\mu)}{a^{2}} (\beta_{i-nw-nu,\xi}) (\beta_{j-nw-nu,\xi}) \right\} \right] d\xi d\eta \\ \\ \left[K_{33}^{m}\right] &= \frac{E_{p}t_{p}(ab)}{2(1-\mu^{2})} \sum_{j=nw+nu+1}^{nw+nv+nv+nv+nv+nv+nv+nv+nv+nv+nv+nv} \int_{0}^{1} \int_{0}^{1} \left[\left\{ \frac{2}{b^{2}} (\beta_{i-nw-nu,\eta}) (\beta_{j-nw-nu,\eta}) \right\} \right] d\xi d\eta \\ \\ \\ \left[K_{33}^{m}\right] &= \frac{E_{p}t_$$

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| DOI:10.15680/IJIRSET.2021.1007070 |

$$\begin{split} \left[K_{11}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=1}^{nw} \sum_{i=1}^{nw} \int_{0}^{1} \left\{ \frac{2(l_{x}^{q} + A_{x}^{q} e_{y}^{q})}{b^{4}} (\phi_{i,\eta\eta}) (\phi_{j,\eta\eta}) - Q_{x}^{q} \frac{1}{b^{4}} (\sum_{i=1}^{nw} d_{i}\phi_{i,\eta}) (\phi_{i,\eta}) (\phi_{j,\eta}) + A_{x}^{q} \frac{1}{b^{4}} (\sum_{i=1}^{nw} d_{i}\phi_{i,\eta}) (\phi_{i,\eta}) (\phi_{j,\eta}) + A_{x}^{q} \frac{1}{b^{4}} (\sum_{i=1}^{nw} d_{i}\phi_{i,\eta}) (\phi_{i,\eta}) (\phi_{j,\eta}) + A_{x}^{q} \frac{1}{b^{3}} (\sum_{i=nw+nu+1}^{nw} d_{i}\beta_{i-nw-nu,\eta})^{2} (\phi_{i,\eta}) (\phi_{j,\eta}) \right\} d\eta \\ \left[K_{22}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=1}^{nw} \sum_{i=1}^{nw} \int_{0}^{1} \left\{ \frac{2(l_{xx}^{q} + A_{x}^{q} (a\xi_{sf}^{q}))^{2}}{b^{4}} (\alpha_{i-nw,\eta\eta}) (\alpha_{j-nw,\eta\eta}) \right\} d\eta \\ \left[K_{33}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=nw+nu+1}^{nw+nu+nv} \sum_{i=nw+nu+1}^{nw+nu+nv} \int_{0}^{1} \left\{ \frac{2(A_{y}^{q})}{b^{2}} (\beta_{i-nw-nu,\eta}) (\beta_{j-nw-nu,\eta}) \right\} d\eta \\ \left[K_{31}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=nw+nu+1}^{nw+nu+nv} \sum_{i=1}^{nw+nu+nv} \int_{0}^{1} \left\{ \frac{2(Q_{x}^{q})}{b^{3}} (\phi_{i,\eta\eta}) (\beta_{j-nw-nu,\eta}) + A_{x}^{q} \frac{1}{b^{3}} (\sum_{i=1}^{nw} d_{i}\phi_{i})^{2} (\phi_{i,\eta}) (\beta_{i-nw-nu,\eta}) \right\} d\eta \\ \left[K_{13}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=1}^{nw} \sum_{i=1}^{nw+nu+nv} \int_{0}^{1} \left\{ -\frac{2(Q_{x}^{q})}{b^{3}} (\beta_{i-nw-nu,\eta}) (\phi_{j,\eta\eta}) \right\} d\eta \right] \\ \left[K_{13}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=1}^{nw} \sum_{i=nw+nu+1}^{nw+nu+nv} \int_{0}^{1} \left\{ -\frac{2(Q_{x}^{q})}{b^{3}} (\beta_{i-nw-nu,\eta}) (\phi_{j,\eta\eta}) \right\} d\eta \right] \\ \left[K_{13}^{sy} \right] &= \sum_{q=1}^{nsy} \left[\frac{E_{sb}}{2} \sum_{j=1}^{nw} \sum_{i=nw+nu+1}^{nw+nu+nv} \int_{0}^{1} \left\{ -\frac{2(Q_{x}^{q})}{b^{3}} (\beta_{i-nw-nu,\eta}) (\phi_{j,\eta\eta}) \right\} d\eta \right] \\ \left[K_{12}^{sy} \right] &= \left[K_{21}^{sy} \right] \left[K_{23}^{sy} \right] = \left[K_{32}^{sy} \right] = 0 \end{aligned}$$

The load vector $\{f\}$ is of the form $\{f_{11}, f_{12}, f_{13}\}^{T}$

 $\{f_{11}\} = \sum_{j=i}^{nw} P\phi_j|_{\xi,\eta} + p(ab) \sum_{j=1}^{nw} \int_0^1 \int_0^1 \phi_j d\xi d\eta, \{f_{12}\} = \{f_{13}\} = 0$



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569|

|| Volume 10, Issue 7, July 2021 ||

| DOI:10.15680/IJIRSET.2021.1007070 |

3.5 SOLUTION METHODOLOGY FOR STATIC DISPLACEMENT FIELD





3.6 RESULT AND DISCUSSION

The objective of the present study is to investigate the effect of large deflection on the static behaviour of stiffened plates and also to determine the influence of different stiffener crosssections on the static behaviour. The present analysis is carried out only for uni-axially single stiffened plate, where a single y-direction stiffener is present along the center line of the plate. However, six different types of classical flexural boundary conditions, namely, SSSS, SCSS, CCSS, CCSS, CCCS and CCCC, arising out of the combinations of clamped (C) and simply supported (S) end conditions are considered. In the present work, three different types of stiffener cross-section, namely, rectangular, inverted-T and I sections, are taken into account. In all the cases uniformly distributed transverse loading has been considered.



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569|

|| Volume 10, Issue 7, July 2021 ||

| DOI:10.15680/IJIRSET.2021.1007070 |

3.6.1 DEFLECTION OF RECTANGULAR PLATE UNDER DIFFERENT FLEXURAL BOUNDARY CONDITIONS

The results for static response of the system under transverse loading are presented in normalized load-displacement plane. Here the ordinate represents normalized maximum displacement (wmax/tp) and abscissa represents normalized load, which is defined as pa4 / (16Dtp), for uniformly distributed load. These results are generated for a rectangular stiffened plate with a = 0.60 m, b = 0.41 m and tp = 0.00633 m. The material properties used for this purpose are Ep = 211 GPa, $\rho p = 7830$ kg/m3. Figures. present the static deflection of rectangular plate under uniformly distributed load. The figure contain six normalized load-displacement curves corresponding to various flexural boundary conditions, namely, SSSS, SCSS, CSCS, CCSS, CCCS and CCCC. It indicates that clamped boundary condition provides maximum stiffness, whereas for simply supported case stiffness is minimum. It is also important to notice that location of maximum deflection is not same for all cases.



(Figure 5.2. Static deflection of a Rectangular plate under uniformly distributed load for different flexural boundary conditions.)

3.6.2 DEFLECTION OF STIFFENED RECTANGULAR PLATE UNDER DIFFERENT FLEXURAL BOUNDARY CONDITIONS

Figure 5.3 has been plotted for rectangular plate stiffened with inverted-T and I stiffener for different flexural boundary condition. Figure 5.3 contain six normalized load-displacement curves corresponding to various flexural boundary conditions, namely, SSSS, SCSS, CSCS, CCSS, CCCS and CCCC, arising out of the combinations of clamped (C) and simply supported (S) end conditions are considered. Here the ordinate represents normalized maximum displacement (wmax/tp) and abscissa represents normalized load, which is defined as pa4 / (16Dtp), for uniformly distributed load. These results are generated for a rectangular stiffened plate with a = 0.60 m, b = 0.41 m and tp = 0.00633 m. The dimensions for inverted-T stiffener is mentioned in Figure 4.1(b) and for I bar stiffener dimensions are as follows Flange dimension are Height (hf) = 0.0095 m, Width (bf) = 0.01 m, web dimensions are Height (hw) = 0.015 m, Width (bw) = 0.005 m. The material properties of stiffener used for this purpose are Es = 211 GPa, $\rho s = 7830$ kg/m3. Figures indicate that clamped boundary condition provides maximum stiffness, whereas



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569|

|| Volume 10, Issue 7, July 2021 ||

| DOI:10.15680/IJIRSET.2021.1007070 |

for simply supported case stiffness is minimum while it is important to reveal that maximum deflection is not same for all boundary condition.



(Figure 5.3. Deflection of Stiffened Rectangular Plate under Different Flexural Boundary Conditions with inverted-T stiffener and I stiffener)

IV.CONCLUSION

Large displacement static analysis of beams and stiffened plates are carried out using energy method, the underlying principle being the extermination of total energy of the system in its equilibrium state. The mathematical formulation of the static problem is based on the principle of minimum potential energy. A direct substitution method with successive relaxation scheme is used to solve the governing set of nonlinear equations. Results generated by the new methodology are compared with those available in literature and they show excellent agreement, thus establishing the accuracy of the present method. Results in the form of normalized load-displacement curve are presented for different combinations of flexural boundary conditions and stiffener crosssections. Also, three dimensional deflected shape plots are furnished along with the contour plots of the deflected system for all the cases. The results demonstrate varying degrees of nonlinearity corresponding to different flexural boundaries. From the present analysis of rectangular plates comparison has been made between inverted-T stiffener, rectangular stiffener and I stiffener. It is found that inverted-T stiffener shows the best stiffness or resistance against deflection.

ACKNOWLEDGMENTS

The authors are grateful to the V M INSTITUTE OF ENGINEERING & TECHNOLOGY, NAGPUR, MAHARASHTRA, INDIA, for providing guidances and resources to carry out this work.

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| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.569|

|| Volume 10, Issue 7, July 2021 ||

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