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# Pairwise $\Omega^*$ -Closed Sets and $\Omega_p$ -Closed Sets in Bitopological Spaces

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**ABSTRACT:** In this paper we introduce the concepts of **pairwise  $\Omega^*$ -closed sets** and  **$\Omega_p$ -closed sets** in bitopological spaces. Also we introduce and study **pairwise  $\Omega^*$ -continuous functions** and  **$\Omega_p$ -continuous functions** and prove pasting lemma for these functions. Moreover, we introduce new classes of bitopological spaces called  **$(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$**  and  **$(\tau_i, \tau_j)$ - $\Omega^*$ - $T_p$**  spaces and obtain their characterizations.

**KEYWORDS:**  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed,  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed,  $(\tau_i, \tau_j)$ - $\Omega^*$ -continuity,  $(\tau_i, \tau_j)$ - $\Omega_p$ -continuity,  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$  space and  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T_p$ .

## I. INTRODUCTION

Kelley[6] defined a bitopological space as a set equipped with two topologies on a set and initiated a systematic study of bitopological spaces. Mashhour[9] and Njastad[11] introduced preopen sets and  $\alpha$ -open sets respectively.

New classes of sets called  $\Omega^*$ -closed sets and  $\Omega_p$ -closed sets in topological spaces was introduced and studied by Mercy and Arockiarani[8]. The purpose of this present paper is to introduce the concepts of pairwise  $\Omega^*$ -closed sets and  $\Omega_p$ -closed sets in bitopological spaces and study their basic properties.

## II. PRELIMINARIES AND BASIC DEFINITIONS:

Throughout this paper  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$  means a bitopological space on which no separation axioms are assumed, unless otherwise mentioned and the integers  $i, j, k \in \{1, 2\}$ . For a subset  $A$  of  $X$ ,  $\tau_i$ -Cl  $(A)$ ,  $\tau_i$ -Int  $(A)$  and  $\tau_i$ -D  $[A]$  denotes the closure, the interior and the prederived set of  $A$  with respect to the topology  $\tau_i$ . Here we recall some known definitions and properties which are needed in this paper.

**Definition 2.1.** Let  $(X, \tau)$  be topological space. A subset  $A \subset X$  is said to be

- (1) **Preopen**[9] if  $A \subset \text{Int}(\text{cl}(A))$  and **Preclosed** if  $\text{cl}(\text{int}(A)) \subset A$ .
- (2)  **$\alpha$ -open** [11] if  $A \subset \text{Int}(\text{cl}(\text{Int}(A)))$ .
- (3) **Regular open** [12] if  $A = \text{Int}(\text{cl}(A))$  and **Regular closed** if  $A = \text{cl}(\text{Int}(A))$ .

**Definition 2.3**[8] Let  $(X, \tau)$  be a topological space,  $A \subset X$  and  $x \in X$ , then  $x$  is said to be a **pre limit** of  $A$  if every preopen set containing  $x$  contains a point of  $A$  different from  $x$ .

**Definition 2.3[8]** A bitopological space  $(X, \tau_i)$  be a topological space. The set of all prelimit points of  $A$  is said to be the **prederived set** of  $A$  and is denoted by  $Dp[A]$ .

**Definition 2.5[5]** Let  $(X, \tau)$  be a topological space,  $A \subset X$ . The **preinterior** of  $A$ , denoted by  $pInt(A)$  is the union of all preopen subsets of  $A$ .

**Definition 2.2[5]** Let  $(X, \tau)$  be a topological space,  $B \subset X$ . The **preclosure** of  $B$ , denoted by  $pcl(B)$  is the intersection of all preclosed sets containing  $B$ .

**Definition 2.6** A subset  $A \subset X$  is said to be

- (1) **Generalized closed** (briefly  $g$ -closed)[7] if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open.
- (2) **Pregeneralized closed** [5] (briefly  $pg$ -closed)[7] if  $pcl(A) \subset U$  whenever  $A \subset U$  and  $U$  is preopen.
- (3) **Generalized preclosed** (briefly  $gp$ -closed)[7] if  $pcl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open.

**Definition 2.7** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be

- (1)  **$g$ -continuous** [3] if  $f^{-1}(V)$  is  $g$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (2)  **$pg$ -continuous** [9] if  $f^{-1}(V)$  is  $pg$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (3)  **$gp$ -continuous** [1] if  $f^{-1}(V)$  is  $gp$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (4) **Pre-continuous** [9] if  $f^{-1}(V)$  is preopen in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ .
- (5) **contra-continuous** [4] if  $f^{-1}(V)$  is closed in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ .
- (6) **perfectly-continuous** [2] if  $f^{-1}(V)$  is both open and closed in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ .

**Definition 3.1.[8]** A subset  $A$  of  $(X, \tau)$  is said to be  **$\Omega^*$ -Closed** in  $(X, \tau)$  if  $pcl(A) \subset Int(U)$  Whenever  $A \subset U$  and  $U$  is preopen in  $(X, \tau)$ .

**Definition 3.2.[8]** A subset  $A$  of  $(X, \tau)$  is said to be  **$\Omega_p$ -Closed** in  $(X, \tau)$  if  $pcl(A) \subset Int(cl(U))$  Whenever  $A \subset U$  and  $U$  is preopen in  $(X, \tau)$ .

**Definition 4.1.[8]** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be  **$\Omega^*$ -continuous** (resp  **$\Omega_p$ -continuous**) if  $f^{-1}(V)$  is  $\Omega^*$ -Closed (resp  $\Omega_p$ -closed) in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

**Definition 4.2.[8]** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be  **$\Omega^*$ -irresolute** (resp  **$\Omega_p$ -irresolute**) if  $f^{-1}(V)$  is  $\Omega^*$ -Closed (resp  $\Omega_p$ -closed) in  $(X, \tau)$  for every  $\Omega^*$ -Closed (resp  $\Omega_p$ -Closed) set  $V$  of  $(Y, \sigma)$ .

### III. PAIRWISE $\Omega^*$ -CLOSED SETS AND PAIRWISE $\Omega_p$ -CLOSED SETS.

**Definition 3.1.** A subset  $A$  of  $(X, \tau)$  is said to be  **$(\tau_i, \tau_j)$ - $\Omega^*$ -closed** in  $(X, \tau_i, \tau_j)$  if  $\tau_j-pcl(A) \subset Int(U)$  whenever  $A \subset U$  and  $U$  is preopen in  $(X, \tau_i)$ .

#### Pairwise $\Omega^*$ -closed sets and $\Omega_p$ -closed sets in bitopological spaces

**Definition 3.2.** A subset  $A$  of  $(X, \tau)$  is said to be  **$(\tau_i, \tau_j)$ - $\Omega_p$ -closed** in  $(X, \tau_i, \tau_j)$  if  $\tau_j-pcl(A) \subset Int(cl(U))$  whenever  $A \subset U$  and  $U$  is preopen in  $(X, \tau_i)$ .

**Proposition 3.3** If  $D[E] \subset Dp[E]$  for each subset  $E$  of  $(X, \tau_i, \tau_j)$  then the union of  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed sets (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed sets) is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed).

**Proof:** Let  $A$  and  $B$  be  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp.  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) subsets of  $X$ . and Let  $U$  be a  $\tau_i$ -preopen set such that  $A \cup B \subset U$ . Then  $\tau_j-pcl(A) \subset Int(U)$  and

$\tau_j-pcl(B) \subset Int(U)$  (resp  $\tau_j-pcl(A) \subset Int(cl(U))$  and  $\tau_j-pcl(B) \subset Int(cl(U))$ ).

Since  $D[A] \subset Dp[A]$  and  $D[B] \subset Dp[B]$  then we have that  $\tau_j-cl(A) = \tau_j-pcl(A)$  and  $\tau_j-cl(B) = \tau_j-pcl(B)$ . Therefore

$\tau_j-cl(A \cup B) = \tau_j-cl(A) \cup \tau_j-cl(B) = \tau_j-pcl(A) \cup \tau_j-pcl(B) \subset Int(U)$

$\tau_j-pcl(A \cup B) \subset Int(U)$  (resp  $\tau_j-pcl(A \cup B) \subset Int(cl(U))$ ). Hence,  $A \cup B$  is to be  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed).



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**Proposition 3.4.** (1) Every  $\tau_i$ -open and  $\tau_j$ -preclosed subset of a bitopological space  $(X, \tau_i, \tau_j)$  is to be  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed. (2) Every  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed subset of  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed and  $(\tau_i, \tau_j)$ -gp-closed.

**Proof:** (1) Let  $A$  be  $\tau_i$ -open and  $\tau_j$ -preclosed subset of  $(X, \tau_i, \tau_j)$  and  $A \subset U$ , where  $U$  is  $\tau_i$ -preopen subset of  $X$ . Then  $\tau_j$ -pcl( $A$ ) =  $A = \text{Int}(A) \subset \text{Int}(U)$ . i.e.  $\tau_j$ -pcl( $A$ )  $\subset$   $\text{Int}(U)$ .

Hence  $A$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed.

(2) Let  $A$  be an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed subset of  $(X, \tau_i, \tau_j)$  and  $A \subset U$ , where  $U$  is  $\tau_i$ -preopen subset of  $X$ . Then  $\tau_j$ -pcl( $A$ )  $\subset$   $\text{Int}(U) \subset \text{Int}(\text{cl}(U))$ . i.e.  $\tau_j$ -pcl( $A$ )  $\subset$   $\text{Int}(\text{cl}(U))$ .

Hence  $A$  is  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed subset of  $X$ .

To prove the second part. Let  $A$  be an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed subset of  $(X, \tau_i, \tau_j)$  and  $A \subset U$ , where  $U$  is  $\tau_i$ -open subset of  $X$ , then  $\tau_j$ -pcl( $A$ )  $\subset$   $\text{Int}(U) \subset U$ . i.e.  $\tau_j$ -pcl( $A$ )  $\subset U$ . Hence  $A$  is  $(\tau_i, \tau_j)$ -gp-closed.

**Proposition 3.4.** If a subset  $A$  of  $(X, \tau_i, \tau_j)$  is  $\tau_i$ -preopen and  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed then it is  $\tau_j$ -preclosed.

**Proof:** Since  $A$  is  $\tau_i$ -preopen and  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed then  $\tau_j$ -pcl( $A$ )  $\subset$   $\text{Int}(A) \subset A$ . Hence  $\tau_j$ -pcl( $A$ ) =  $A$  and  $A$  is  $\tau_j$ -preclosed.

**Theorem 3.5.** An  $\tau_i$ -open set of  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ -gp-closed if and only if it is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed.

**Proof** Let  $A$  be an  $\tau_i$ -preopen and  $(\tau_i, \tau_j)$ -gp-closed set. Assume that  $A \subset U$  and  $U$  is  $\tau_i$ -preopen in  $X$ . Thus  $A = \text{Int}(A) \subset \text{Int}(U)$ . Since  $\text{Int}(U)$  is  $\tau_i$ -open in  $X$  and  $A$  is

$(\tau_i, \tau_j)$ -gp-closed, then  $\tau_j$ -pcl( $A$ )  $\subset$   $\text{Int}(U)$ . Hence  $A$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set. Conversely, it is obvious that every  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set is  $(\tau_i, \tau_j)$ -gp-closed.

**Theorem 3.2.** Let  $B$  and  $Y$  be subsets of a bitopological space  $(X, \tau_i, \tau_j)$  such that  $B \subset Y \subset X$ .

(1) In  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in the subspace  $Y$  and  $Y$  is an open and  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in  $(X, \tau)$  then  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in  $(X, \tau_i, \tau_j)$

(2) If  $Y$  is open and  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ . Then  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in the subspace  $Y$ .

**Proof:** (1) Let  $U$  be a  $\tau_i$ -preopen set of  $(X, \tau_i, \tau_j)$  such that  $B \subset U$ . Since  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in the subspace  $Y$ . then we have  $\tau_j$ -pcl $_Y(B) = \text{Int}_Y(U \cap Y)$ . then we obtain that  $Y \cap$

$\tau_j$ -pcl( $B$ )  $\subset \tau_j$ -pcl $_Y(B) \subset \text{Int}_Y(U \cap Y) = \text{Int}(U \cap Y)$ . Hence  $\text{Int}(U \cap Y) \cup \{X - (\tau_j$ -pcl( $B$ ))\} is

$\tau_i$ -preopen in  $(X, \tau)$  and it contains  $Y$ . Since  $Y$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$  then we have  $\tau_j$ -pcl( $B$ )  $\subset$   $\text{pcl}(Y) \subset \text{Int}[\text{Int}(U \cap Y) \cup \{X - (\tau_j$ -pcl( $B$ ))\}]  $\subset \text{Int}(U \cap Y) \cup \{X - (\tau_j$ -pcl( $B$ ))\}. Thus  $\tau_j$ -pcl( $B$ )  $\subset \text{Int}(U)$  and  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ .

(2) Let  $B \subset U$ , where  $U$  is  $\tau_i$ -preopen in the subspace  $Y$ . Since  $Y$  is open in  $X$ , then by lemma 3.14,  $U$  is  $\tau_i$ -preopen in  $(X, \tau_i, \tau_j)$ . Since  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in  $(X, \tau_i, \tau_j)$ , then  $\tau_j$ -pcl( $B$ )  $\subset \text{Int}(U)$ . Since  $Y$  is  $\tau_i$ -open in  $X$ , then  $\text{Int}(U) = \text{Int}_Y(U)$  and  $\tau_j$ -pcl $_Y(B) =$

$Y \cap \tau_j$ -pcl( $B$ )  $\subset \text{Int}(U) \subset \text{Int}_Y(U)$ . Hence  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in the subspace  $Y$ .

**Corollary 3.1.** Let  $B \subset Y \subset X$  and  $Y$  be  $\tau_i$ -open and  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$  then the following are equivalent. (1)  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$  (2)  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in the subspace  $Y$ .

**Theorem 3.3.** Let  $B$  and  $Y$  be subsets of a space  $(X, \tau_i, \tau_j)$  such that  $B \subset Y \subset X$ . (1) If  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set in the subspace  $Y$  and  $Y$  is  $\tau_i$ -open and

$(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in  $(X, \tau_i, \tau_j)$  then  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set in  $(X, \tau_i, \tau_j)$

(2) If  $Y$  is  $\tau_i$ -open and  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed in  $(X, \tau_i, \tau_j)$ , then  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set in the subspace  $Y$ .

**Proof:** (1) Let  $U$  be a  $\tau_i$ -preopen set in  $(X, \tau_i, \tau_j)$  such that  $B \subset U$ . Since  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set in the subspace  $Y$ , then  $\tau_j$ -pcl $_Y(B) \subset \text{Int}_Y(\text{cl}_Y(U \cap Y))$ . So,  $Y \cap (\tau_j$ -pcl( $B$ ))  $\subset$

$\tau_j$ -pcl $_Y(B) \subset \text{Int}_Y(\text{cl}_Y(U \cap Y)) = \text{Int}(\text{cl}(U \cap Y)) \cap Y$ . Hence  $\text{Int}(\text{cl}(U \cap Y)) \cup \{X - (\tau_j$ -pcl( $B$ ))\} is

$\tau_i$ -preopen in  $(X, \tau_i, \tau_j)$  and it contains  $Y$ . Since  $Y$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in  $(X, \tau_i, \tau_j)$ . Then we have  $\tau_j$ -pcl( $B$ )  $\subset \tau_j$ -pcl( $Y$ )  $\subset \text{Int}[\text{Int}(\text{cl}(U \cap Y)) \cup \{X - (\tau_j$ -pcl( $B$ ))\}]  $\subset \text{Int}(U \cap Y) \cup \{X -$

$(\tau_j$ -pcl( $B$ ))\}; thus  $\tau_j$ -pcl( $B$ )  $\subset \text{Int}(\text{cl}(U))$  and  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed in  $(X, \tau)$ .

(2) Let  $B \subset U$ , where  $U$  is  $\tau_i$ -preopen in the subspace  $Y$ . Since  $Y$  is  $\tau_i$ -open in  $X$ , then by lemma 3.14  $U$  is  $\tau_i$ -preopen



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in  $(X, \tau_i, \tau_j)$ . Since  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set in  $(X, \tau_i, \tau_j)$ , then  $\tau_j\text{-pcl}(B) \subset \text{Int}(\text{cl}(U))$ . Since  $Y$  is  $\tau_i$ -open in  $X$ , then  $\text{Int}(U) = \text{Int}_Y(U)$  and  $\tau_j\text{-pcl}_Y(B) = Y \cap \tau_j\text{-pcl}(B) \subset \text{Int}(\text{cl}(U)) \cap Y = \text{Int}_Y(U)$ . Hence  $B$  is an  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set in the subspace  $Y$ .

#### IV. PAIRWISE $\Omega^*$ -CONTINUITY AND PAIRWISE $\Omega_p$ -CONTINUITY.

**Definition 4.1.** A function  $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  is said to be  **$(\tau_i, \tau_j)$ - $\Omega^*$ -continuous** (resp  **$(\tau_i, \tau_j)$ - $\Omega_p$ -continuous**) if  $f^{-1}(V)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -Closed (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) in  $(X, \tau_i, \tau_j)$  for every closed set  $V$  of  $(Y, \sigma_i, \sigma_j)$ .

**Definition 4.2.** A function  $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$  is said to be  **$(\tau_i, \tau_j)$ - $\Omega^*$ -irresolute** (resp  **$(\tau_i, \tau_j)$ - $\Omega_p$ -irresolute**) if  $f^{-1}(V)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) in  $(X, \tau_i, \tau_j)$  for every  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) set  $V$  of  $(Y, \sigma_i, \sigma_j)$ .

**Proposition 4.1.** (1) Every  $(\tau_i, \tau_j)$ - $\Omega^*$ -continuous function is  $(\tau_i, \tau_j)$ -gp-continuous.

(2) Every  $(\tau_i, \tau_j)$ - $\Omega^*$ -continuous function is  $(\tau_i, \tau_j)$ - $\Omega_p$ -continuous.

(3) Every contra-continuous and pre-continuous function is  $(\tau_i, \tau_j)$ - $\Omega^*$ -continuous.

#### Pairwise $\Omega^*$ -closed sets and $\Omega_p$ -closed sets in bitopological spaces

#### 1. APPLICATIONS:

**Definition 5.1.** A bitopological space  $(X, \tau_i, \tau_j)$  is said to be

(1)  **$(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$**  if for every  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set is  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ .

(2)  **$(\tau_i, \tau_j)$ - $\Omega^*$ - $T_p$**  if for every  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed set is  $\tau_j$ -closed in  $(X, \tau_i, \tau_j)$ .

(3)  **$(\tau_i, \tau_j)$ -Pre- $T^{1/2}$**  if for every  $(\tau_i, \tau_j)$ -pg-closed set is  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ .

**Proposition 5.2.** Let  $(X, \tau_i, \tau_j)$  be a bitopological space.

(1) For each  $x \in X$ ,  $\{x\}$  is  $\tau_j$ -preclosed (or) its complement  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ .

(2) For each  $x \in X$ ,  $\{x\}$  is  $\tau_i$ -open and  $\tau_j$ -preclosed (or) its complement  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ .

**Proof:** (1) Suppose that  $\{x\}$  is not  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ . Then  $\{x\}$  is not  $\tau_i$ -preopen and the only  $\tau_i$ -preopen set containing  $X - \{x\}$  is  $X$ . Therefore,  $\tau_j\text{-pcl}(X - \{x\}) \subset \text{Int}(X) = X$ . So,  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ .

(2) Suppose that  $\{x\}$  is not  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ . Then by (1),  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$  and then  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed. Suppose that  $\{x\}$  is not  $\tau_i$ -open and let  $U$  be a  $\tau_i$ -preopen set such that  $X - \{x\} \subset U$ . If  $U = X$  then  $\tau_j\text{-pcl}(X - \{x\}) \subset \text{Int}(\text{cl}(U)) = U$ .

If  $U = X - \{x\}$  then we have that  $\text{Int}(\text{cl}(U)) = \text{Int}(\text{cl}(X - \{x\})) = \text{Int}(X) = X$ . Hence  $\tau_j\text{-pcl}(X - \{x\}) \subset \text{Int}(\text{cl}(U))$ . Therefore,  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed in  $(X, \tau_i, \tau_j)$ .

**Theorem 5.1.** For a bitopological space  $(X, \tau_i, \tau_j)$  the following are equivalent:

(1) Every  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed is  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ .

(2) For each  $x \in X$ ,  $\{x\}$  is  $\tau_i$ -preopen (or)  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ .

(3)  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ -pre- $T^{1/2}$ .

**Proof:** (1)  $\Rightarrow$  (2) Suppose that for a point  $x \in X$ ,  $\{x\}$  is not  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ . By Proposition 5.2, (1)  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ . By assumption,

$X - \{x\}$  is  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$  and hence  $\{x\}$  is  $\tau_i$ -preopen. Therefore, each singleton is  $\tau_i$ -preopen (or)  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ .

(2)  $\Rightarrow$  (1) Let  $A$  be an  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set in  $(X, \tau_i, \tau_j)$ . We want to prove that  $\tau_j\text{-pcl}(A) = A$ . Suppose that  $x \in \tau_j\text{-pcl}(A)$ .

**Case 1:**  $\{x\}$  is  $\tau_i$ -preopen in  $(X, \tau_i, \tau_j)$ . Then  $\{x\} \cap A \neq \emptyset$  which implies  $x \in A$ .

**Case 2:**  $\{x\}$  is  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$  and  $x \notin A$ . Then  $\tau_j\text{-pcl}(A) - A$  contains a  $\tau_j$ -preclosed set

$\{x\}$ . And this contradicts [10 Proposition 3.3] Hence  $x \in A$  and  $A$  is  $\tau_j$ -preclosed. Therefore, every  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed set is  $\tau_j$ -preclosed.



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(2)  $\Rightarrow$  (3) This is shown in [13, Theorem 4.8].

**Theorem 5.2:** For a bitopological space  $(X, \tau_i, \tau_j)$  the following properties hold.

- (1) If  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -Tp, then for each  $x \in X$ , the singleton  $\{x\}$  is  $\tau_i$ -open (or)  $\tau_j$ -preclosed.
- (2)  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$ . if and only if for each  $x \in X$ ,  $\{x\}$  is  $\tau_i$ -preopen (or)  $\tau_j$ -preclosed in  $(X, \tau_i, \tau_j)$ .
- (3) If  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -Tp, then, it is  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$ .
- (4) If  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$ , then it is  $(\tau_i, \tau_j)$ -pre- $T^{1/2}$ .

**Proof:** (1) Suppose that for some  $x \in X$ ,  $\{x\}$  is not  $\tau_j$ -preclosed. By Proposition 5.2,  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed in  $(X, \tau_i, \tau_j)$ . Hence  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed in  $(X, \tau_i, \tau_j)$ . Since  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -Tp, then  $X - \{x\}$  is  $\tau_j$ -closed in  $(X, \tau_i, \tau_j)$ . Thus  $\{x\}$  is  $\tau_i$ -open.

(2) **Necessity:** Suppose that a singleton  $\{x\}$  is not  $\tau_j$ -preclosed. By Proposition 5.2,  $X - \{x\}$  is  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed in  $(X, \tau_i, \tau_j)$ . Using the assumption we have that  $\{x\}$  is  $\tau_i$ -preopen.

**Sufficiency:** It follows from the assumption that every subset is  $\tau_i$ -preopen and  $\tau_j$ -preclosed. Then  $(X, \tau_i, \tau_j)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$ .

(3) It is straight forward from the definitions of  $(\tau_i, \tau_j)$ - $\Omega^*$ -Tp spaces and  $(\tau_i, \tau_j)$ - $\Omega^*$ - $T^{1/2}$  spaces.

(4) It is obvious from (2) above and Theorem 5.3.

V. PASTING LEMMA

Let  $X = A \cup B$  and let  $f : A \rightarrow Y$  and  $h : B \rightarrow Y$  be two functions. We say that  $f$  and  $h$  are compatible if  $f(x) = h(x)$  for every  $x \in A \cap B$ . The combination  $f \vee h : X \rightarrow Y$  is defined by  $(f \vee h)(x) = f(x)$  if  $x \in A$  and  $(f \vee h)(x) = h(x)$  if  $x \in B$ . The function  $f \vee h$  is called the combination of  $f$  and  $h$ .

**Theorem 6.1:** Suppose that  $A$  and  $B$  are both  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed and  $\tau_i$ -open subsets of

$X$ . Let  $f : (A, \tau_1/A, \tau_2/A) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $h : (B, \tau_1/B, \tau_2/B) \rightarrow (Y, \sigma_1, \sigma_2)$  be compatible functions. Assume that  $D[E] \subset D_p[E]$  for any  $E \subset X$ . If  $f$  and  $h$  are

$(\tau_i, \tau_j)$ - $\Omega^*$ -continuous (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -continuous) then the combination  $f \vee h : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is also  $(\tau_i, \tau_j)$ - $\Omega^*$ -continuous (resp  $(\tau_i, \tau_j)$ - $\Omega_p$ -continuous).

**Proof:** Let  $F$  be a  $\tau_j$ -closed subset of  $Y$ . Then  $(f \vee h)^{-1}(F) = [(f \vee h)^{-1}(F) \cap A] \cup [(f \vee h)^{-1}(F) \cap B] = f^{-1}(F) \cup h^{-1}(F)$ . By the assumption,  $f^{-1}(F)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp.  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) in  $(A, \tau_1/A, \tau_2/A)$  and  $h^{-1}(F)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp.  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) in  $(B, \tau_1/B, \tau_2/B)$ . Since  $A$  and  $B$  are both  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed and  $\tau_i$ -open subsets of  $(X, \tau_i, \tau_j)$ , by Theorem 3.15, (Theorem 3.17)  $f^{-1}(F)$  and  $h^{-1}(F)$  are both  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp.  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) sets in  $(X, \tau_i, \tau_j)$ . Then by proposition 3.3,  $f^{-1}(F) \cup h^{-1}(F)$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -closed (resp.  $(\tau_i, \tau_j)$ - $\Omega_p$ -closed) in  $(X, \tau_i, \tau_j)$ . Hence  $f \vee h$  is  $(\tau_i, \tau_j)$ - $\Omega^*$ -continuous (resp.  $(\tau_i, \tau_j)$ - $\Omega_p$ -continuous).

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