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Families of Disjoint Sets Colouring Technique and Concept of Common and Non Common Edge

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ABSTRACT: Families of disjoint sets colouring technique is trial to generalize all type of colouring, such as edge colouring, vertex colouring and faces colouring, in this paper we introduce the concept of common edges between two vertices each of degree greater than one. This concept has two special cases, edge colouring satisfies these two special cases ,which explain why edge colouring is not complicated compare with vertex colouring. We used concept of common edge, non common edge to introduce some results which are generalization to theorems of edge colouring.

KEYWORDS: Families of disjoint sets colouring technique, common edge, non common edge, proper vertex, improper vertex.

I. INTRODUCTION

Colouring of families of disjoint sets technique depend on the number n, generally in case of sets n is degree of intersection, for colouring edges of the graph n is the largest degree of vertex (maximum number of edges such that any two are adjacent). For colouring vertices of the graph, n is the clique number (maximum number of vertices such that any two are adjacent).

In paper [1] we introduced the concept of families of disjoint setscolouring technique as a topic of set theory. We also introduce definitions of degree of intersection, meeting and standard of meeting, minimum number of families of disjoint sets and maximum number of families of disjoint sets. In paper [2] we introduce some concepts in graph theory to introduce new method for colouring edges and vertices of graph. This method has two types first related to vertices of the graph and second related to cycles. This method depend on the theory in our paper "Family of disjoint sets and Its Applications", with suitable changes. In paper [3] we prove minimum number of families of disjoint sets with known degree of intersection (minimum number of colours) and we introduce another proof for maximum number of families of disjoint sets (maximum number of colours). In paper[4] we assume both sets and graphs are finite. Results in that paper use families of disjoint setscolouring technique for finite sets and finite graphs. Using results in that paper we can compare between families of disjoint setscolouring technique and colouring of graphs in graph theory.

In paper [5] we prove some results in graph theory using colouring of families of disjoint sets technique. We introduced the concept ofset labeled by families of disjoint sets and the degree of this set, as generalization to thedegree of intersection and proved some results related to this concept. In paper [6]we prove some results of edge'scolouring (trivial colouring and nontrivialcolouring). We prove some results of edge'scolouring for many graphs connected by bridges. In many examples we determine minimum colours to colour edges of the graphs only by calculation when we know number of the edges , number of the vertices and largest degree of vertex, whatever the graph is huge. We used the concept of trivial colouring and nontrivialcolouring to prove results of edge'scolouring for trees, bipartitegraphs and complete graphs.

In paper [7] we modify the definitions of local maximum families of disjoint sets and local minimum families of disjoint sets, which we introduced in paper [1], and prove some propositions related to these two concepts.

In paper [8] we used families of disjoint setscolouringtechnique, to colour vertices of the graph depend on the clique number, and we introduce some results of vertex'scolouring (trivial colouring and nontrivial colouring) depend on clique number. We introduce the concept maximum and minimumnumber of edges in the graph to colour k vertices by m colours, and we used this concept to prove a result about number of all possible ways to colour k vertices by m colours, and a result about the range of edges in which we can colour k vertices by m colours.

In paper e[9] we introduce the concept of common set to establish some results forfamilies of disjoint setscolouringtechnique. The concept of common set is essential concept forfamilies of disjoint setstechnique, and we have to modify some definitions in previous papers due to common set concept.

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This paper analogue to paper [9], the difference in paper [9] we partition sets into families of disjoint sets, herewe partition edges into colour classes. In section two we recall some definitions and results related to colouring in graph theory and families of disjoint sets colouring technique. In section three we introduce concept of common edge between two common vertices, as essential concept for edge colouring, and in section four weintroduce some results related to this concept.

In section five we introduce method is called construction of minimum number of colour classessome results related to this method.

In section six we introduce some results for addition of common edges and non common edges, such that the proper vertices of these k edges (standard of partition) remains unchanged.

In section seven we used concept of common edge, non common edge to introduce some results which are generalization to theorems of edge colouring.

II. RELATED WORK

In this section we recall some definitions and results related to colouring in graph theory and families of disjoint setscolouring technique.

Definition 2.1. Let $\chi_1, \chi_2, \chi_3, ..., \chi_m$ be *m* families of disjoint sets and $X_{i_1}, X_{i_2}, X_{i_3}, ..., X_{i_n}$ be *n* sets such that any two sets belong to two different families of disjoint sets, and *X* is nonempty set, $X = X_{i_1} \cap X_{i_2} \cap X_{i_3} \cap ... \cap X_{i_n}$, $X = \{x : x \in X_{i_j}, 1 \le i \le r, 1 \le j \le n, n \le r, X_{i_j} \in \chi_i\}$ then the positive integer *n* is called degree of intersection of *X* [1].

Proposition 2.2. Let $\chi_1, \chi_2, \chi_3, ..., \chi_r$ be *r* families of disjoint sets and $\chi_1, \chi_2, \chi_3, ..., \chi_w$ be *W* nonempty intersections, the family of disjoint sets χ_i appear w_i times at *W* intersections and the family of disjoint sets χ_j appear w_j time at *W* intersections, then we say the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these *W* intersections if and only if $w+1 \le w + w_i$.

Remark 2.3.In this paper we will labelled a vertex using its degree and edges. If v is vertex of degree three we can used the notation $v = (e_i, e_j, e_k)$ pointing to v as common point and any two of edges e_i, e_j, e_k are adjacent. The notation v = (i, j, k) pointing to colours i, j, k, where i, j, k are integers. If v of degree n we write $v = (e_1, e_2, e_3, ..., e_n)$ for pointing to edges $e_1, e_2, e_3, ..., e_n$, or we can used notation v = (1, 2, 3, ..., n) forpointing to colours (or colour classes) 1, 2, 3, ..., n, and we say v represented (or labelled) by 1, 2, 3, ..., n. [2].

Definition 2.4. Let 1,2,3,...,m be *m* colours used for colouring edges of the graph *G* at the vertex, and $v_1, v_2, v_3,..., v_w$ be *W* vertices each of degree *n*, and *W* be the minimum number of vertices such thatany two colours be two different colours at *W* vertices, then these *W* vertices are called standard of different *m* colours (colour classes).[2].

Definition 2.5. Let $\chi_1, \chi_2, \chi_3, ..., \chi_m$ be *m* families of disjoint sets, $X_1, X_2, X_3, ..., X_w$ each of degree *n*, and *W* be the minimum number of nonempty intersections, such that any two families of disjoint sets meet each other at these *W* intersections, then these *W* intersections called standard of meeting for these *m* families of disjoint sets [1]. Theorem 2.6. The sum of degrees of vertices equals twice the number of edges [15].

Definition 2.7. A walk in a graph G is a finite sequence $W = v_0 e_1 v_1 e_2 v_2 \dots e_k v_k$ whose term are alternately vertices and edges, we say W is a walk from v_0 to v_k . If $v_0, v_1, v_2, v_3, \dots, v_k$ are distinct, W is called a pathin a graph G. If $e_1, e_2, e_3, \dots, e_k$ are distinct, W is called a trail in a graph G. A closed trial whose $v_0 = v_k$ is called a cycle.

Definition2.8. A graph is called acyclic if it has no cycles. A tree is a connected acyclic graph [14].

Definition 2.9. An edge with identical ends is called a loop [10].

Proposition 2.10 Let $A_1, A_2, A_3, ..., A_k$ be k common sets, and $X_1, X_2, X_3, ..., X_w$ be all proper intersections of these k sets, with maximum degree of intersection equal n, if we partition all sets (common sets and non common sets) into n+1=m minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, ..., \chi_m$, and $X_1, X_2, X_3, ..., X_w$ is not standard of partition these k sets into n+1=m minimum families of disjoint sets. If $X_1, X_2, X_3, ..., X_r$ is standard partition (with maximum degree of intersection equal n) $A_1, A_2, A_3, ..., A_r$ into n+1=m minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, ..., \chi_m$ and $X_1, X_2, X_3, ..., X_r$ is standard function (with maximum degree of intersection equal n) $A_1, A_2, A_3, ..., A_r$ into n+1=m minimum families of disjoint sets $\chi_1, \chi_2, \chi_3, ..., \chi_r$

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where $\{A_1, A_2, A_3, ..., A_r\} \subset \{A_1, A_2, A_3, ..., A_k\}$, and $\{X_1, X_2, X_3, ..., X_t\} \subset \{X_1, X_2, X_3, ..., X_w\}$, then the sets $A_1, A_2, A_3, ..., A_k$ partition into n + 1 = m minimum families of disjoint sets [9].

III. COMMON EDGES AND NONCOMMON EDGES

In this section we introduce the concept of common edges and proper vertices and introduce some results related to these two concepts.

Definition 3.1. Let G be a graph consisting of nonempty set V(G) of vertices and nonempty set E(G) of edges, an edge e is called common edge between two common vertices v_i and v_j if e joins v_i and joins v_j such that v_i link to v_1 and link to v_2 , and v_j link to v_3 and link to v_4 , where $v_1 \neq v_2$ and link to $v_3 \neq v_4$.

Remark3.2. As abbreviation instate of common edge between two common vertices, we saycommon edge.

Definition 3.3. Let G be a graph consisting of nonempty set V(G) of vertices and nonempty set E(G) of edges, an edge e is called non common edge between two common vertices v_i and v_j if e joins v_i and joins v_j such that at least one of v_i and v_j is of degree one.

Example 3.4.If v_1, v_2, v_3 , are three vertices each of degree four, v_1 is end point of edges e_1, e_2, e_3, e_4, v_2 is end point of edges e_2, e_5, e_6, e_7 , and v_3 is end point of edges e_3, e_5, e_6, e_8 , we used the notation $v_1 \equiv (e_1, e_2, e_3, e_4)$, $v_2 \equiv (e_2, e_5, e_6, e_7)$, and $v_3 \equiv (e_3, e_5, e_6, e_8)$, or as abbreviation we used the notation $v_1 \equiv (1, 2, 3, 4), v_2 \equiv (2, 5, 6, 7)$, and $v_3 \equiv (3, 5, 6, 8)$, The edge e_2 is common edge between the two vertices v_1, v_2 , the edges e_5, e_6 are common edges between the two vertices v_1, v_2 , the edges e_5, e_6 are common edges between the two vertices v_1, v_3 . If e_4 joins links v_1 and v_4, e_1 links v_1 and v_5, e_7 links v_2 and v_7, e_8 links v_3 and v_6 , and $d(v_4) = d(v_5) = d(v_6) = d(v_7) = 1$, the common edges at the vertices v_1, v_2, v_3 are e_2, e_3, e_5, e_6 , and non common edges at the vertices v_1, v_2, v_3 are e_1, e_4, e_7, e_8 . The edges e_5, e_6 are multiple edges between the two vertices v_2, v_3 .

Definition 3.5. Let G be a graph consisting of nonempty set V(G) of vertices and nonempty set E(G) of edges, a vertex v is called proper vertex if at least two common edges intersect at v.

Definition 3.6. Let G be a graph consisting of nonempty set V(G) of vertices and nonempty set E(G) of edges, a vertex v is called improper vertex if not more than one common edge intersect at v. Also a vertex v is called improper vertex if all edges intersect at v are non common edges.

Definition 3.7. Let *G* be a of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ and with maximum degree of vertex equal *n*, if these *k* edgespartition into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, then these *w* intersections calledstandard of partition *k* edges into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, if for any twocolour classes χ_i and χ_j we have $w + 1 \le t_i + t_j$ and $t_i \le w, t_j \le w$, where $(t_i$ appearance of χ_i at $v_1, v_2, v_3, ..., v_w$).

Proposition 3.8. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ and with maximum degree of vertex equal *n*, suppose these *k* edges and non common edges partitioned into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, and $e_1, e_2, e_3, ..., e_p$ are edges (common and non common) belong to one colour class χ_j , then $t_j = w_1 + w_2 + w_3 + ... + w_p \le w$ where w_i be number of times the edge e_i appear at these *w* vertices, $1 \le i \le p$, $1 \le j \le m$, and t_i be number of times the colour class χ_i appear at these *w* vertices.

Proof: SupposeLet $e_1, e_2, e_3, ..., e_p$ belong to one colour class χ_j and χ_j labelled by j, then each of $e_1, e_2, e_3, ..., e_p$ labelled by j and the label j appear $w_1 + w_2 + w_3 + ... + w_p$ times at $v_1, v_2, v_3, ..., v_w$ then whatever we exchange partition of all edges (common and non common) into m colour classes, and for the edges $e_1, e_2, e_3, ..., e_p$ belong to one colour class, any two edges of $e_1, e_2, e_3, ..., e_p$ are nonadjacent, then the label j not appear twice at any intersection, if



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j appear once at any one of $v_1, v_2, v_3, \dots, v_w$ then $t_j = w_1 + w_2 + w_3 + \dots + w_p = w$, if *j* appear once at some of $v_1, v_2, v_3, \dots, v_w$ then $t_j = w_1 + w_2 + w_3 + \dots + w_p < w$, therefore we have $t_j = w_1 + w_2 + w_3 + \dots + w_p \le w$.

Remark 3.9. For vertex $v \equiv (1,2,3,...,n)$ the label *i* representance colour class, where $1 \le i \le n$, each *i* at the vertex *v* either it is common edgebelong to this colour class or non common edgebelong to this colour class. If it is common edge, the label *i* depend on two factor (1) the labels 1,2,3,...,i-1,i+2,...,n, as each label represent an edge adjacent to the edge *i*. (2) *t* proper vertices, as common edge between *t* vertices , means this common edge appear at *t* vertices and with same label. If it is non common edge, the label *i* depend on one factor, the labels 1,2,3,...,i-1,i+2,...,n, as each labelrepresent an edge adjacent to the edge *i*. Therefore when we partition *k* edges and non common edges into *m* colour classes, to deal with non common edge is easy than common edge.

Example 3.10. Suppose all of v_1, v_2, v_3, v_4, v_5 are proper vertices, each vertex is intersection of some of edges $e_1, e_2, e_3, \dots, e_{10}$ such that each one of the edges e_1, e_2, e_3, e_4, e_5 is common edge between two proper vertices, and each one of the edges $e_6, e_7, e_8, e_9, e_{10}$ is non common edge between two proper vertices. $v_1 \equiv (e_1, e_2, e_8)$, $v_2 \equiv (e_1, e_4, e_{10}), v_3 \equiv (e_7, e_4, e_5), v_4 \equiv (e_6, e_3, e_5), v_5 \equiv (e_2, e_3, e_9)$, From these five vertices we can partition these ten edges intofive colour classes as follows: $\chi_1 \equiv (e_1, e_6), \ \chi_2 \equiv (e_2, e_7), \ \chi_3 \equiv (e_3, e_8), \ \chi_4 \equiv (e_4, e_9),$ $\chi_5 \equiv (e_5, e_{10})$. We use the label *a* for χ_1 , *b* for χ_2 , *c* for χ_3 , *d* for χ_4 , *e* for χ_5 , and we write these vertices as follows: $v_1 \equiv (a, b, c), v_2 \equiv (a, d, e), v_3 \equiv (\overline{b}, d, e), v_4 \equiv (\overline{a}, c, e), v_5 \equiv (b, c, \overline{d}),$ we use dash formon common edge,to differentiate betweencommon edge and non common edge. UsingRemark 3.9. we first deal with common sets, since non common sets is easy, the common edge e_1 is not adjacent to the common edge e_5 , so we consider e_1, e_5 belong one colour class, and we label this colour class by 1, the common edge e_2 is not adjacent to the common edge e_4 so we consider e_2, e_4 belong onecolour class, and we label this colour class by 2, we label colour class contained the edge e_3 by 3, the non common edge e_6 is adjacent to common edges e_3 , e_5 so we label e_6 by 2, continue in this mannerwe label e_7 by $\overline{3}$, we label e_8 by $\overline{3}$, we label e_9 by $\overline{1}$, and we label e_{10} by $\overline{3}$, so we write these proper vertices as $v_1 \equiv (1,2,\bar{3}), v_2 \equiv (1,2,\bar{3}), v_3 \equiv (\bar{3},2,1), v_4 \equiv (\bar{2},3,1), v_1 \equiv (2,3,\bar{1}),$ Therefore these ten edges partition into threecolour classes. The label 1 points to a common edge belongs to the colour class labeled by 1, and the label 1 points to a non common edge belongs to the colour class labeled by 1.

Here we modify Definition 2.5 and Definition 2.9. using concept of proper intersection.....

Definition 3.11. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ with maximum degree of vertex equal *n*, if these *k* edgespartition into *m* colour classes

 $\chi_1, \chi_2, \chi_3, ..., \chi_m$, and at vertex v_j intersect the edges $e_1, e_2, e_3, ..., e_n$ where $1 \le j \le w, e_1 \subseteq \chi_1, e_2 \subseteq \chi_2$,

 $e_3 \subseteq \chi_3, ..., e_n \subseteq \chi_n$, then the vertex v_i iscalled degree *n*.

Definition 3.12. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ with maximum degree of vertex equal *n*, if these *k* edgespartition into *m* colour classes, the colour class χ_i is said to be meet the colour class χ_j at these *w* vertices, if whatever we exchange any edge e_i by an edge e_i , where $e_i \in \chi_i$ and $e_i \in \chi_i$, $\chi_i \neq \chi_i$, $\chi_j \neq \chi_i$, and e_i intersects at least one member of colour class χ_j , such that the

number t_i not changed (t_i appearance of χ_i at $v_1, v_2, v_3, ..., v_w$) and χ_i remains a colour class, then e_i intersects at least one member of colour class χ_j .

Definition 3.13. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ with maximum degree of vertex equal *n*, if these *k* edgespartition into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, a colour class χ_i is said to be not meet the colour class χ_j at these *w* vertices, where $1 \le i < j \le m$,

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if for any two edges e_r , e_s such that $e_r \in \chi_i$, $e_s \in \chi_j$ where $1 \le r < s \le k$, then e_r and e_s are not adjacent, and if $t_i + t_j \le w$.

Definition 3.14. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ with maximum degree of vertex equal *n*, if these *k* edgespartition into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, then these *w* vertices is calledstandard of partition these *k* sets into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, if for any twocolour classes χ_i and χ_j where $1 \le i < j \le m$, then $w + 1 \le t_i + t_j$ and then $t_i \le w, t_j \le w$.

Definition 3.15. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ (standard of partition), with maximum degree of vertex equal *n*, if these *k* edges partition into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, then *m* (where $n + 1 \le m$,) iscalled minimum number of colour classes if we try to partition these *k* edges into *m*-1 colour classes, there exists acolour class χ_i such that $w + 1 \le t_i$, where $1 \le i \le m-1$, and t_i appearance of χ_i at $v_1, v_2, v_3, ..., v_w$.

Definition 3.16. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$ (standard of partition), with maximum degree of vertex equal *n*, if these *k* edges partition into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, then *m* iscalled maximum number of colour classes, if we partition these *k* edges into any *m*+1 colour classes, there exist two colour classes χ_i, χ_j such that $t_i + t_j \le w$, where $1 \le i < j \le m+1$, and t_i, t_j appearance of χ_i, χ_j at $\chi_1, \chi_2, \chi_3, ..., \chi_w$, respectively.

Example 3.17.Let *G* be a graph of sixcommon edgesto $e_1, e_2, e_3, e_4, e_5, e_6$ and these sixcommon edges intersect at five proper vertices v_1, v_2, v_3, v_4, v_5 with maximum degree of vertex equal five, are proper intersections of the edges $e_1, e_2, e_3, e_4, e_5, e_6$ where $v_1 \equiv (e_1, e_2, e_7, e_8, e_9)$, $v_2 \equiv (e_1, e_3, e_4)$, $v_3 \equiv (e_3, e_6)$, weall $v_4 \equiv (e_6, e_5)$, and $v_5 \equiv (e_2, e_4, e_5)$, where e_7, e_8, e_9 are non common edges. $d(v_1) = 5$, $d(v_2) = d(v_5) = 3$, $d(v_3) = d(v_4) = 2$. Then the proper vertices v_1, v_2, v_3, v_4, v_5 can not formstandard of partition to $e_1, e_2, e_3, e_4, e_5, e_6$ and any subset of v_1, v_2, v_3, v_4, v_5 can not formstandard of equation to a subset of $e_1, e_2, e_3, e_4, e_5, e_6$.

Proposition 3.18.Let G be a graph of k common edges $e_1, e_2, e_3, ..., e_k$ and these k common edges intersect at w proper vertices $v_1, v_2, v_3, ..., v_w$ (each vertex is intersection of at least two common edges), with maximum degree of vertex equal n, if all edges (common edges and non common edges) partition into m colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$,

labelled by 1,2,3,..., *m*, and if t_i be number of times the colour class χ_i appear at these *w* vertices, where $1 \le i \le m$,

and for all $1 \le j \le w$ then we have $\sum_{l=1}^{k} c_l + \sum_{l=1}^{w} c_l = \sum_{j=1}^{w} d(v_j)$, where c_l appearance of edge e_l at w vertices, $1 \le l \le k$,

 $1 \le l \le k_1, k_1$ number of non common edges.

Proof: We labelled edges by same labels of colour classes, such that two adjacent deges have two different labels, and labelled any vertex v_j , $1 \le j \le w$, as $v_j \equiv (a_1, a_2, a_3, ..., a_p)$ where $\{a_1, a_2, a_3, ..., a_p\} \subseteq \{1, 2, 3, ..., m\}$, $2 \le p \le n$, and p is degree of vertex v_j . Using Theorem2.5. the result holds.

Remark 3.19.Families of disjoint setscolouringtechnique is trial to generalize all type of colouring, edges, vertices, surfaces, each of common edge, common vertex and common face is essential concept forfamilies of disjoint setscolouringtechnique. Common set can be between two intersections or more than two intersections [9], common vertex can be between two cliques or more than two cliques [19], in this paper we see common edge only between two vertices, this explaincommon edge is special case and why colouring edges is easier thancolouring vertices colouring faces.

Proposition 3.20.In a graph G any cycle $v_0e_1v_1e_2v_2e_3,...,e_kv_k$ of vertices and edges, then $e_1, e_2, e_3,...,e_k$ are common edges and $v_1, v_2, v_3,...,v_w$ are proper vertices.

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Proof: UsingDefinition2.7. ,Definition3.1. and Definition3.5. edges, $e_1, e_2, e_3, ..., e_k$ are common edges and $v_0, v_1, v_2, v_3, ..., v_k$ are proper vertices.

Proposition 3.21. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are proper intersections of these k sets, and each vertex is intersection of at least two common edges, then there is at least one cycle include all the edges $e_1, e_2, e_3, ..., e_k$ or some of them.

Proof: Suppose $u_0a_1u_1a_2u_2a_3...a_lu_l$ is an alternating sequence of proper vertices $u_0, u_1, u_2, ..., u_l$ and common edges $a_1, a_2, a_3, ..., a_l$ where $\{u_0, u_1, u_2, ..., u_l\} \subseteq \{v_0, v_1, v_2, ..., v_k\}$ and $\{a_1, a_2, a_3, ..., a_l\} \subseteq \{e_1, e_2, e_3, ..., e_k\}$ if $u_0a_1u_1a_2u_2a_3...a_lu_l$ is path and not cycle, where u_0 is origin and u_l is terminus, since each vertex is intersection of at least two common edges, then u_0 is not origin, since u_l is not terminus, since the sequence $u_0a_1u_1a_2u_2a_3...a_lu_l$ is finite and if $u_0 = u_l$ then $u_0a_1u_1a_2u_2a_3...a_lu_l$ is cycle, if $u_0 \neq u_l$ and since each vertex is intersection of at least two common edges, then there is at least one cycle include all the edges $e_1, e_2, e_3, ..., e_k$ or some of them.

Proposition 3.22. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are proper intersections of these k edges, with maximum degree of vertex equal n, suppose these k sets and W intersections form a walk

 $u_1a_1u_2a_2u_3a_3...u_{t-1}a_{t-1}u_t$ of proper vertices and common edges where $\{u_1, u_2, u_3, ..., u_t\} = \{v_1, v_2, v_3, ..., v_w\}$ and $\{a_1, a_2, a_3, ..., a_{t-1}\} = \{e_1, e_2, e_3, ..., e_k\}$ and if there is no cycle of $\{e_1, e_2, e_3, ..., e_k\}$ or subset of $\{e_1, e_2, e_3, ..., e_k\}$, if we partition all edges(common and non common) into *m* minimum colour classes, then m = n.

Proof: From definition of path any two edges of $\chi_1 = \{a_1, a_3, a_5, ..., a_{t-2}\}$ are disjoint, and same thing for $\chi_2 = \{a_2, a_4, a_6, ..., a_{t-1}\}$ when *t* is odd, or any two f $\chi_1 = \{a_1, a_3, a_5, ..., a_{t-1}\}$ are disjoint, and same thing for $\chi_2 = \{a_2, a_4, a_6, ..., a_t\}$ when *t* is even. Let the colour class χ_1 be labeled by 1 and letthe colour class χ_2 be labeled by 2. The remaining edges are non common edges and any one of them can belabeled by one of the sets $\{3,4,5,...,n\}$. Therefore m = n.

Proposition 3.23. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose these k edges and W vertices form two paths $u_1a_1u_2a_2u_3a_3...u_{t-1}a_{t-1}u_t$ and $q_1b_1q_2b_2q_3b_3...q_{s-1}b_{s-1}q_s$ of proper vertices and common edges where

$$\{u_1, u_2, u_3, ..., u_t\} \cup \{q_1, q_2, q_3, ..., q_s\} = \{v_1, v_2, v_3, ..., v_w\}$$
 and

 $\{a_1, a_2, a_3, ..., a_{t-1}\} \cup \{b_1, b_2, b_3, ..., b_{s=1}\} = \{e_1, e_2, e_3, ..., e_k\}$ and if there is no cycle of $\{e_1, e_2, e_3, ..., e_k\}$ or subset of $\{e_1, e_2, e_3, ..., e_k\}$, if we partition all edges(common and non common) into *m* minimum colour classes, then m = n. Proof: If there is no common edge common between the two paths, and if there is no proper vertex common between the two paths, using Proposition 3.22. we have m = n. If there is common edge common between the two paths, from definition of path, two colour classes of $\chi_1, \chi_2, \chi_3, \chi_4$ can form one colour class (there is a common edge common between the two colour classes), where $\chi_1 = \{a_1, a_3, a_5, ...\}$, $\chi_2 = \{a_2, a_4, a_6, ...\}$, $\chi_3 = \{b_1, b_3, b_5, ...\}$,

 $\chi_4 = \{b_2, b_4, b_6, \dots\}$ and $\chi_1 \cup \chi_2 \cup \chi_3 \cup \chi_4 = \{e_1, e_2, e_3, \dots, e_k\}$. Let the colour class χ_1 be labeled by 1, χ_2 be labeled

by 2, χ_3 be labeled by 3, and χ_4 be labeled by 4. Let us consider χ_1 and χ_4 form one colour class, then the colour class χ_4 be labeled by 1. Therefore maximum number of colur classes each consist of non common edges only, equal n-3, therefore of remaining non common edges can be labeled by one element of the set $\{4,5,6,...,n\}$. Therefore m=n. If there is common proper vertex between the four colour classes $\chi_1, \chi_2, \chi_3, \chi_4$ therefore maximum number of colur classes each consist of non common edges can be labeled by one element of the set $\{5,6,7,...,n\}$. Therefore m=n. Therefore remaining non common edges only, equal n-4, therefore remaining non common edges can be labeled by one element of the set $\{5,6,7,...,n\}$. Therefore m=n. Then the result holds.

Proposition 3.24.Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose these k edges and W vertices form three paths if we partition all edges (common and non common) into m minimum families of disjoint sets, then m = n.

Proposition 3.25. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose these k edges and w vertices form q paths if we partition all edges (common and non common) into m minimum families of disjoint sets, then m = n.

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Remark3.26. Proof of Proposition 3.24.and Proposition 3.25. is the same as proof of Proposition 3.22. and proof of Proposition 3.23.

Proposition 3.27. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose if we partition all these edges(common and non common edges) into m edges) into m edges, where m edges where m edges m edges

edges)into *m* colour classes where m = n + 1 it is not necessary $v_1, v_2, v_3, ..., v_w$ isstandard of partition.

Proof: Example 3.18. is enough to prove this proposition.

Remark .3.28. From definition of common edge and non common edge, multiple edge is common edge and loop is non common edge.

Remark .3.29. From definition of non common edge, and definition of loop, any vertex v have a loop (an edge with identical ends) with d(v) = n, then we consider d(v) equal define n-1.

IV. SOME RESULTS USING COMMON EDGES

In this section we used the concept of common edge between two proper vertices to introduce some results of edgecolouring.

Remark4.1.When we partition all edges (common edges and non common edges) into *m* colour classes, by words change and exchange we mean change an edge from acolour class to another and exchange two different edges between two different colour classes, such that any two adjacent edges belong to two different colour classes, and two non adjacent edges belong to two different colour classes.

Proposition 4.2. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose all edges (common edges and non common edges) partition into m colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, for any two colour classes χ_j and χ_j (i) if χ_i meets χ_j at these w vertices, then $w+1 \le t_i + t_j$ where $1 \le i < j \le m$, and $\chi_i \cup \chi_j$ can't form one colour class. (ii) if χ_i dose not meet χ_j at these w vertices, then $t_i + t_j \le w$ and $\chi_i \cup \chi_j$ can form a colour class.

Proof: (i) If these *w* vertices isstandard of partition of all edges (common edges and non common edges), the result holds using only definitionstandard of partition. Suppose these *w* vertices isstandard of partition, let χ_i meet χ_j at these *w* vertices, if all edges belong to $\chi_i \bigcup \chi_j$ arrange such that appear once at each vertex of $v_1, v_2, v_3, ..., v_w$ then there exist at least one vertex at which appear two edges of $\chi_i \bigcup \chi_j$, then we have then $w+1 \le t_i + t_j$ and $\chi_i \bigcup \chi_j$ can't form one colour class.(ii) If these *w* vertices isstandard of partition. Suppose these *w* vertices is not standard of partition, let χ_i does not meet χ_j at these *w* vertices, if all edges belong to $\chi_i \bigcup \chi_j$ arrange such that appear once at each vertex of $v_1, v_2, v_3, ..., v_w$ then there exist holds using only definitionstandard of partition. Suppose these *w* vertices is not standard of partition, let χ_i does not meet χ_j at these *w* vertices, if all edges belong to $\chi_i \bigcup \chi_j$ arrange such that appear once at each vertex of $v_1, v_2, v_3, ..., v_w$ then from definition of χ_i does not meet χ_j , there no vertex at which appear two edges of $\chi_i \bigcup \chi_j$, then we have then $t_i + t_j \le w$ and $\chi_i \bigcup \chi_j$ can form a colour class.

Proposition 4.3. Suppose we have common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose all edges (common edges and non common edges) partition into m colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, for any two colour classes χ_i meet χ_j at these w vertices, if t_i, t_j be number of times the colour classes χ_i, χ_j appear at these w vertices respectively, where these w vertices isstandard of partition of all edges (common edges and non common edges) into m colour classes, then we have (i) $w+1 \le t_i + t_j$ for w odd or even. (ii) $\frac{w+1}{2} \le t_i \le w$ (If there is special case existence of colour class χ_i such that $2 \le t_i \le \frac{w-1}{2}$ it will be only one colour class) for w odd, where $1 \le i < j \le m$. (iii) $\frac{w+2}{2} \le t_i \le w$ (If there is special case

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existence of colour class χ_i such that $2 \le t_i \le \frac{w}{2}$ it will be only one colour class) for w even, where $1 \le i < j \le m$.

(iv) If there exist two colour classes χ_i, χ_j such that $t_i \le t_j \le \frac{w-1}{2}$ for w odd or $t_i \le t_j \le \frac{w}{2}$ for w even, then χ_i and χ_i they will form only one colour class. Proof: (i) It holds from definition ofstandard ofpartition. (ii)Suppose $t_i = t_j < \frac{w+1}{2}$ for w odd, then $t_i = t_j \le \frac{w-1}{2}$ for w odd, therefore $t_i + t_j \le w-1$ for w odd, this contradict (i), therefore $\frac{w+1}{2} \le t_i$ for w odd, from Proposition 4.2. we have $t_i \le w$ for w odd or even, therefore $\frac{w+1}{2} \le t_i \le w$, for special case let $t_i = r$ where $2 \le r \le \frac{w-1}{2}$ and for two special case we have $t_i + t_j = r + r$ and $4 \le t_i + t_j \le w - 1$ a contradiction, if $\frac{w+1}{2} \le t_i \le w$ and $2 \le t_j \le \frac{w-1}{2}$ we can find $w+1 \le t_i + t_j$ such as $t_i + t_j \le 2 + (w-1) = w+1$ and $t_i + t_j = (\frac{w-1}{2}) + (\frac{w+3}{2}) = w+1$, therefore χ_i and χ_i form only one colour class. (iii) The proof is similar to proof of part (ii). (iv)Suppose we have only one t_i such that $t_i \le \frac{w-1}{2}$ for w odd or $t_i \le \frac{w}{2}$ for w even, therefore for all other families of disjoint sets χ_j there is t_j such that $\frac{W+3}{2} \le t_j \le w$ for w odd or $\frac{W}{2} \le t_j \le w$ for w even, we have $w+1 \le t_i + t_j \le \frac{3w-1}{2}$ for w odd, and $w+1 \le t_i + t_j \le \frac{3w}{2}$ for w even, therefore we have $w+1 \le t_i + t_j$ for wodd or even. Suppose we have t_i and t_j such that $t_i = t_j \le \frac{w-1}{2}$ for w odd or $t_i = t_j \le \frac{w}{2}$ for w even, therefore $t_i + t_j \le w - 1$ for w odd and $t_i + t_j \le w$ for w even, this contradict $w + 1 \le t_i + t_j$ therefore (iv) holds Proposition 4.4. Suppose we have common edges $e_1, e_2, e_3, \dots, e_k$ and $v_1, v_2, v_3, \dots, v_w$ are propervertices of these k edges, with maximum degree of vertex equal n, suppose these k edges and non common edges partitioned into mcolour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, labelled by 1,2,3,...,*m*, and if t_i be number of times the colour class χ_i appear at these *w* vertices, and for all $1 \le i \le m$, $1 \le l \le k$, $1 \le j \le w$ and k_1 is number of non common edges, then we have $\sum_{i=1}^{m} t_i = k_1 + \sum_{i=1}^{k} w_i = k_1 + \sum_{i=1}^{k} c_i = k_1 + 2k = \sum_{i=1}^{w} d(v_i).$ Proof: We labelled all edges belong one colour class by label of this colour class. Any two different colour classes have two different colour, we labeled colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$ by 1,2,3,..., *m* respectively, and two adjacent edges have two different labels, (two different colours). We labelled any vertex v_j $(1 \le j \le w)$ by number of labels equal its degree, and same labels of all edges intersect at v_i . Let e_i be common edgebetween c_i proper vertices (two proper vertices), since $d(v_j)$ equal number of edges intersect at vertex v_j , therefore vertex $k_1 + \sum_{i=1}^{n} c_i = k_1 + 2k = \sum_{i=1}^{n} d(v_j)$, using Proposition 3.8. then we have vertex $k_1 + \sum_{i=1}^{k} w_i = k_1 + \sum_{i=1}^{k} c_i = k_1 + 2k = \sum_{i=1}^{w} d(v_i)$. Since these edges (common

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and non common) partitioned into *m* colour classes, then we have $k_1 + \sum_{i=1}^{k} c_i = k_1 + 2k = \sum_{i=1}^{m} t_i$, therefore we have

vertex $\sum_{i=1}^{m} t_i = k_1 + \sum_{i=1}^{k} w_i = k_1 + \sum_{i=1}^{k} c_i = k_1 + 2k = \sum_{j=1}^{w} d(v_j).$

Remark: 4.5. As inProposition 4.4. for edge colouring $c_i = w_i$ always has fixed value $c_i = w_i = 2$. In case of vertex colouring fase colouring, and set colouring $c_i = w_i$ is variable, this explainedges colouring is not complicated compare with other types of colouring.

The following result show maximum or minimum number of colour classes sometimes is not depend directly on maximum degree of vertex equal n.

Proposition 4.6. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$ and there is no non common edge, these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal *n*, let vertices $v_1, v_2, v_3, ..., v_w$ be standard of partitionthese *k* common edges into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, if for all *i*, $1 \le i \le k$ we have $\frac{w+1}{2} \le w_i$ for wodd, or $m < k, \frac{w+2}{2} \le w_i$ for weven, where w_i appearance of the edge e_i at $v_1, v_2, v_3, ..., v_w$, then these *k* edges partition into *k* colour classes.

Proof: Suppose the edges $e_1, e_2, e_3, ..., e_k$ partition into M colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, and suppose $k \le m$, then there exists a colour class χ_i at least consists of two edges, therefore $w + 1 \le t_i$ for w odd, $w + 2 \le t_i$ for w even, which is a contradiction to Proposition 6.22. therefore $k \le m$, since k can't be less than M, therefore k = m. Proposition 4.7. Let G be a graph of k common edges $e_1, e_2, e_3, ..., e_k$, these k common edges intersect at w proper vertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal n, let vertices $v_1, v_2, v_3, ..., v_w$ be standard of partition edges(common and non common edges) into m colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, then (i)maximum value of m

equal
$$2n-1$$
. (ii) if $\sum_{j=1}^{w} d(v_j) = m(\frac{w+1}{2}) + r$ where $0 \le r \le \frac{w-1}{2}$, and wis odd, or if

 $\sum_{\substack{j=1\\\dots\dots\dots}}^{w} d(v_j) = (m-1)(\frac{w+2}{2}) + \frac{w}{2} + r \text{ where } 0 \le r \le \frac{w}{2}, \text{ and } w \text{ is even, then } m \text{ is maximum number families of } m = 0$

disjoint sets.

Proof: (i) Suppose *m* is maximum number of colour classes, let m = 2n, and *w* is odd, suppose minimum value of

$$t_i, 1 \le i \le m, \ t_1 = t_2 = t_3 = \dots = t_m = \frac{w+1}{2}, \ \text{since } \sum_{j=1}^w d(v_j) = \sum_{i=1}^m t_i \le nw \text{ then we have}$$

$$\sum_{j=1}^w d(v_j) = \sum_{i=1}^m t_i = 2n(\frac{w+1}{2}) \le nw \text{ it is a contradiction, therefore } m < 2n, \text{ and suppose } m = 2n-1, \text{ then we have}$$

$$\sum_{j=1}^w d(v_j) = \sum_{i=1}^m t_i = (2n-1)(\frac{w+1}{2}) \le nw \text{ thereforwith condition } 2n-w-1 \le 0, \text{ maximum value of } m \text{ equal}$$

$$2n-1. \text{ For } w \text{ even, let } m = 2n, \text{ suppose minimum value of } t_i \text{ as } t_1 = t_2 = t_3 = \dots = t_{m-1} = \frac{w+2}{2}, \ t_m = \frac{w}{2}, \text{ where}$$

$$1 \le i \le m \text{ since } \sum_{j=1}^w d(v_j) = \sum_{i=1}^m t_i = (2n-1)(\frac{w+2}{2}) + \frac{w}{2} \le nw \text{ it is a contradiction, therefore } m < 2n, \text{ suppose}$$

$$m = 2n-1, \text{ then we have } \sum_{j=1}^w d(v_j) = \sum_{i=1}^m t_i = (2n-2)(\frac{w+2}{2}) + \frac{w}{2} \le nw \text{ therefore with the condition}$$

$$2n-2 \le \frac{w}{2}, \text{ maximum value of } m \text{ equal } 2n-1.$$

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(ii) First suppose we partition these common edges and non common edges into number of colour classes equal m + 1, and let *w* be odd, such that there are $m - r_1$ colour classes each colour class appears $\frac{w+1}{2}$ times, and there are r_1 colour classes,

each colour class appear $\frac{w-1}{2}$ times, and there is one colour class appears $r + r_1$ times, where $1 \le r_1$, such that

$$1 \le r + r_1 \le \frac{w-1}{2}, \text{ therefore } \sum_{j=1}^w d(v_j) = m(\frac{w+1}{2}) + r = (m - r_1)(\frac{w+1}{2}) + r_1(\frac{w-1}{2}) + (r + r_1) \text{ and there exist colour } r_1 \le \frac{w-1}{2}$$

classes χ_i, χ_j such that $t_i + t_j \le w$, where $1 \le i < j \le m+1$, and t_i, t_j appearance of χ_i, χ_j at $X_1, X_2, X_3, ..., X_w$, respectively, hence *m* ismaximum numbercolour classes.

Second suppose we partition these k sets into number of colour classes equal m+1, and let w is even, such that there are $m-r_1$ colour classes each colour class appear $\frac{w}{2}+1$ times, and there are r_1 colour classes appear $\frac{w}{2}-1$ times, and there is one

colour class appears $r + r_1$ times, where $1 \le r_1$, such that $1 \le r + r_1 \le \frac{w}{2} - 1$, therefore

 $\sum_{j=1}^{w} d(v_j) = m(\frac{w}{2}+1) + r = (m-r_1)(\frac{w}{2}+1) + r_1(\frac{w}{2}-1) + (r+r_1) \text{ and}$ $\sum_{j=1}^{w} d(X_j) = m(\frac{w}{2}+1) + r = (m-r_1)(\frac{w}{2}+1) + r_1(\frac{w}{2}-1) + (r+r_1) \text{ and there exist two colour classes } \chi_i, \chi_j \text{ such that } t_i + t_j \le w,$

where $1 \le i < j \le m+1$, and t_i, t_j appearance of χ_i, χ_j at $X_1, X_2, X_3, ..., X_w$, respectively, hence *m* is maximum number colour classes. Therefore the result holds.

V. CONSTRUCTION MINIMUM NUMBER OF COLOUR CLASSES TO COLOUR EDGES

In this section we introduce a method to construct minimum number of colour classes. Here we give some examples to explain this method, and introduce some results related to this method.

Definition 5.1. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$, these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal *n*, let vertices $v_1, v_2, v_3, ..., v_w$ be standard of partition edges(common and non common edges) into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, using the following method: In firs step we try partition edges intomaximum number of colour classes, such that eachcolour class has appearance equal *w*, (we partition till there is no colour class its appearance equal *W*), supposemaximum number of colour classes each has appearance *W* is q_0 . In second step we try partition remaining edges intomaximum number of families of disjoint sets, such that eachcolour class has appearance equal w - 1, supposemaximum number of colour classes each has appearance w - 1 is q_1 . In third step we try partition edges intomaximum number of colour classes, such that eachcolour classes, such that eachcolour classes, such that eachcolour classes, such that eachcolour classes each has appearance w - 1 is q_1 . In third step we try partition edges intomaximum number of colour classes, such that eachcolour classes, such that eachcolour classes, such that w - 2, supposemaximum number of colour classes each has appearance w - 1 is q_2 . We continue in this process, till in last stepsuppose maximum number of colour classes each has appearance w - 2 is q_2 . We continue in this process, till in last stepsuppose maximum number of colour classes each has appearance w - 8 is q_s , and we have define $\sum_{i=0}^{s} (w - i)q_i = wq_0 + (w - 1)q_1 + (w - 2)q_2 + ... + (w - s)q_s = \sum_{j=1}^{w} d(v_j)$ where

 $0 \le q_i, 2 \le w - i \le w$, this method is called construction of minimum number of colour classes.

Remark 5.2. From Definition 5.1. we have all $q_0 + q_1 + q_2 + q_3 + \ldots + q_s = m$.

Remark 5.3. Generally when we partition k common edges $e_1, e_2, e_3, ..., e_k$ into m colour classes, we have four cases: (1) These m colour classes, are maximum number of colour classes. (2) These m colour classes, are minimumnumber of colour classes. (3) These m colour classes, are neither satisfy maximum number of colour classes, norsatisfy minimum number of colour classes. (4) These m colour classes, satisfying both maximum number of colour classes, and minimum number of colour classes. Proposition 5.4. Suppose we have k common edges $e_1, e_2, e_3, ..., e_k$ and $v_1, v_2, v_3, ..., v_w$ be all

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proper vertices of these k sets(standard of partition), with maximum degree of vertex equal n, where 2 < n, and there is a vertex of degree two, all edges (common edges and non common edges) partitioned into m colour classes, then the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ not more than two.

Proof: Suppose thenumber of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ equal three or more than three, let χ_1, χ_2, χ_3 be three colour classes each appear w times at $v_1, v_2, v_3, ..., v_w$ from definition of colour class, each one of χ_1, χ_2, χ_3 appear at all of intersections $v_1, v_2, v_3, ..., v_w$ then there is no intersection of degree less than three, a contradiction since there is one intersection out of degree two, it is also a contradiction for more than three colour classes each appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ therefore the number of colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ the set $v_1, v_2, v_3, ..., v_$

 $v_1, v_2, v_3, \dots, v_w$ not more than two.

We generalized Proposition 5.4. as follows:

Proposition 5.5. Suppose we have k common edges $e_1, e_2, e_3, \dots, e_k$ and $v_1, v_2, v_3, \dots, v_w$ be all proper vertices of these

k edges (standard of partition), with maximum degree of vertex equal *n*, andminimum degree of intersection equal *q*, where q < n, all edges (common edges and non common edges) partitioned into *m* colour classes, then the number of colour classes appear *w* times at $v_1, v_2, v_3, ..., v_w$ not more than *q*.

Proof of Proposition 5.5. is the same as in Proposition 5.4.

Proposition 5.5. Suppose we have k common sets $A_1, A_2, A_3, ..., A_k$, and $X_1, X_2, X_3, ..., X_w$ be all proper intersections of these k sets(standard of partition), with maximum degree of intersection equal n, and minimum degree of intersection equal q, where q < n, all sets (common sets and non common sets) partitioned into m families of disjoint sets, then the number of families of disjoint sets appear w times at $X_1, X_2, X_3, ..., X_w$ not more than q.

Proof of Proposition 5.5. is the same as in Proposition 5.4.

Proposition: 5.6. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$, these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal *n*, let vertices $v_1, v_2, v_3, ..., v_w$ be standard of partition edges(common and non common edges) into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, using method introduced by Definition 5.1. then *m* is minimum number of colour classes.

Proof: Step I: Let
$$\sum_{j=1}^{w} d(v_j) = wq_0 + r$$
 where $0 \le r \le w-1$, then we have $\sum_{j=1}^{w} d(v_j) = wm \le wn$, or

 $\sum_{j=1}^{w} d(v_j) = w(m-1) + r \le wn$, then $m \le n$, since m < n is not true, therefore if we partition edges(common and non

common edges) instate of *m* colour classes into m-1 colour classes m=n and *m* is minimum number of colour classes.

Step II: Let
$$\sum_{j=1}^{w} d(v_j) = wq_0 + (w-1)q_1 + r$$
 where $0 \le r \le w-2$, then we have $q_0 + q_1 + 1 = m$ if $r \ne 0$ and

 $q_0 + q_1 = m$ if r = 0, and suppose partition these k sets into m - 1 colour classes, such that $q_0 + q_1 + 1 = m - 1$ if $r \neq 0$ and $q_0 + q_1 = m - 1$ if r = 0, then either there is $q_0 + x$ colour classes appear w times at $v_1, v_2, v_3, ..., v_w$ or there is $q_1 + x$ colour classes appear w - 1 times at $v_1, v_2, v_3, ..., v_w$ where x is an integer $1 \le x$ which is a contradiction, because there is at least one family of disjoint sets χ_i such that $w + 1 \le t_i$, therefore m is minimum number of colour classes.

Step III: Let
$$\sum_{j=1}^{w} d(v_j) = wq_0 + (w-1)q_1 + (w-2)q_2 + r$$
 where $0 \le r \le w-3$, then we have

$$\sum_{j=1}^{w} d(v_j) = wq_0 + (w-1)q_1 + (w-2)q_2 + r \le wn \text{ and } q_0 + q_1 + q_2 + 1 = m \text{ if } r \ne 0 \text{ and } q_0 + q_1 + q_2 = m \text{ if } r = 0,$$

and suppose partition these k sets into m-1 colour classes, such that

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r = 0, and suppose partition these k sets into m - 1 colour classes, such that $q_0 + q_1 + q_2 + 1 = m - 1$ if $r \neq 0$ and $q_0 + q_1 + q_2 = m - 1$ if r = 0, then either there is $q_0 + x$ families of disjoint sets appear w times at $v_1, v_2, v_3, ..., v_w$ or there is $q_1 + x$ colour classes appear w - 1 times orthere is $q_2 + x$ colour classes appear w - 2 times at $v_1, v_2, v_3, ..., v_w$ where x is an integer $1 \le x$ which is a contradiction, because there is at least onecolour class χ_i such that $w + 1 \le t_i$, therefore m is minimum number of colour classes. For the remaining stepswhere inlast step we have

 $\sum_{j=1}^{n} d(X_{i}) = wq_{0} + (w-1)q_{1} + (w-2)q_{2} + (w-3)q_{3} + \dots + (w-s)q_{s} + r$, in each following step, the proof is same as in step III. Therefore the result holds.

Proposition 5.7. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$, these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal *n*, let vertices $v_1, v_2, v_3, ..., v_w$ be standard of partition edges(common and non common edges) into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, any colour class appears not more than w-1 times at $v_1, v_2, v_3, ..., v_w$ and if $n(w-1) + r = \sum_{j=1}^w d(v_j)$ where $2 \le r \le w-2$, then $n+1 \le m$.

Proof: Letthese edges (common edges and non common edges) partitioned into number of colour classes equal m, if we consider appearance for any colour class at w vertices $v_1, v_2, v_3, ..., v_w$ equal w-1, and since $n(w-1) + r = \sum_{j=1}^{w} d(v_j)$ therefore m = n+1 and any two colour classes meet each otherat $v_1, v_2, v_3, ..., v_w$ and if we consider maximum appearance for any colour class less than w-1 and equal w-t, where $\frac{w+1}{2} \le w-t < w-1$ for w be odd, and

$$\frac{w}{2} + 1 \le w - t < w - 1 \text{ for } w \text{ be even, and since } n(w - 1) + r = \sum_{j=1}^{w} d(v_j) \text{ we have}$$

 $n(w-t) + n(t-1) + r = n(w-1) + r = \sum_{j=1}^{w} d(v_j)$ since maximum appearance for any colour class equal w-ttherefore m = n+1 if $1 \le n(t-1) + r \le w-t$, and $n+1 \le m$ if $(w-t)+1 \le n(t-1)+r$. Therefore $n+1 \le m$. Proposition 5.8 Let *G* be a graph of *k* common edges $e_1, e_2, e_3, \dots, e_k$, these *k* common edges intersect at *w* proper vertices $v_1, v_2, v_3, \dots, v_w$, with maximum degree of vertex equal *n*, let vertices $v_1, v_2, v_3, \dots, v_w$ be standard of partition

edges(common and non common edges) into *m* colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$, such that *m* isminimum numbercolour classes. Let q_0 be number of colour classes appear *w* times at $v_1, v_2, v_3, ..., v_w$ where $0 \le q_0 \le 2$ and $n-q_0$ be maximum number of colour classes appear w-1 times at $v_1, v_2, v_3, ..., v_w$ if $wn - n + q_0 + r = \sum_{j=1}^w d(V_j)$ where

 $2 \le r < w - 1$, then n + 1 = m.

Proof: Letthese k setspartitioned into number of colour classes equal m, using Proposition 3.19. we can writ

 $wq_0 + (w-1)(n-q_0) + r = \sum_{i=1}^k c_i = \sum_{j=1}^w d(v_j)$ where c_i number of times the edge e_i appear at w proper vertices, since $wq_0 + (w-1)(n-q_0) + r = wn - n + q_0 + r$ then for minimum number of colour classes we have to take maximumappearance, suppose we have q_0 colour classes each one appear W times, and $n - q_0$ colour classes each one appear w-1 times, and one colour class appear r times, since $q_0 + (n-q_0) + 1 = n + 1$ then n+1=m. Using method of Definition 5.1. m = n + 1 is minimum number of colour classes. To prove \mathcal{M} is minimum number of colour classes, suppose we partition these k sets into m-1 colour classes, suppose we distribute colour class appear r times at $v_1, v_2, v_3, ..., v_w$ to colour classes each one appear w times, it is a contradiction, since there is no family of disjoint sets appear more than w times. Suppose we distribute colour class appear r times to colour class each one appear w-1



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times, it is a contradiction, sincenumber of colour classes each one appear w times not more than q_0 , and there is no colour class appear more than w times. Therefore \mathcal{M} is minimum number of colour classes.

VI. ADDITION OF COMMON EDGES AND NON COMMON EDGES

In this section we suppose G be a graph of k common edges $e_1, e_2, e_3, \dots, e_k$, these k common edges intersect at w proper vertices $v_1, v_2, v_3, \dots, v_w$, with maximum degree of vertex equal n, these vertices $v_1, v_2, v_3, \dots, v_w$ to arestandard of partition, all edges (common edges and non common edges) partitioned into m colour classes. Here we introduce some results for addition of common edges and non common edges, such that the proper vertices of these k edges (standard of partition) remains unchanged.

Proposition 6.1. Let G be a graph of k common edges $e_1, e_2, e_3, ..., e_k$, these k common edges intersect atvertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal n, if we add only one non common sets, without changing maximum degree of vertices, and without addition of any proper vertex, only degree of oneproper vertex changes, then these k common edges and non common edges can be partitioned into m minimum colour classes.

Proof: : Let these *m* minimum colour classeslabeled by 1,2,3,...,*m*, sincestandard of partition these common edges and non common edges after additional of a non common edgeremains $v_1, v_2, v_3, ..., v_w$, since additional of noncommon edge appears once at aproper vertex v_j where $1 \le j \le w$, suppose $d(v_j) = x$, where $2 \le x \le n-1$, if $v_j \equiv (1,2,3,...,x)$ then after addition of the noncommon edge we labelled v_j by $v_j \equiv (1,2,3,...,x,y)$ where $x+1 \le y \le m$, and additional noncommon edge belongs to colour class χ_y and $x+1 \le y \le m$, since before we add the non common edge $d(v_j) \le n-1$ the additional noncommon edge can belongs to one of the following colour classes

 $\chi_{x+1}, \chi_{x+2}, \chi_{x+3}, ..., \chi_m$. then these common edges and non common edges partitioned into *m* colour classes.

Proposition 6.2. Let G be a graph of k common edges $e_1, e_2, e_3, ..., e_k$, these k common edges intersect atvertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal n, these edges(common and non common edges) can be partitioned into m colour classes, if we remove only one non common edge, without changing maximum degree of vertex, and without removal of any proper vertex of $v_1, v_2, v_3, ..., v_w$, then after this removal, common edges and non common edges partitioned into m minimum colour classes.

Proof: Let these *m* colour classeslabeled by 1,2,3,...,m, suppose after removal a non common edge these edges(common and non common edges) can not partition into *m* colour classes, and partitioned into m-1 colour classes, if we add the removal edge, we return to the case before removal, these edges(common and non common edges) can partition into *m* colour classes, which contradictProposition 6.1. then thesecommon edges and non common edgespartitioned into *m* colour classes.

Proposition 6.3. Let G be a graph of k common edges $e_1, e_2, e_3, ..., e_k$, these k common edges intersect atvertices $v_1, v_2, v_3, ..., v_w$, (standard of partition for these edges, common and non common), with maximum degree of vertex equal n, if all edges partitioned into m minimum colour classes, then addition of q non common edges, without changing maximum degree of vertex, and without addition of any proper vertex, such that the standard of partition after addition for common edges and non common edges remains $v_1, v_2, v_3, ..., v_w$, then all edges partitioned into m minimum colour classes.

Proof: : Let these *m* minimum colour classeslabeled by 1,2,3,...,*m*, sincestandard of partition afteraddition of *q* non common edgesremains $v_1, v_2, v_3, ..., v_w$, we have three cases: case one any noncommon edge appears once at oneproper vertex then any additional noncommon edge appears at one of the vertices $v_{j+1}, v_{j+2}, v_{j+3}, ..., v_{j+q}$ before addition of a non common edge we have $v_{j+l} \equiv (1,2,3,...,x)$ and after addition of a non common edge we have $v_{j+l} \equiv (1,2,3,...,x)$ and after addition of a non common edge and $d(v_{j+l}) = x + 1$ after addition of a non common edge, where for all l, $1 \le l \le q$, $1 \le j \le w$, $2 \le x \le n-1$, thenanyadditional noncommon edgelabeled by one element of the set $\{x+1, x+2, x+3,...,m\}$. We have two cases

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for addition q non common edges: Case one we add q noncommon edges appear at v_i such that any two noncommonedges out of these q noncommon edges have two differentlabels, where $1 \le q \le y$, $1 \le y \le n-2$, $3 \le x + y \le n$, the symbol v_i denote the vertex v_i before addition of q noncommon edges. The symbol v_i denote the vertex v_i after addition of q noncommon edges, we can abeled v_i and v_i as $v_i \equiv (1, 2, 3, ..., x)$, and $v_i = (1, 2, 3, ..., x, x + 1, x + 2, x + 3, ..., x + q)$, where $x + q \le n$, since $n \le m$ then all edges partitioned into m colour classes. Case three: At a proper vertex one out of these q non common edges appears once and at another proper vertex two or more than two out of these q non common edges appears at this proper vertex. To proof used in case one and case two. The result holds.

Proposition 6.4. Let G be a graph of k common edges $e_1, e_2, e_3, \dots, e_k$, these k common edges intersect atvertices $v_1, v_2, v_3, \dots, v_w$, (standard of partition for these edges, common and non common), with maximum degree of vertex equal n, if all edges partitioned into m minimum colour classes, and there is at least oneproper vertex represent intersection only of common edges with degree of intersection equal n, if all non common edges in G equal q, such that the standard of partition after removal of these q non common edges remains $v_1, v_2, v_3, \dots, v_w$, if all edges partitioned into m minimum colour classes, then after removal of these q non common edges, all edges (k common edges) partitioned into *m* minimum colour classes.

Proof: Step I: Here conditions of Proposition 6.2. satisfied, suppose we remove one non common edge,

usingProposition 6.2, these k edges partitioned into m minimum colour classes.

Step II: Conditions of Proposition 6.2. satisfied, suppose we remove the second non common edge, usingProposition 6.2, these k sets partitioned into m minimum colour classes. Continue in this process till last step (Step q) Step q: Conditions of Proposition 6.2. satisfied, suppose we remove the last non common edge, using Proposition 6.2, these k edge partitioned into m minimum colour classes.

VII. THE MAIN RESULT

Remark 7.1. In this paper some times we saystandard of partition edges in G, instate of standard of partition for these edges, common and non common, because both have the same meaning.

Proposition 7.2 Let $A_1, A_2, A_3, \dots, A_k$ be k common

There is a mistake in statement of Proposition 2.10. as it appear in [9]. The following proposition is modification of Proposition 2.10.

Proposition 7.2. Let $A_1, A_2, A_3, \dots, A_k$ be k common sets, and $X_1, X_2, X_3, \dots, X_k$ be all proper intersections of these k sets, with maximum degree of intersection equal n, if we try to partition all sets (k common sets and non common sets) into *m* minimum families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ will not be not standard of partition. If $X_1, X_2, X_3, \dots, X_t$ with maximum degree of intersection equal n, and be standard partition $A_1, A_2, A_3, \dots, A_r$ and non common sets at $X_1, X_2, X_3, \dots, X_t$, into *m* minimum families of disjoint sets($n+1 \le m$), where

 $\{A_1, A_2, A_3, ..., A_r\} \subset \{A_1, A_2, A_3, ..., A_k\}$, and $\{X_1, X_2, X_3, ..., X_r\} \subset \{X_1, X_2, X_3, ..., X_w\}$, then all sets (common sets and non common sets) be partitioned into m minimum families of disjoint sets where $n+1 \le m$. Proposition 7.3. Let G be a graphwith maximum degree of vertex equal n, if we try to partition all edges in G into m minimum colour classes, $v_1, v_2, v_3, \dots, v_w$ will not standard of partition. If $v_1, v_2, v_3, \dots, v_t$ is proper vertices of common edges $e_1, e_2, e_3, \dots, e_r$ and $v_1, v_2, v_3, \dots, v_t$ is standard f partition these t common edges and noncommon edges at $v_1, v_2, v_3, ..., v_t$ into *m* minimum colour classes $(n+1 \le m)$ where $\{e_1, e_2, e_3, ..., e_r\} \subset \{e_1, e_2, e_3, ..., e_k\}$, $\{v_1, v_2, v_3, ..., v_t\} \subset \{v_1, v_2, v_3, ..., v_w\}, t < w \text{ and } r < k, \text{ then alledges in } G \text{ partitioned into } m \text{ minimum colour classes}$ equal $n+1 \le m$.

Proof: Since $v_1, v_2, v_3, ..., v_t$ be standard partition $e_1, e_2, e_3, ..., e_r$ into m

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minimum colour classes $\chi_1, \chi_2, \chi_3, ..., \chi_m$ such that

 $\chi_{1} = \{e_{11}, e_{12}, e_{13}, \dots, e_{1r_{1}}\}$ $\chi_{2} = \{e_{21}, e_{22}, e_{23}, \dots, e_{2r_{2}}\}$ $\chi_{3} = \{e_{31}, e_{32}, e_{33}, \dots, e_{3r_{3}}\} \text{ and so on till we define}$ $\chi_{m} = \{e_{m1}, e_{m2}, e_{m3}, \dots, e_{mr_{m}}\}$

From definition of standard of partition we have $t + 1 \le r_i + r_j$, where $1 \le i \le m - 1$, $2 \le j \le m$, if

$$r_{1} + r_{2} + r_{3} + \dots + r_{m} = r, \ 1 \le i < j \le m, \ 1 \le l \le m.$$

Since $\{e_{1}, e_{2}, e_{3}, \dots, e_{r}\} \subset \{e_{1}, e_{2}, e_{3}, \dots, e_{k}\}$ we can write
 $\chi_{1} = \{e_{11}, e_{12}, e_{13}, \dots, e_{1r_{1}}\} \subseteq \{e_{11}, e_{12}, e_{13}, \dots, e_{1k_{1}}\}$ if
 $\chi_{2} = \{e_{21}, e_{22}e_{23}, \dots, e_{2r_{2}}\} \subseteq \{e_{21}, e_{22}, e_{23}, \dots, e_{2k_{2}}\}$
 $\chi_{3} = \{e_{31}, e_{32}, e_{33}, \dots, e_{3r_{3}}\} \subseteq \} \{e_{31}, e_{32}, e_{33}, \dots, e_{3k_{3}}\}$ and so on till we can write
 $\chi_{m} = \{e_{m1}, e_{m2}, e_{m3}, \dots, e_{mr_{m}}\} \subseteq \{e_{m1}, e_{m2}, e_{m3}, \dots, e_{mk_{m}}\}.$

Suppose alledges in G partitioned into minimum colour classes equal m-1, since $v_1, v_2, v_3, ..., v_w$ is not standard of partition for all edges in G, then there are two colour classes χ_i and χ_j can form one colour class, then

 $w+1 \le k_i + k_j$ not holds for some i, j where $1 \le i < j \le m$, if i = 1 and j = 2, then from

 $\chi_1 = \{e_{11}, e_{12}, e_{13}, \dots, e_{1r_1}\} \subseteq \{e_{11}, e_{12}, e_{13}, \dots, e_{1k_1}\}$ and $\chi_2 = \{e_{21}, e_{22}e_{23}, \dots, e_{2r_2}\} \subseteq \{e_{21}, e_{22}, e_{23}, \dots, e_{2k_2}\}$ we have $k_1 + k_2 \leq w$ this means $r_1 + r_2 \leq t$, it is contradict Since $v_1, v_2, v_3, \dots, v_t$ be standardof partition $e_1, e_2, e_3, \dots, e_r$. Then since $k_1 + k_2 \leq w$, and since χ_1 and χ_2 can not form one colour class, therefore alledges in G partitioned into m minimum colour classes equal $n + 1 \leq m$.

Remark 7.4. Let *G* be a graph all its common edges intersect atvertices $v_1, v_2, v_3, ..., v_w$, if all edges partitioned into *m* minimum colour classes equal n + 1, if $v_1, v_2, v_3, ..., v_w$ is standard of partition all edges into *m* minimum colour classes, to show χ_i and χ_j are two different where $1 \le i < j \le m$, it is enough to show $w + 1 \le t_i + t_j$. If $v_1, v_2, v_3, ..., v_w$ is not standard of partition all edges of *G* into *m* minimum colour classes, to show χ_i and χ_j are two different colour classes at least one pair of edges e_i, e_j adjacent.

Proposition 7.5. Let *G* be a graph of *k* common edges $e_1, e_2, e_3, ..., e_k$, these *k* common edges intersect atvertices $v_1, v_2, v_3, ..., v_w$, if all edges partitioned into *m* minimum colour classes equal n + 1, then vertices $v_1, v_2, v_3, ..., v_w$ is standard of partition all edges into *m* minimum colour classes or there are some vertices of $v_1, v_2, v_3, ..., v_w$ is standard of partition some edges of *G* into *m* minimum colour classes.

Proof: Suppose all edges in *G* partitioned into *m* minimum colour classes or for any two different colour classes χ_i and χ_j where $1 \le i < j \le m$ there exists at least one pair of edges e_i, e_j adjacent, such that $e_i \in \chi_i$ and $e_j \in \chi_j$. If for any twocolour classes χ_i and χ_j we have $w+1 \le t_i + t_j$ then $v_1, v_2, v_3, ..., v_w$ is standard of partition. Suppose $v_1, v_2, v_3, ..., v_w$ is not standard of partition, and let $e_1, e_2, e_3, ..., e_r$, where $\{u_1, u_2, u_3, ..., u_t\} \subset \{v_1, v_2, v_3, ..., v_w\}$ then since any twocolour classes χ_i and χ_j then we have $w+1 \le t_i + t_j$ where $3 \le t \le w-1$, then $u_1, u_2, u_3, ..., u_t$ be standard of partition. Let $u_1, u_2, u_3, ..., u_t$ be not standard of partition where $3 \le t \le w-1$, and $v_1, v_2, v_3, ..., v_w$ is standard of partition. Suppose the not standard of partition where $3 \le t \le w-1$, and $v_1, v_2, v_3, ..., v_w$ is standard of partition. Let $u_1, u_2, u_3, ..., u_t$ be not standard of partition if $1 \le t \le 2$. Therefore the result holds.

In this paperProposition 7.6. is the main result.

Proposition 7.6. Let G be a graph, and G^* is the graph G after removal of all non common edges. G^* is the graph of of k common edges $e_1, e_2, e_3, ..., e_k$, these k common edges intersect atvertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of intersection equal n, and if all edges partitioned into m minimum colour classes, then $n+1 \le m$ if and only if w

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isodd and $nw - n + 2 \le \sum_{j=1}^{w} d(v_j)$, or there exists definestandardof partition $u_1, u_2, u_3, \dots, u_t$ with maximum

degree of intersection equal *n*, and $\{u_1, u_2, u_3, ..., u_t\} \subset \{v_1, v_2, v_3, ..., v_w\}$, *t* is odd and $nt - n + 2 \le \sum_{j=1}^t d(u_j)$.

Proof: Let *G* be a graph all of its edgespartitioned into *m* minimum colour classes equal n+1, using Proposition 7.5.then vertices $v_1, v_2, v_3, ..., v_w$ is standard of partition all edges into *m* minimum colour classes or there are somevertices of $v_1, v_2, v_3, ..., v_w$ is standard of partition some edges of *G* into *m* minimum colour classes. Since *w* isodd, then for any colour class we have $2 \le t_i \le w-1$, where $1 \le i \le m$, since m = n+1 by using method of construction of minimum number of colour classes, then we have and $n(w-1)+2 \le \sum_{j=1}^{w} d(v_j) \le (n+1)(w-1)$. Since *t* isodd, then for any colour class we have $2 \le t_i \le w-1$, where $1 \le i \le m$, since $n+1 \le m$ by using method of construction of minimum number of colour classes, then we have and $n(w-1)+2 \le \sum_{j=1}^{w} d(v_j) \le (n+1)(w-1)$.

Therefore first part is holds.

Conversely: We want to show $n+1 \le m$. From w oddwe have, $2 \le t_i \le w-1$, and

 $n(w-1)+2 = nw-n+2 \le \sum_{j=1}^{w} d(v_j)$, using method of construction of minimum number of colour classes, then we

have odd $n+1 \le m$. From t oddwe have, $2 \le t_i \le w-1$, and $n(t-1)+2 = nt-n+2 \le \sum_{j=1}^{t} d(v_j)$, using method of construction of minimum number of colour classes, then we have $n+1 \le m$.

Proposition 5.9 Let G be a graph of k common edges $e_1, e_2, e_3, ..., e_k$, these k common edges intersect at w proper vertices $v_1, v_2, v_3, ..., v_w$, with maximum degree of vertex equal n, let all edgespartitioned into minimum number of colour classes equal m, then either m = n or m = n+1.

Proof: Since maximum degree of vertex equal n, we have $n \le m$, using construction of minimum number of colour classes method, and since any edge appears once or twice at $v_1, v_2, v_3, ..., v_w$, then evert colour class appears w or

w-1 at $v_1, v_2, v_3, ..., v_w$, for n+1 colour classes we have $n(w-1)+2 \le \sum_{j=1}^{w} d(v_j) \le (n+1)(w-1) \le n_w$ where

 $w \le n+1$ and generally we have $n(w-1)+2 \le \sum_{j=1}^{w} d(v_j) \le n(w-1)+r \le nw$ where $2 \le r \le n$. For $n+2 \le m$ and

for minimum value of
$$\sum_{j=1}^{w} d(v_j)$$
 we have $\sum_{j=1}^{w} d(v_j) = (n+1)(w-1) + 2 \le wn$ which is a contradiction since

 $w+1 \le n$ is not true for all w and n. Generally we have $\sum_{j=1}^{w} d(v_j) = (n+1)(w-1) + r \le wn$ where $2 \le r \le w-1$, then $w+r-1 \le n$ which is a contradiction for all w, n, r. Therefore minimum number of colour classes m either m=n or m=n+1.

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